Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE A Level Mathematics

Pure Mathematics Paper 1 (9MA01 01)

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Summer 2018
Publications Code 9MA0_01_1806_MS
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\,|pq|=|c|$, leading to x = …

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\,|pq|=|c|\ \text{and}\ |mn|=|a|$, leading to x = …

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

General Instructions for Marking

1. The total number of marks for the paper is 100

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- **bod** – benefit of doubt
- **ft** – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- **cao** – correct answer only
- **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
- **isw** – ignore subsequent working
- **awrt** – answers which round to
- **SC**: special case
- **o.e.** – or equivalent (and appropriate)
- **d** or **dep** – dependent
- **indep** – independent
- **dp** decimal places
- **sf** significant figures
- ***** The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$\rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify.

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}=\frac{8\theta^2}{6\theta}=\frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

.....

Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}=\frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta \sin 3\theta}=\frac{2\sin^2 2\theta}{2\theta \sin 3\theta}=\frac{2\times(2\theta)^2}{2\theta \times 3\theta}=\frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2}=2+6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x=4$ into their $\frac{dy}{dx}=2\times4-2-12\times4^{-\frac{1}{2}}=...$	M1	1.1b
	Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x=4$ into their $\frac{d^2y}{dx^2}=2+6\times4^{-\frac{3}{2}}=(2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2}=2.75>0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx}=Ax+B+Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x=4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4}=...$

Alternatively substitutes $x=4$ into an equation resulting from $\frac{dy}{dx}=0$ Eg. $\frac{36}{x}=(x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓ , QED for a minimal conclusion

Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe

Alt Shows that $x=4$ is a root of the resulting equation and states "hence there is a stationary point"

All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x=4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x=4\Rightarrow\frac{d^2y}{dx^2}>0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x=4$ or calculates the value of y either side of $x=4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x=4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Notes:

B1: States or uses $\frac{1}{2}r^2\theta = 11$ This may be implied with an embedded found value for θ

B1: States or uses $2r + r\theta = 4r\theta$ or equivalent

M1: Full method to find $r = ...$ This involves combining the equations to eliminate θ or find θ

The initial equations must be of the same "form" (see **) but condone slips when attempting to solve.

It cannot be scored from impossible values for θ Hence only score if $0 < \theta < 2\pi$ FYI $\theta = \frac{2}{3}$ radians

Allow this to be scored from equations such as $...r^2\theta = 11$ and ones that simplify to $...r = ...r\theta$ **

Allow their $2r + r\theta = 4r\theta \Rightarrow \theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $2r + r\theta = 4r\theta \Rightarrow r\theta = ..$ then substitute this into their $\frac{1}{2}r^2\theta = 11$

Allow their $\frac{1}{2}r^2\theta = 11 \Rightarrow \theta = \frac{..}{r^2}$ then substitute into their $2r + r\theta = 4r\theta \Rightarrow r = ..$

A1: $r = \sqrt{33}$ only but isw after a correct answer.

.....

The whole question can be attempted using θ in degrees.

B1: States or uses $\frac{\theta}{360} \times \pi r^2 = 11$

B1: States or uses $2r + \frac{\theta}{360} \times 2\pi r = 4 \times \frac{\theta}{360} \times 2\pi r$

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3)=$ and $f(4)=$ where $f(x)=\pm\left(2\ln(8-x)-x\right)$	M1	2.1
	$f(3)=\left(2\ln(5)-x\right)=(+)0.22$ and $f(4)=\left(2\ln(4)-4\right)=-1.23$ Change of sign and function continuous in interval $[3,4]\Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts $f(3)=$ and $f(4)=$ where $f(x)=\pm\left(2\ln(8-x)-x\right)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

