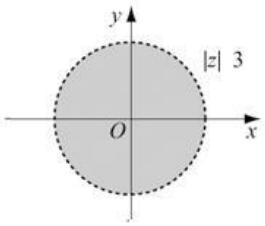
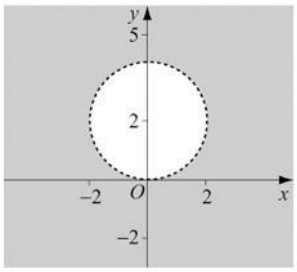
Argand diagrams 2F

1 a

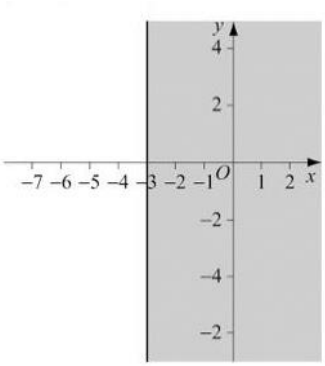


$|z| = 3$ represent a circle centre $(0, 0)$, radius 3

b $|z - (2\mathrm{i})| > 2$

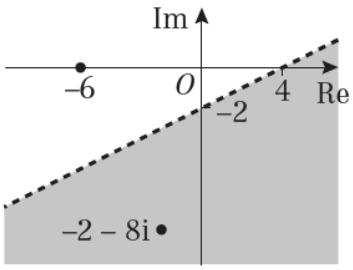


c $|z + 7| \geq |z - 1|$



$|z + 7| = |z - 1|$ represents the perpendicular bisector of the line joining $(-7, 0)$ to $(1, 0)$ which has equation $x = -3$.

d $|z + 6| > |z + 2 + 8\mathrm{i}|$



$|z + 6| = |z + 2 + 8\mathrm{i}|$ represents a perpendicular bisector of the line joining $(-6, 0)$ to $(-2, -8)$.

$|x + \mathrm{i}y + 6| = |x + \mathrm{i}y + 2 + 8\mathrm{i}|$

$\Rightarrow |x + 6 + \mathrm{i}y| = |(x + 2) + \mathrm{i}(y + 8)|$

$\Rightarrow |(x + 6) + \mathrm{i}y|^2 = |(x + 2) + \mathrm{i}(y + 8)|^2$

$\Rightarrow (x + 6)^2 + y^2 = (x + 2)^2 + (y + 8)^2$

$\Rightarrow x^2 + 12x + 36 + y^2 = x^2 + 4x + 4 + y^2 + 16y + 64$

$\Rightarrow 12x + 36 = 4x + 16y + 68$

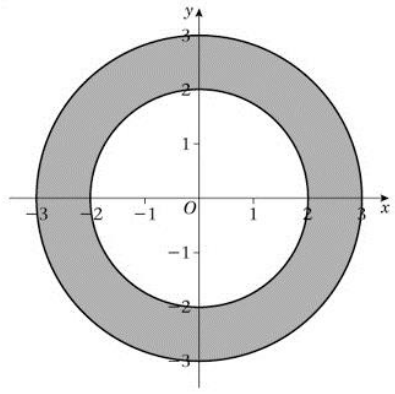
$\Rightarrow 8x + 36 - 68 = 16y$

$\Rightarrow 8x - 32 = 16y$

$\Rightarrow y = \frac{1}{2}x - 2$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

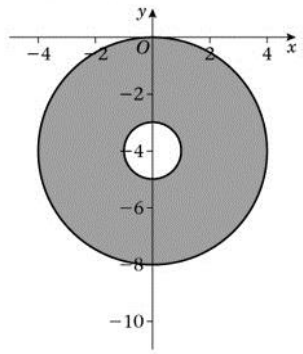
1 e $2 \leqslant |z| \leqslant 3$



$|z| = 2$ represents a circle centre (0, 0), radius 2

$|z| = 3$ represents a circle centre (0, 0), radius 3

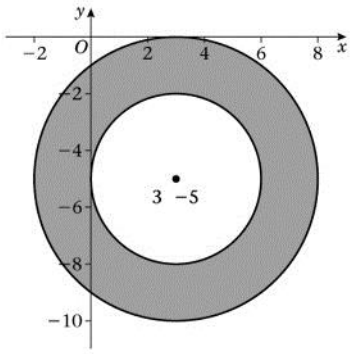
f $1 \leqslant |z + 4i| \leqslant 4$



$|z + 4i| = 1$ represents a circle centre (0, 4), radius 1.

