Integration 13C

1 a $\frac{dy}{dx} = 3x^2 + 2x$

$\Rightarrow y = \frac{3}{3}x^3 + \frac{2}{2}x^2 + c$

So $y = x^3 + x^2 + c$

$x = 2,\ y = 10 \Rightarrow 10 = 8 + 4 + c$

So $c = -2$

So the equation is $y = x^3 + x^2 - 2$

b $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$

$\Rightarrow y = \frac{4}{4}x^4 + \frac{2}{-2}x^{-2} + 3x + c$

So $y = x^4 - x^{-2} + 3x + c$

$x = 1,\ y = 4 \Rightarrow 4 = 1 - 1 + 3 + c$

So $c = 1$

So the equation is $y = x^4 - x^{-2} + 3x + 1$

or $y = x^4 - \frac{1}{x^2} + 3x + 1$

c $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$

$\Rightarrow y = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{4}\frac{x^3}{3} + c$

So $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + c$

$x = 4,\ y = 11 \Rightarrow 11 = \frac{2}{3} \times 2^3 + \frac{1}{12} \times 4^3 + c$

So $c = \frac{16}{3} - \frac{16}{3} = 0$

So the equation is $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3$

d $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$

$\Rightarrow y = 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2 + c$

So $y = 6\sqrt{x} - \frac{1}{2}x^2 + c$

$x = 4,\ y = 0 \Rightarrow 0 = 6 \times 2 - \frac{1}{2} \times 16 + c$

So $c = -4$

So the equation is $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$

e $\frac{dy}{dx} = (x + 2)^2$

$= x^2 + 4x + 4$

$\Rightarrow y = \frac{1}{3}x^3 + 2x^2 + 4x + c$

$x = 1,\ y = 7 \Rightarrow 7 = \frac{1}{3} + 2 + 4 + c$

So $c = \frac{2}{3}$

So the equation is $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$

f $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$\Rightarrow y = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

So $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + c$

$x = 0,\ y = 1 \Rightarrow 1 = \frac{2}{5} \times 0 + 6 \times 0 + c$

So $c = 1$

So the equation is $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$

2 $f'(x) = 2x^3 - \frac{1}{x^2}$

$= 2x^3 - x^{-2}$

So $f(x) = \frac{2}{4}x^4 - \frac{x^{-1}}{-1} + c = \frac{1}{2}x^4 + \frac{1}{x} + c$

But $f(1) = 2$

So $2 = \frac{1}{2} + 1 + c$

$\Rightarrow c = \frac{1}{2}$

So $f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$

3 $\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2}$

$= x^{-\frac{3}{2}} + 3x^{-2}$

$\Rightarrow y = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 3\frac{x^{-1}}{-1} + c$

So $y = -2x^{-\frac{1}{2}} - 3x^{-1} + c$

$= -\frac{2}{\sqrt{x}} - \frac{3}{x} + c$

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3 $x = 9,\ y = 0 \Rightarrow 0 = -\frac{2}{3} - \frac{3}{9} + c$

So $c = \frac{2}{3} + \frac{1}{3} = 1$

So the equation is $y = 1 - \dfrac{2}{\sqrt{x}} - \dfrac{3}{x}$

4 $y = \int (9x^2 + 4x - 3)\mathrm{d}x$

$= \dfrac{9x^3}{3} + \dfrac{4x^2}{2} - 3x + c$

$= 3x^3 + 2x^2 - 3x + c$

When $x = -1$ and $y = 0$,

$0 = 3(-1)^3 + 2(-1)^2 - 3(-1) + c$

$-3 + 2 + 3 + c = 0$

$c = -2$

$\mathrm{f}(x) = 3x^3 + 2x^2 - 3x - 2$

5 $y = \int (3x^{-\frac{1}{2}} - 2x\sqrt{x})\mathrm{d}x$

$= \int (3x^{-\frac{1}{2}} - 2x^{\frac{3}{2}})\mathrm{d}x$

$= \dfrac{3x^{\frac{1}{2}}}{\frac{1}{2}} - \dfrac{2x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= 6x^{\frac{1}{2}} - \frac{4}{5}x^{\frac{5}{2}} + c$