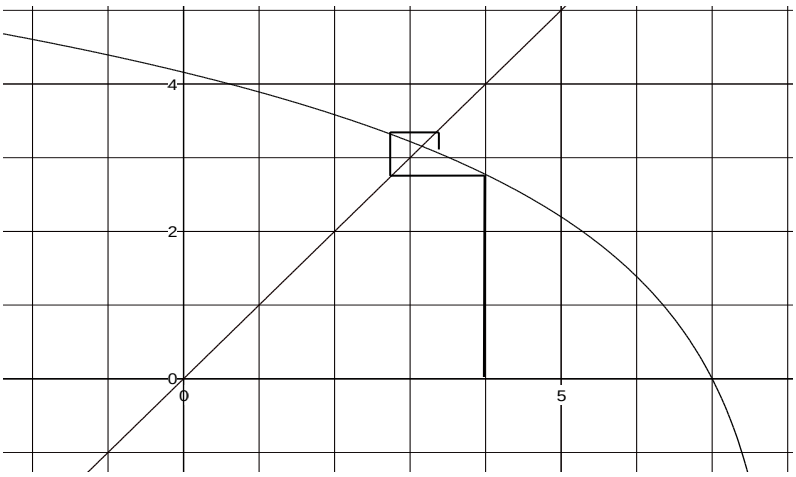
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2ln8=3.21>3,\ 2\ln4=2.77<4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



Question	Scheme	Marks	AOs
5	$\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.....C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{dy}{d\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
			(5 marks)

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{dy}{d\theta}$ (condone it being stated as $\frac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\frac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\frac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\frac{dy}{d\theta}$ condoning it appearing as $\frac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.....C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to k =...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as c = ... to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=\left(\sqrt{20}\right)^2$

(c)

M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$

or**, in the alternative**, for solving their $(4k-34)^2-4\times 5\times\left(k^2-10k+54\right)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

.....

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find *k*. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times 5\times\left(k^2-10k+54\right)=0\Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

.....

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding *k*

A1: $k=-19$ only

……

Other method are possible using trigonometry.

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\,dx = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \to A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\,dx = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right)\times\frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to *k*.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x - k \to \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30 \times 6.5)° = \text{awrt } 4.48\,\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt\ 4.48\text{m}$. The units must be seen somewhere in (a) . So allow when $D = 4.482.. = 4.5\text{ m}$

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k, \quad |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for *t* such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of *t* for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{inv}\sin\ their - 0.6$ to give the first value of *t* where 30 *t* > 255

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$	M1	2.1
	$2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\frac{dy}{dx}=2y-2x$	M1	2.1
	$\frac{dy}{dx}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{dy}{dx}\to\infty \Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\frac{dy}{dx}+2y$ unless you see evidence that they have used the incorrect law vu'-uv'

A1: Fully correct derivative $2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

Allow attempts where candidates write $2x\mathrm{d}x-2x\mathrm{d}y-2y\mathrm{d}x+6y\mathrm{d}y=0$

but watch for students who write $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$

M1: For a valid attempt at making $\frac{dy}{dx}$ the subject. with two terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$

Look for $(...\pm...)\frac{dy}{dx}=.....$ It is implied by $\frac{dy}{dx}=\frac{2y-2x}{6y-2x}$

This cannot be scored from attempts such as $\frac{dy}{dx}=2x-2x\frac{dy}{dx}-2y+6y$ which only has one correct term.

A1*: $\frac{dy}{dx}=\frac{y-x}{3y-x}$ with no errors or omissions.

The previous line $\frac{dy}{dx}=\frac{2y-2x}{6y-2x}$ or equivalent must be seen.

(b)

M1: Deduces that $3y-x=0$ oe

M1: Attempts to find either the *x* or *y* coordinates of *P* and *Q* by solving their $y=\frac{1}{3}x$ with $x^2-2xy+3y^2=50$ simultaneously. Allow for finding a quadratic equation in *x* or *y* and solving to find at least one value for *x* or *y*.

This may be awarded when candidates make the numerator = 0 ie using $y=x$

A1: $\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$

dM1: Dependent upon the previous M, it is for finding the *y* coordinate from their *x* (or vice versa)

This may also be scored following the numerator being set to 0 ie using $y=x$

A1: Deduces that $P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$ OE. Allow to be *x* =... *y* =...