$|z + 4i| = 4$ represents a circle centre (0, –4), radius 4.

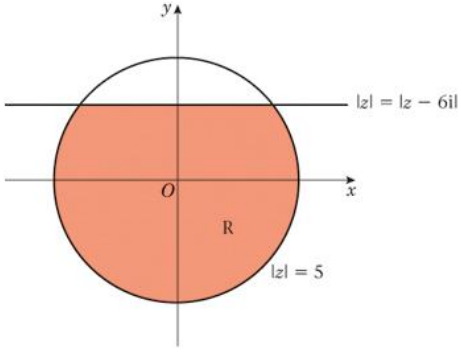
g $3 \leqslant |z - 3 + 5i| \leqslant 5$



$|z - (3 - 5i)| = 3$ represents a circle centre (3, –5), radius 3.

$|z - (3 - 5i)| = 3$ represents a circle centre (3, –5), radius 5.

2 $|z| \leqslant 5$ and $|z| \leqslant |z - 6i|$



$|z| = 5$ represents a circle centre (0, 0), radius 5

$|z| = |z - 6i|$ represents a perpendicular bisector of the line joining (0, 0), to (0, 6) and has the equation $y = 3$.

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

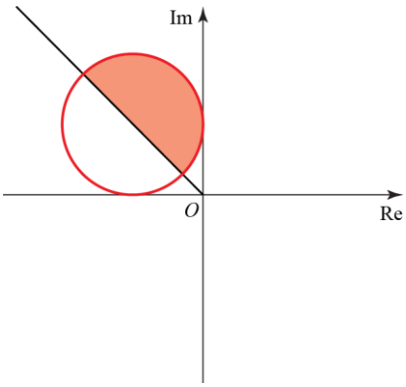
3 $|z+1-\mathrm{i}| \leqslant 1$

$|z-(-1+\mathrm{i})| \leqslant 1$

This represents the region on the boundary and inside the circle with centre $(-1, 1)$ and radius 1.

$0 \leqslant \arg z \leqslant \frac{3\pi}{4}$ This represents the half-line from $(0, 0)$ with angle θ, where $0 \leqslant \theta \leqslant \frac{3\pi}{4}$.

The shaded area shown is where these two regions overlap.



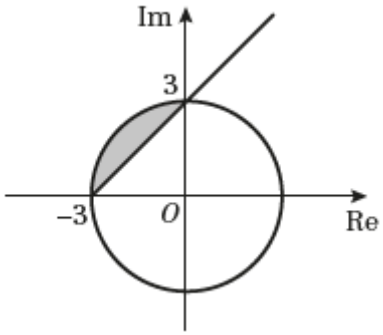
4 $z \in \mathbb{C} : |z| \leqslant 3$

This represents the region on the boundary and inside the circle with centre $(0, 0)$ and radius 3.

$z \in \mathbb{C} : \frac{\pi}{4} \leqslant \arg(z+3) \leqslant \pi$

This represents the half-line from $(-3, 0)$ with angle θ, where $\frac{\pi}{4} \leqslant \theta \leqslant \pi$.

The shaded area shown is where these two regions overlap.



5 a i $|z-2| = |z-6-8\mathrm{i}|$ represents the perpendicular bisector of the line joining (2, 0) to (6, 8).

Write $z = x+\mathrm{i}y$ to find a Cartesian form for this perpendicular bisector.

