Trigonometric identities and equations 10E

1 a $\sin 4\theta = 0 \quad 0° \le \theta \le 360°$

Let $X = 4\theta$ so $0° \le X \le 1440°$

Solve $\sin X = 0$ in the interval $0° \le X \le 1440°$

From the graph of $y = \sin X$, $\sin X = 0$ where

$X = 0°, 180°, 360°, 540°, 720°, 900°, 1080°, 1260°, 1440°$

$\theta = \frac{X}{4}$

$= 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°$

b $\cos 3\theta = -1 \quad 0° \le \theta \le 360°$

Let $X = 3\theta$ so $0° \le X \le 1080°$

Solve $\cos X = 0$ in the interval $0° \le X \le 1080°$

From the graph of $y = \cos X$, $\cos X = -1$ where

$X = 180°, 540°, 900°,$

$\theta = \frac{X}{3}$

$= 60°, 180°, 300°$

c $\tan 2\theta = 1 \quad 0° \le \theta \le 360°$

Let $X = 2\theta$

Solve $\tan X = 1$ in the interval $0° \le X \le 720°$.

A solution is $X = \tan^{-1}(1) \quad = 45°$

As $\tan X$ is +ve, X is in the first and third quadrants.

So $X = 45°, 225°, 405°, 585°$

$\theta = \frac{X}{2}$

$= 22\frac{1}{2}°, 112\frac{1}{2}°, 202\frac{1}{2}°, 292\frac{1}{2}°$

d $\cos 2\theta = \frac{1}{2} \quad 0 \le \theta \le 360°$

Let $X = 2\theta$

Solve $\cos X = \frac{1}{2}$ in the interval $0 \le X \le 720°$.

A solution is $X = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

As $\cos X$ is +ve, X is in the first and fourth quadrants.

So $X = 60°, 300°, 420°, 660°$

d $\theta = \frac{X}{2}$

$= 30°, 150°, 210°, 330°$

e $\tan\frac{1}{2}\theta = -\frac{1}{\sqrt{3}} \quad 0 \le \theta \le 360°$

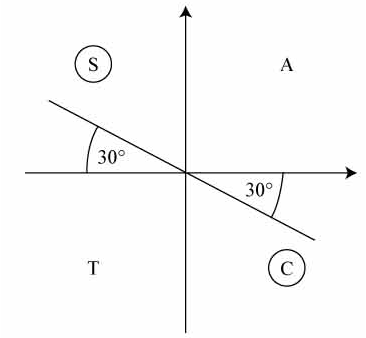
Let $X = \frac{1}{2}\theta$

Solve $\tan X = -\frac{1}{\sqrt{3}}$ in the interval $0° \le X \le 180°$.

A solution is $X = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30°$

(This is not in the interval.)

As $\tan X$ is −ve, X is in the second and fourth quadrants.



Read off solution in the interval $0° \le X \le 180°$.

$X = 150°$

So $\theta = 2X$

$= 300°$

f $\sin(-\theta) = \frac{1}{\sqrt{2}} \quad 0° \le \theta \le 360°$

Let $X = -\theta$

Solve $\sin X = \frac{1}{\sqrt{2}}$ in the interval $0° \le X \le 360°$.

A solution is $X = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

As $\sin X$ is +ve, X is in the first and second quadrants.

$X = -315°, -225°$

So $\theta = -X$

$= 225°, 315°$

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2 a $\tan(45° - \theta) = -1 \quad 0° \le \theta \le 360°$

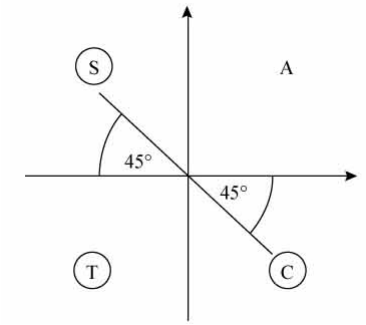
Let $X = 45° - \theta$ so $0° \ge -\theta \ge -360°$

Solve $\tan X = -1$ in the interval

$45° \ge X \ge -315°$

A solution is $X = \tan^{-1}(-1) = -45°$

As $\tan X$ is −ve, X is in the second and fourth quadrants.



X = −225°, −45°

So $\theta = 45° - X = 90°, 270°$

b $2\sin(\theta - 20°) = 1$

So $\sin(\theta - 20°) = \frac{1}{2} \quad 0° \le \theta \le 360°$

Let $X = \theta - 20°$

Solve $\sin X = \frac{1}{2}$ in the interval

$-20° \le X \le 340°$

A solution is $X = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

As $\sin X$ is +ve, solutions are in the first and second quadrants.