(c)

B1ft: Explains that this is where $\frac{dy}{dx}=0$ and so you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution (or larger solution).

Allow a follow through for candidates who mix up parts (b) and (c)

Alternatively candidates could complete the square $(x-y)^2+2y^2=50$ and state that *y* would reach a maximum value when $x=y$ and choose the positive solution from $2y^2=50$

Question	Scheme	Marks	AOs
10(a)	$\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{H\cos 0.25t}{40}\Rightarrow\int\frac{1}{H}\mathrm{d}H=\int\frac{\cos 0.25t}{40}\mathrm{d}t$	M1	3.1a
	$\ln H=\frac{1}{10}\sin 0.25t\left(+c\right)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5\Rightarrow c=\ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right)=\frac{1}{10}\sin 0.25t\Rightarrow H=5\mathrm{e}^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5\mathrm{e}^{0.1}=5.53\,\mathrm{m}$ (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t=\frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	
			(8 marks)

(a)

M1: Separates the variables to reach $\int\frac{1}{H}\mathrm{d}H=\int\frac{\cos 0.25t}{40}\mathrm{d}t$ or equivalent.

The integral signs need to be present on both sides and the $\mathrm{d}H$ AND $\mathrm{d}t$ need to be in the correct positions.

M1: Integrates both sides to reach $\ln H=A\sin 0.25t$ or equivalent with or without the + c

A1: $\ln H=\frac{1}{10}\sin 0.25t+c$ or equivalent with or without the + c. Allow two constants, one either side

If the 40 was on the lhs look for $40\ln H=4\sin 0.25t+c$ or equivalent.

dM1: Substitutes $t=0, H=5\Rightarrow c=..$ There needs to have been a single " + c " to find.

It is dependent upon the previous M mark. You may allow even if you don't explicitly see "$t=0, H=5$" as it may be implied from their previous line

If the candidate has attempted to change the subject and made an error/ slip then condone it for this M but not the final A. Eg. $40\ln H=4\sin 0.25t+c\Rightarrow\Rightarrow H^{40}=\mathrm{e}^{4\sin 0.25t}+\mathrm{e}^{c}\Rightarrow 5^{40}=1+\mathrm{e}^{c}\Rightarrow c=...$

Also many students will be attempting to get to the given answer so condone the method of finding c =... These students will lose the A1* mark

A1*: Proceeds via $\ln H=\frac{1}{10}\sin 0.25t+\ln 5$ or equivalent to the given answer $H=5\mathrm{e}^{0.1\sin 0.25t}$ with at least one correct intermediate line and no incorrect work.

DO NOT condone c's going to c's when they should be e^{c} or A

Accept as a minimum $\ln H = \frac{1}{10}\sin 0.25t + \ln 5 \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + \ln 5}$ or $H = e^{\frac{1}{10}\sin 0.25t} \times e^{+\ln 5}$ before sight of the given answer

If the only error was to omit the integration signs on line 1, thus losing the first M1, allow the candidate to have access to this mark following a correct intermediate line (see above).

If they attempt to change the subject first then the constant of integration must have been adapted if the A1* is to be awarded. $\ln H = \frac{1}{10}\sin 0.25t + c \Rightarrow H = e^{\frac{1}{10}\sin 0.25t + c} \Rightarrow H = Ae^{\frac{1}{10}\sin 0.25t}$

The dM1 and A1* under this method are awarded at virtually the same time.

Also, for the final two marks, you may see a proof from $\int_0^H \frac{40}{H}\,dH = \int_5^t \cos 0.25t\,dt$

……

There is an alternative via the use of an integrating factor.

……

(b)

B1: States that the maximum height is 5.53 m Accept $5e^{0.1}$ Condone a lack of units here, but penalise if incorrect units are used.