$|x+\mathrm{i}y-2| = |x+\mathrm{i}y-6-8\mathrm{i}|$

$\Rightarrow |(x-2)+\mathrm{i}y| = |(x-6)+\mathrm{i}(y-8)|$

$\Rightarrow |(x-2)+\mathrm{i}y|^2 = |(x-6)+\mathrm{i}(y-8)|^2$

$\Rightarrow (x-2)^2 + y^2 = (x-6)^2 + (y-8)^2$

$\Rightarrow x^2 - 4x + 4 + y^2 = x^2 - 12x + 36 + y^2 - 16y + 64$

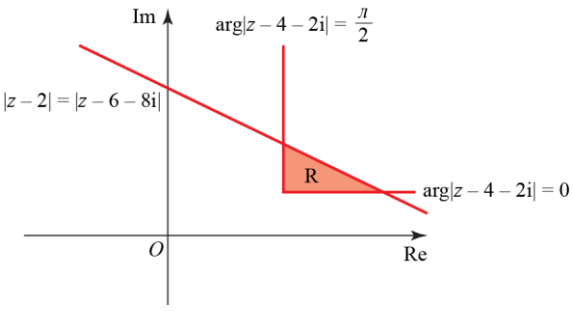
$\Rightarrow -4x + 4 = -12x - 16y + 100$

$\Rightarrow 8x + 16y - 96 = 0 \quad (\div 8)$

$\Rightarrow x + 2y - 12 = 0$

$\Rightarrow 2y = -x + 12$

$\Rightarrow y = -\frac{1}{2}x + 6$



ii $\arg(z-4-2\mathrm{i}) = 0$

$\arg(z-(4+2\mathrm{i})) = 0$

This is a half line from (4, 2) making an angle of 0 with the positive Re axis.

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

5 a iii $\arg(z-4-2\mathrm{i})=\dfrac{\pi}{2}$

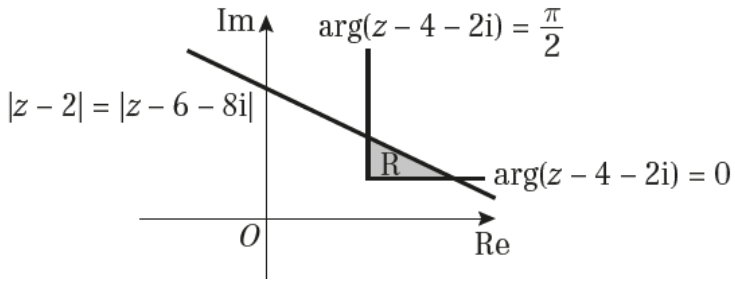
$\arg\left(z-(4+2\mathrm{i})\right)=\dfrac{\pi}{2}$

This is a half line from (4, 2) making an angle of $\dfrac{\pi}{2}$ with the positive Re axis.

b $\{z\in\mathbb{C}:|z-2|\leqslant|z-6-8\mathrm{i}|\}$ represents the region $y\leqslant-\dfrac{1}{2}x+6$

$\left\{z\in\mathbb{C}:0\leqslant\arg(z-4-2\mathrm{i})\leqslant\dfrac{\pi}{2}\right\}$ represented by the region in between and including the two half-lines $\arg\left(z-(4+2\mathrm{i})\right)=0$ and $\arg\left(z-(4+2\mathrm{i})\right)=\dfrac{\pi}{2}$

The region satisfied by both of these is shaded.



6 a i $|x+\mathrm{i}y+10|=|x+\mathrm{i}y-6-4\sqrt{2}\mathrm{i}|$

so $(x+10)^2+y^2=(x-6)^2+(y-4\sqrt{2})^2$

$x^2+20x+100+y^2=x^2-12x+36+y^2-8\sqrt{2}y+32$

$32x=-8\sqrt{2}\,y-32$

$8\sqrt{2}y+32(x+1)=0$

$y+2\sqrt{2}(x+1)=0$

$y=-2\sqrt{2}(x+1)$

ii $(x+1)^2+y^2=9$

$(x^2+2x+y^2=8)$

b Substitute $y=-2\sqrt{2}(x+1)$ into $(x+1)^2+y^2=9$

$(x+1)^2+8(x+1)^2=9$

$9(x+1)^2=9$

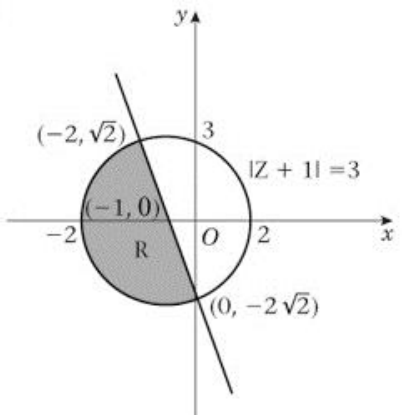
$x+1=\pm1$

$x=0,-2$ $\quad(0,\ -2\sqrt{2})$ and $(-2,\ 2\sqrt{2})$

$z=-2\sqrt{2}\mathrm{i}$ and $z=-2+2\sqrt{2}\mathrm{i}$

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

6 c



The region $|z+10| \leqslant |z-6-4i\sqrt{2}|$ is given by $y \leqslant -2\sqrt{2}(x+1)$

$|z+1| \leqslant \sqrt{3}$ is the area inside the circle $(x+1)^2 + y^2 = 9$

The shaded area satisfies both of these inequalities.

© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.

Challenge

$A=\{z\in\mathbb{C}:|z+5+8\mathrm{i}|\leqslant 5\}$

$A=\{z\in\mathbb{C}:|z-(-5-8\mathrm{i})|\leqslant 5\}$

This represents the region on the boundary and inside the circle with centre $(-5,-8)$ and radius 5.

$B=\{z\in\mathbb{C}:|z+8+4\mathrm{i}|\leqslant|z+2+12\mathrm{i}|\}$

$B=\{z\in\mathbb{C}:|z-(-8-4\mathrm{i})|\leqslant|z-(-2-12\mathrm{i})|\}$

This represents the region of all points closer to $(-8,-4)$ than $(-2,-12)$.

To find this region, first find the perpendicular bisector of $(-8,-4)$ than $(-2,-12)$.

The gradient of the line joining $(-8,-4)$ than $(-2,-12)$ is $-\frac{8}{6}=-\frac{4}{3}$

The gradient of the perpendicular bisector is $\frac{3}{4}$

The midpoint of $(-8,-4)$ than $(-2,-12)$ is $(-5,-8)$

The equation of the perpendicular bisector is found by using $y-y_1=m(x-x_1)$ with $m=\frac{3}{4}$ and $(x_1,y_1)=(-5,-8)$

$y+8=\frac{3}{4}(x+5)$

$y+8=\frac{3}{4}x+\frac{15}{4}$

$y=\frac{3}{4}x-\frac{17}{4}$

As $(-8,-4)$ is above this line, the shaded region will also be above the line.

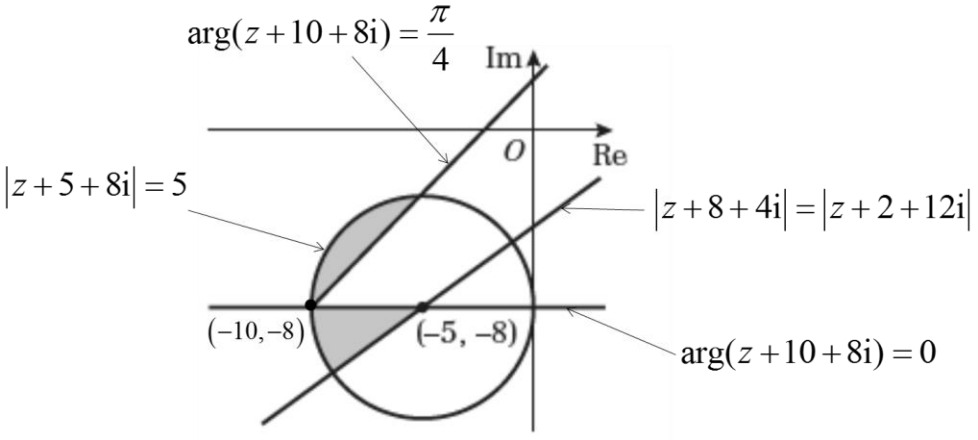
$C=\left\{z\in\mathbb{C}:0\leqslant\arg(z+10+8\mathrm{i})\leqslant\frac{\pi}{4}\right\}$

$C=\left\{z\in\mathbb{C}:0\leqslant\arg(z-(-10-8\mathrm{i}))\leqslant\frac{\pi}{4}\right\}$

The region is the half-line from $(-10,-8)$ which forms an angle θ with the positive real axis, where $0\leqslant\theta\leqslant\frac{\pi}{4}$

Remember, we require the region C', so shade the area outside of C.

The diagram shows the correct shaded region together with labels of each locus.



© Pearson Education Ltd 2017. Copying permitted for purchasing institution only. This material is not copyright free.