Integration 13B

1 a $\int x^3 \mathrm{d}x = \frac{x^4}{4} + c$

b $\int x^7 \mathrm{d}x = \frac{x^8}{8} + c$

c $\int 3x^{-4} \mathrm{d}x = \frac{3x^{-3}}{-3} + c = -x^{-3} + \mathrm{c}$

d $\int 5x^2 \mathrm{d}x = \frac{5x^3}{3} + c$

2 a $\int (x^4 + 2x^3)\,\mathrm{d}x = \frac{5x^5}{5} + \frac{2x^4}{4} + c$

$= x^5 + \frac{x^4}{2} + c$

b $\int (2x^3 - x^2 + 5x)\,\mathrm{d}x = \frac{2x^4}{4} - \frac{x^3}{3} + \frac{5x^2}{2} + c$

$= \frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$

c $\int (5x^{\frac{3}{2}} - 3x^2)\,\mathrm{d}x = \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^3}{3} + c$

$= 2x^{\frac{5}{2}} - x^3 + c$

3 a $\int (4x^{-2} + 3x^{-\frac{1}{2}})\,\mathrm{d}x = \frac{4x^{-1}}{-1} + \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -4x^{-1} + 6x^{\frac{1}{2}} + c$

b $\int (6x^{-1} - x^{-\frac{1}{2}})\,\mathrm{d}x = \frac{6x^{-1}}{-1} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -6x^{-1} - 2x^{\frac{1}{2}} + c$

c $\int (2x^{-\frac{3}{2}} + x^2 - x^{-\frac{1}{2}})\,\mathrm{d}x$

$= \frac{2x^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{x^3}{3} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -4x^{-\frac{1}{2}} + \frac{x^3}{3} - 2x^{\frac{1}{2}} + c$

4 a $\int (4x^3 - 3x^{-4} + r)\,\mathrm{d}x = \frac{4x^4}{4} - \frac{3x^{-3}}{-3} + rx + c$

$= x^4 + x^{-3} + rx + c$

b $\int (x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}})\,\mathrm{d}x = \frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= \frac{x^2}{2} + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$

c $\int (px^4 + 2t + 3x^{-2})\,\mathrm{d}x$

$= \frac{px^5}{5} + \frac{4tx^2}{2} + \frac{3x^{-1}}{-1} + c$

$= \frac{px^5}{5} + 2tx^2 - 3x^{-1} + c$

5 a $\int (3t^2 - t^{-2})\,\mathrm{d}t = \frac{3t^3}{3} - \frac{t^{-1}}{-1} + c$

$= t^3 + t^{-1} + c$

b $\int (2t^2 - 3t^{-\frac{3}{2}} + 1)\,\mathrm{d}t = \frac{2t^3}{3} - \frac{3t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$

$= \frac{2t^3}{3} + 6t^{-\frac{1}{2}} + t + c$

c $\int (pt^3 + q^2 + px^3)\,\mathrm{d}t$

$= \frac{pt^4}{4} + q^2 t + px^3 t + c$

6 a $\int \frac{(2x^2 + 3)}{x^2}\,\mathrm{d}x = \int \left(\frac{2x^2}{x^2} + \frac{3}{x^2}\right)\mathrm{d}x$

$= \int \left(2 + 3x^{-2}\right)\mathrm{d}x$

$= 2x + \frac{3x^{-1}}{-1} + c$

$= 2x - \frac{3}{x} + c$

b $\int (2x + 3)^2 \mathrm{d}x = \int \left(4x^2 + 12x + 9\right)\mathrm{d}x$

$= \frac{4x^3}{3} + 6x^2 + 9x + c$

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6 c $\int (2x+3)\sqrt{x}\,dx = \int (2x+3)x^{\frac{1}{2}}\,dx$

$= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right) dx$

$= \frac{2x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{4x^{\frac{5}{2}}}{5} + 2x^{\frac{3}{2}} + c$

7 a $\int \left(x + \frac{1}{x}\right)^2 dx = \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$

$= \int \left(x^2 + 2 + x^{-2}\right) dx$

$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c$

$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$

b $\int \left(\sqrt{x} + 2\right)^2 dx = \int \left(x + 4\sqrt{x} + 4\right) dx$

$= \int \left(x + 4x^{\frac{1}{2}} + 4\right) dx$

$= \frac{x^2}{2} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$

$= \frac{x^2}{2} + \frac{8x^{\frac{3}{2}}}{3} + 4x + c$

c $\int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x}\right) dx = \int (x^{-\frac{1}{2}} + 2x^{\frac{1}{2}})\,dx$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= 2x^{\frac{1}{2}} + \frac{4x^{\frac{3}{2}}}{3} + c$

8 a $\int \left(x^{\frac{2}{3}} + \frac{4}{x^3}\right) dx = \int \left(x^{\frac{2}{3}} + 4x^{-3}\right) dx$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{4x^{-2}}{-2} + c$

$= \frac{3x^{\frac{5}{3}}}{5} - \frac{2}{x^2} + c$

8 b $\int \left(\frac{2+x}{x^3} + 3\right) dx = \int \left(2x^{-3} + x^{-2} + 3\right) dx$