$X = 30°, 150°$

So $\theta = X + 20°$

$= 50°, 170°$

c Solve $\tan X = \sqrt{3}$ where $X = (\theta + 75°)$.

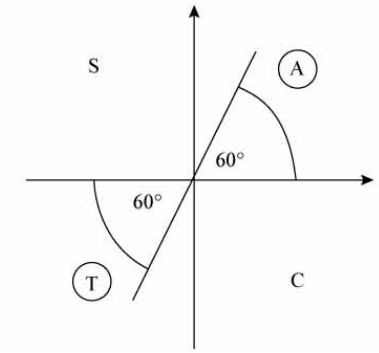
The interval for X is $75° \le X \le 435°$

One solution is $\tan^{-1}\left(\sqrt{3}\right) = 60°$

(This is not in the interval)

As $\tan X$ is +ve, solutions are in the first and third quadrants.

c



$X = 240°, 420°$

So $\theta = X - 75°$

$= 165°, 345°$

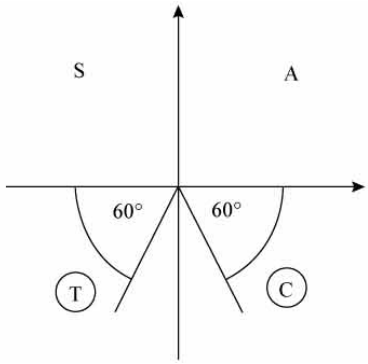
d Solve $\sin X = \frac{\sqrt{3}}{2}$ where $X = (\theta - 10°)$.

The interval for X is $-10° < X \le 350°$

First solution is $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60°$

(This is not in the interval)

As $\sin X$ is –ve, X is in the third and fourth quadrants.



Read off solutions in the interval

$-10° < X \le 350°$

$X = 240°, 300°$

So $\theta = X + 10°$

$= 250°, 310°$

e Solve $\cos X° = 0.6$ where $X = (70° - x)$.

The interval for X is

$180 + 70 \ge X \ge -180 + 70$

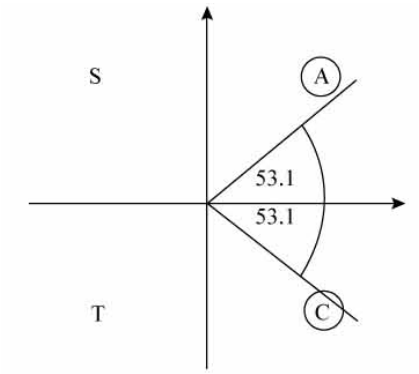
i.e. $-110 \le X \le 250$.

First solution is $\cos^{-1}(0.6) = 53.1°$

As $\cos X°$ is + ve, X is in the first and fourth quadrants.

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2 e



$X = -53.1°, +53.1°$
So $\theta = 70° - X$
$= 16.9°, 123°$ (3 s.f.)

3 a Let $X = 3\theta$
So $3 \sin X = 2 \cos X$
$\frac{\sin X}{\cos X} = \frac{2}{3}$
$\tan X = \frac{2}{3}$
As $X = 3\theta$, then as $0^\circ \leq \theta \leq 180°$
So $3 \times 0^\circ \leq X \leq 3 \times 180°$
So the interval for X is $0^\circ \leq X \leq 540°$.
$X = 33.7°, 213.7°, 393.7°$
i.e. $3\theta = 33.7°, 213.7°, 393.7°$
So $\theta = 11.2°, 71.2°, 131.2°$

b Let $X = \theta + 45°$
So $4 \sin X = 5 \cos X$
$\frac{\sin X}{\cos X} = \frac{5}{4}$
$\tan X = \frac{5}{4}$
As $X = \theta + 45°$, then as $0^\circ \leq \theta \leq 450°$
so $0 + 45 \leq X \leq 450° + 45$
So the interval for X is $45° \leq X \leq 495°$.
$X = 51.3°, 231.3°, 411.3°$
i.e. $\theta + 45° = 51.3°, 231.3°, 411.3°$
So $\theta = 6.3°, 186.3°, 366.3°$

c Let $X = 2x$
$2 \sin X - 7 \cos X = 0$
$2 \sin X = 7 \cos X$
$\frac{\sin X}{\cos X} = \frac{7}{2}$
$\tan X = \frac{7}{2}$
As $X = 2x$, then as $0^\circ \leq x \leq 180°$
So $2 \times 0^\circ \leq X \leq 2 \times 180°$