(c)

M1: For identifying that it would reach the maximum height for the 2nd time when $0.25t = \frac{5\pi}{2}$ or 450

A1: Accept awrt 31.4 or 10π Allow if units are seen

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\frac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ **and** $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=(1+2x-2x^2.....)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2...\right)$ $=1+\frac{1}{2}x+\frac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\frac{5}{2}x-\frac{5}{8}x^2.....$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\frac{1}{4}$ Should not use $x=\frac{1}{2}$ as $\frac{1}{2}>\frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}}\approx1+\frac{5}{2}x-\frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}}=\frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	
			(10 marks)

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5}=1+0.5\times(4x)+\frac{(0.5)\times(-0.5)}{2}\times(4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1+2x\pm0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1\pm0.5x\pm0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on

the multiplication on one term only. It is dependent upon having scored the first B and one of the other two M's

In the alternative it is for multiplying $\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ and comparing it to $(1+4x)^{0.5}$

It is for the key step in adding 'six' terms to produce the quadratic expression.

A1*: Completes proof with no errors or omissions. In the alternative there must be some reference to the fact that both sides are equal.

(b)

B1: States that the expansion may not / is not valid when $|x|>\frac{1}{4}$

This may be implied by a statement such as $\frac{1}{2}>\frac{1}{4}$ or stating that the expansion is only valid when $|x|<\frac{1}{4}$

Condone, for this mark a candidate who substitutes $x=\frac{1}{2}$ into the $4x$ **and** states it is not valid as 2 > 1 oe

Don't award for candidates who state that $\frac{1}{2}$ is too big without any reference to the validity of the expansion.

As a rule you should see some reference to $\frac{1}{4}$ or $4x$

(c)(i)

M1: Substitutes $x=\frac{1}{11}$ into BOTH sides $\sqrt{\frac{1+4x}{1-x}}\approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ and attempts to find at least one side.

As the left hand side is $\frac{\sqrt{6}}{2}$ they may multiply by 2 first which is acceptable

A1: Finds both sides leading to a correct equation/statement $\sqrt{\frac{15}{10}}=\frac{1183}{968}$ oe $\sqrt{6}=2\times\frac{1183}{968}$

A1: $\sqrt{6}=\frac{1183}{484}$ or $\sqrt{6}=\frac{2904}{1183}$ $\sqrt{6}=2\times\frac{1183}{968}=\frac{1183}{484}$ would imply all 3 marks

……

Watch for other equally valid alternatives for 11(a) including

B1: $(1+4x)^{0.5}\approx\left(1+\frac{5}{2}x-\frac{5}{8}x^2\right)(1-x)^{0.5}$ then the M's are for $(1+4x)^{0.5}$ and $(1-x)^{0.5}$

M1: $(1-x)^{0.5}=1+(0.5)(-x)+\frac{(0.5)\times(-0.5)}{2}(-x)^2$

……

Or

B1: $\sqrt{\frac{1+4x}{1-x}}=\sqrt{1+\frac{5x}{1-x}}=\left(1+5x(1-x)^{-1}\right)^{\frac{1}{2}}$ then the first M1 for one application of binomial and the second would be for both $(1-x)^{-1}$ and $(1-x)^{-2}$

……

Or

B1: $\sqrt{\frac{1+4x}{1-x}}\times\frac{\sqrt{1-x}}{\sqrt{1-x}}=\sqrt{(1+3x-4x^2)}\times(1-x)^{-1}=\left(1+(3x-4x^2)\right)^{\frac{1}{2}}\times(1-x)^{-1}$

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $$p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $$A = \frac{32000}{'1.0658'^4} \text{ or } A = \frac{50000}{'1.0658'^{11}}$$	M1	1.1b
	$A = 24795 \rightarrow 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p / 1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$$'1.0658'^t = \frac{100000}{24800}$$	M1	3.4
	$$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5$, $50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ‘ =24 800’ or ‘ ≈ 24800 ’. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.

The statement must reference **the car**, its **cost/value**, and **''0'' time**

Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.

The statement must reference **a yearly rate** and **an increase in value or multiplier**.

They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."

Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'

Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their $'1.0658'^t = k$

Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\frac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times '1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

Question	Scheme for Substitution		Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • Using a valid substitution u =..., changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=2u$ oe	Substitution $u=x+2\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u^2\pm 2\right)u^2\mathrm{d}u$	$\int 2x\sqrt{x+2}\,\mathrm{d}x$ $=\int A\left(u\pm 2\right)\sqrt{u}\mathrm{d}u$	M1	1.1b
	$=Pu^5\pm Qu^3$	$=Su^{\frac{5}{2}}\pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$ *		A1*	2.1
			(7)	
				(7 marks)

M1: For attempting to integrate using substitution. Look for

- terms and limits changed to u's. Condone slips and errors/omissions on changing $\mathrm{d}x\to\mathrm{d}u$
- attempted multiplication of terms and raising of at least one power of u by one. Condone slips
- Use of at least the top correct limit. For instance if they go back to x's the limit is 2

B1: For substitution it is for giving the substitution and stating a correct $\frac{\mathrm{d}x}{\mathrm{d}u}$

Eg, $u=\sqrt{x+2}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}u}=2u$ or equivalent such as $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x+2}}$

M1: It is for attempting to get all aspects of the integral in terms of 'u'.
All terms must be attempted including the dx. You are only condoning slips on signs and coefficients
dM1: It is for using a correct method of expanding and integrating each term (seen at least once) . It is dependent upon the previous M
A1: Correct answer in x or u See scheme
ddM1: Dependent upon the previous M, it is for using the correct limits for their integral, **the correct way around**

A1*: Proceeds correctly to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**

There must be at one least correct intermediate line between $\left[\frac{4}{5}u^5-\frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$ and $\frac{32}{15}\left(2+\sqrt{2}\right)$

Question Alt	Scheme for by parts	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x=Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$	M1	1.1b
	$=Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$=\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x\to x^2\times\frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$

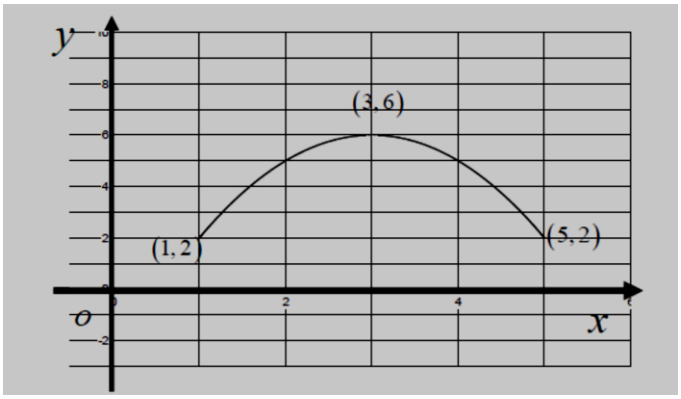
dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

A1*: Proceeds to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**.

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow \quad 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for k = $\left\{k : 7 \leqslant k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1 - 2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y=6-(x-3)^2$ without any errors

Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why *C* does not include all points of $y=6-(x-3)^2, \quad x\in\mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on *x* or *y***.

For example

'As $-1\le \sin t\le 1$ then $1\leqslant x\leqslant 5$' Condone in words '*x* lies between 1 and 5' and strict inequalities

'As $\sin t\le 1$ then $x\leqslant 5$' Condone in words '*x* is less than 5'

'As $-1\le \cos(2t)\le 1$ then $2\leqslant y\leqslant 6$ ' Condone in words '*y* lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2\leqslant x\leqslant 5$"

Do not allow a statement on the top limit of *y* as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$

  $x+y=k\Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0\Rightarrow 25-35+k+3=0$ $\Rightarrow k=7$

- or deduces that $k<\frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for *k*.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The = 0 may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2...4ac$ where ... is any inequality leading to a critical value for *k*. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0\Rightarrow k=\frac{37}{4}$

A1: Range of values for *k* = $\left\{k:7\leqslant k<\frac{37}{4}\right\}$ Accept $k\in\left[7,\frac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1\Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of *k*. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for *k* = $\{k:7\leqslant k<9.25\}$	A1	2.5

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