$= \frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c$

$= -\frac{1}{x^2} - \frac{1}{x} + 3x + c$

c $\int \left(x^2 + 3\right)\left(x - 1\right) dx = \int \left(x^3 - x^2 + 3x - 3\right) dx$

$= \frac{x^4}{4} - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + c$

d $\int \frac{(2x+1)^2}{\sqrt{x}} dx = \int \left(\frac{4x^2 + 4x + 1}{x^{\frac{1}{2}}}\right) dx$

$= \int (4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{\frac{-1}{2}})\,dx$

$= \frac{8x^{\frac{5}{2}}}{5} + \frac{8x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + c$

e $\int \left(3 + \frac{\sqrt{x} + 6x^3}{x}\right) dx = \int (3 + x^{\frac{-1}{2}} + 6x^2)\,dx$

$= 3x + 2x^{\frac{1}{2}} + 2x^3 + c$

f $\int \sqrt{x}\left(\sqrt{x} + 3\right)^2 dx = \int x^{\frac{1}{2}}\left(x + 6x^{\frac{1}{2}} + 9\right) dx$

$= \int (x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}})\,dx$

$= \frac{2x^{\frac{5}{2}}}{5} + 3x^2 + 6x^{\frac{3}{2}} + c$

9 a $\int \left(\frac{A}{x^2} - 3\right) dx = \int (Ax^{-2} - 3)\,dx$

$= \frac{Ax^{-1}}{-1} - 3x + c$

$= -\frac{A}{x} - 3x + c$

b $\int \left(\sqrt{Px} + \frac{2}{x^3}\right) dx = \int \left(\sqrt{P}x^{\frac{1}{2}} + 2x^{-3}\right) dx$

$= \frac{\sqrt{P}x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2x^{-2}}{-2} + c$

$= \frac{2\sqrt{P}x^{\frac{3}{2}}}{3} - \frac{1}{x^2} + c$

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9 c $\int\left(\frac{p}{x^2}+q\sqrt{x}+r\right)\mathrm{d}x$

$=\frac{px^{-1}}{-1}+\frac{qx^{\frac{3}{2}}}{\frac{3}{2}}+rx+c$

$=-\frac{p}{x}+\frac{2qx^{\frac{3}{2}}}{3}+rx+c$

10 $\int\left(\frac{6}{x^2}+4\sqrt{x}-3x+2\right)\mathrm{d}x$

$=\int(6x^{-2}+4x^{\frac{1}{2}}-3x+2)\,\mathrm{d}x$

$=\frac{6x^{-1}}{-1}+\frac{4x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{3x^2}{2}+2x+c$

$=-\frac{6}{x}+\frac{8x^{\frac{3}{2}}}{3}-\frac{3x^2}{2}+2x+c$

11 $\int\left(8x^3+6x-\frac{3}{\sqrt{x}}\right)\mathrm{d}x$

$=\int(8x^3+6x-3x^{-\frac{1}{2}})\,\mathrm{d}x$

$=\frac{8x^4}{4}+\frac{6x^2}{2}-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}+c$

$=2x^4+3x^2-6\sqrt{x}+c$

12 a $(2+5\sqrt{x})^2=(2+5\sqrt{x})(2+5\sqrt{x})$

$=4+10\sqrt{x}+10\sqrt{x}+25x$

$=4+20\sqrt{x}+25x$

So $k=20$

b $\int(4+20\sqrt{x}+25x)\,\mathrm{d}x$

$=\int(4+20x^{\frac{1}{2}}+25x)\,\mathrm{d}x$

$=4x+\frac{20x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{25x^2}{2}+c$

$=4x+\frac{40x^{\frac{3}{2}}}{3}+\frac{25x^2}{2}+c$

13 $\int\left(3x^5-\frac{4}{\sqrt{x}}\right)\mathrm{d}x=\int(3x^5-4x^{-\frac{1}{2}})\,\mathrm{d}x$

$=\frac{3x^6}{6}-\frac{4x^{\frac{1}{2}}}{\frac{1}{2}}+c$

$=\frac{x^6}{2}-8\sqrt{x}+c$

14 $\int\left(\frac{p}{2x^2}+pq\right)\mathrm{d}x=\int\left(\frac{px^{-2}}{2}+pq\right)\mathrm{d}x$

$=\frac{px^{-1}}{2\times-1}+pqx+c$

$=-\frac{p}{2x}+pqx+c$

$=\frac{2}{x}+10x+c$

$-\frac{p}{2}=2$ and $pq=10$

$p=-4$ and $q=-2.5$

15 a Using the binomial expansion:

$f(x)=2^{10}+\binom{10}{1}2^9(-x)^1+\binom{10}{2}2^8(-x)^2$

$=1024-5120x+11520x^2$

b $\int(1024-5120x+11520x^2)\mathrm{d}x$

$=1024x-\frac{5120x^2}{2}+\frac{11520x^3}{3}+c$

$=1024x-2560x^2+3840x^3+c$

So $A=1024$, $B=-2560$ and $C=3840$

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