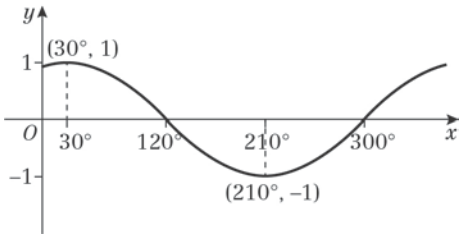
3 c So the interval for X is $0^\circ \leq X \leq 360°$.
$X = 74.05°, 254.05°$
i.e. $2x = 74.05°, 254.05°$
So $x = 37.0°, 127.0°$

d Let $X = \theta - 60°$
So $\sqrt{3} \sin X + \cos X = 0$
$\sqrt{3} \sin X = -\cos X$
$\frac{\sin X}{\cos X} = \frac{-1}{\sqrt{3}}$
$\tan X = \frac{-1}{\sqrt{3}}$
As $X = \theta - 60°$, then as $-180° \leq \theta \leq 180°$
So $-180° - 60 \leq X \leq 180° - 60$
So the interval for X is $-240° \leq X \leq 120°$.
$X = -30°, -210°$
i.e. $\theta - 60° = -210°, -30°$
So $\theta = -150°, 30°$

4 a Let $X = x + 20°$
So $\sin X = \frac{1}{2}$
As $X = x + 20°$, then as $0 \leq x \leq 180°$
So $0 + 20 \leq x \leq 180° + 20$
So the interval for X is $20° \leq X \leq 200°$.
$X = 30°, 150°$
i.e. $x + 20° = 30°, 150°$
So $x = 10°, 130°$

b Let $X = 2x$
So $\cos X = -0.8$
As $X = 2x$, then as $0 \leq x \leq 180°$
So $2 \times 0 \leq X \leq 2 \times 180°$
So the interval for X is $0 \leq X \leq 360°$
$X = 143.13°, 216.87°$
i.e. $2x = 143.13°, 216.87°$
So $x = 71.6°, 108.4°$

5 a



b $\left(0°, \frac{\sqrt{3}}{2}\right)$, (120°, 0), (300°, 0)

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5 c Let $X = x + 60°$
So $\sin X = 0.55$
As $X = x + 60°$, then as $0^\circ \leq x \leq 360°$
So $0^\circ + 60^\circ \leq x \leq 360° + 60^\circ$
So the interval for X is $60° \leq X \leq 420°$.
$X = 33.4°, 146.6°, 393.4°$
i.e. $x + 60° = 33.4°, 146.6°, 393.4°$
So $x = 86.6°, 333.4°$

6 a $4\sin x = 3\cos x$

$$\frac{\sin x}{\cos x} = \frac{3}{4}$$

$$\tan x = \frac{3}{4}$$

b Let $X = 2\theta$

So $\tan X = \frac{3}{4}$

As $X = 2\theta$, then as $0^\circ \leq \theta \leq 360°$
So $2 \times 0^\circ \leq X \leq 2 \times 360°$
So the interval for X is $0^\circ \leq X \leq 720°$.
$X = 36.87°, 216.87°, 396.87°, 576.87°$
i.e. $2\theta = 36.87°, 216.87°, 396.87°, 576.87°$
So $\theta = 18.4°, 108.4°, 198.4°, 288.4°$

7 a $\tan 60k° = -\frac{1}{\sqrt{3}}$

Let $X = 60k°$

So $\tan X = -\frac{1}{\sqrt{3}}$

$X = -30°, 150°, 330°,$
I.e. $60k° = -30°, 150°, 330°,$
$k = -0.5, 2.5,$
$k > 0$ so $k = 2.5$

b No because when $X = 330°$, $k = 5.5$.
As k increases, the period of the tan graph increases.

Challenge

Let $X = 3x - 45°$
So $\sin X = \frac{1}{2}$
As $X = 3x - 45°$, then as $0^\circ \leq x \leq 180°$
So $3 \times 0^\circ - 45° \leq x \leq 3 \times 180° - 45°$
So the interval for X is $-45° \leq X \leq 495°$.
$X = 30°, 150°, 390°$
i.e. $3x - 45° = 30°, 150°, 390°$
So $x = 25°, 65°, 145°$

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