When $x = 4$ and $y = 10$,

$10 = 6(4)^{\frac{1}{2}} - \frac{4}{5}(4)^{\frac{5}{2}} + c$

$12 - \frac{128}{5} + c = 10$

$c = \frac{118}{5}$

$y = 6x^{\frac{1}{2}} - \frac{4}{5}x^{\frac{5}{2}} + \frac{118}{5}$

6 a $\dfrac{6x + 5x^{\frac{3}{2}}}{\sqrt{x}} = \dfrac{6x + 5x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$

$= x^{-\frac{1}{2}}(6x + 5x^{\frac{3}{2}})$

$= 6x^{\frac{1}{2}} + 5x$

$p = \frac{1}{2}$ and $q = 1$

b $y = \int (6x^{\frac{1}{2}} + 5x)\mathrm{d}x$

$= \dfrac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{5x^2}{2} + c$

$= 4x^{\frac{3}{2}} + \dfrac{5x^2}{2} + c$

6 b When $x = 9$ and $y = 100$,

$100 = 4(9)^{\frac{3}{2}} + \dfrac{5(9)^2}{2} + c$

$108 + \frac{405}{2} + c = 100$

$c = -\frac{421}{2}$

$y = 4x^{\frac{3}{2}} + \frac{5}{2}x^2 - \frac{421}{2}$

7 a $\mathrm{f}(t) = \int (10 - 5t)\mathrm{d}t$

$= 10t - \dfrac{5t^2}{2} + c$

When $x = 0$ and $y = 0$,

$\mathrm{f}(0) = 10(0) - \dfrac{5(0)^2}{2} + c = 0$

$c = 0$

$\mathrm{f}(t) = 10t - \frac{5}{2}t^2$

b $\mathrm{f}(3) = 10(3) - \dfrac{5(3)^2}{2}$

$= 7\frac{1}{2}$

8 a $\mathrm{f}(t) = \int (-9.8t)\mathrm{d}t$

$= -\dfrac{9.8t^2}{2} + c$

$= -4.9t^2 + c$

When $x = 0$ and $y = 35$,

$\mathrm{f}(0) = -4.9(0)^2 + c = 35$

$c = 35$

$\mathrm{f}(t) = -4.9t^2 + 35$

b $\mathrm{f}(1.5) = -4.9(1.5)^2 + 35 = 23.975$

The height of the arrow is 23.975 m.

c $\mathrm{f}(0) = 35$

The height of the castle is 35 m.

d The arrow will hit the ground when the height is 0.

$-4.9t^2 + 35 = 0$

$t = \sqrt{\dfrac{-35}{-4.9}} = 2.67$ or -2.67

The time must be positive, so 2.67 seconds.

e The assumption is that the ground is flat.

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Challenge

1 a $f_2'(x) = f_1(x) = x^2$

So $f_2(x) = \frac{1}{3}x^3 + c$

The curve passes through $(0, 0)$.

so $f_2(0) = 0 \Rightarrow c = 0$

So $f_2(x) = \frac{1}{3}x^3$

$f_3'(x) = \frac{1}{3}x^3$

$f_3(x) = \frac{1}{12}x^4 + c$

But $c = 0$ since $f_3(0) = 0$.

So $f_3(x) = \frac{1}{12}x^4$

b $f_2(x) = \frac{1}{3}x^3$, $f_3(x) = \dfrac{x^4}{3 \times 4}$

So the power of x is $n+1$ for $f_n(x)$.

The denominator is $3 \times 4 \times \ldots$ up to $n+1$.

$$f_n(x) = \frac{x^{n+1}}{3 \times 4 \times 5 \times \ldots \times (n+1)}$$

2 $f_2'(x) = f_1(x) = 1$

$\Rightarrow f_2(x) = x + c$

But $f_2(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$

So $f_2(x) = x + 1$

$f_3'(x) = f_2(x) = x + 1$

$\Rightarrow f_3(x) = \frac{1}{2}x^2 + x + c$

But $f_3(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$

So $f_3(x) = \frac{1}{2}x^2 + x + 1$

$f_4'(x) = f_3(x) = \frac{1}{2}x^2 + x + 1$

$\Rightarrow f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + c$

But $f_4(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$

So $f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

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