Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/4A**

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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1. **In this question you must show detailed reasoning.**

Without performing any division, explain why $n = 20\,210\,520$ is divisible by 66

(4)

66 = 2 × 3 × 11, where 2, 3 and 11 are distinct primes, so n is divisible by 66 if it is divisible by 2, by 3 and by 11.

Divisibility by 2: the last digit of n is 0, which is even, so n is even

⇒ 2 divides n

Divisibility by 3: sum of the digits

2 + 0 + 2 + 1 + 0 + 5 + 2 + 0 = 12 = 4 × 3

the digit sum is a multiple of 3 ⇒ 3 divides n

Divisibility by 11: alternating sum of the digits

2 − 0 + 2 − 1 + 0 − 5 + 2 − 0 = 0 = 0 × 11

the alternating sum is a multiple of 11 ⇒ 11 divides n

n is divisible by 2, 3 and 11, which are coprime, so n is divisible by 2 × 3 × 11 = 66

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Question 1 continued

(Total for Question 1 is 4 marks)

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2. A binary operation ★ on the set of non-negative integers, $\mathbb{Z}_0^+$, is defined by

$$m \star n = |m - n| \qquad m, n \in \mathbb{Z}_0^+$$

(a) Explain why $\mathbb{Z}_0^+$ is closed under the operation ★

(1)

(b) Show that 0 is an identity for $(\mathbb{Z}_0^+, \star)$

(2)

(c) Show that all elements of $\mathbb{Z}_0^+$ have an inverse under ★

(2)

(d) Determine if $\mathbb{Z}_0^+$ forms a group under ★, giving clear justification for your answer.

(3)

(a) For $m, n \in \mathbb{Z}_0^+$, $m - n$ is an integer, so $|m - n|$ is an integer ≥ 0

$\Rightarrow$ $m \star n = |m - n| \in \mathbb{Z}_0^+$, hence $\mathbb{Z}_0^+$ is closed under ★

(b) For any $m \in \mathbb{Z}_0^+$: $0 \star m = |0 - m| = |-m| = m$

and $m \star 0 = |m - 0| = |m| = m$

$0 \star m = m \star 0 = m$ for all m, hence 0 is an identity for $(\mathbb{Z}_0^+, \star)$

(c) For any $m \in \mathbb{Z}_0^+$: $m \star m = |m - m| = 0$, the identity

so each m is self-inverse ($m^{-1} = m \in \mathbb{Z}_0^+$), hence every element has an inverse under ★

(d) For a group ★ must be associative: $(m \star n) \star p = m \star (n \star p)$ for all m, n, p

Test $m = 1$, $n = 2$, $p = 3$:

$1 \star (2 \star 3) = 1 \star |2 - 3| = 1 \star 1 = |1 - 1| = 0$

$(1 \star 2) \star 3 = |1 - 2| \star 3 = 1 \star 3 = |1 - 3| = 2$

$0 \neq 2$, so $1 \star (2 \star 3) \neq (1 \star 2) \star 3$ and ★ is not associative

$\therefore$ $\mathbb{Z}_0^+$ does not form a group under ★

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Question 2 continued

(Total for Question 2 is 8 marks)

3. (a) Use the Euclidean Algorithm to find integers a and b such that

$$125a + 87b = 1$$

(5)

(b) Hence write down a multiplicative inverse of 87 modulo 125

(1)

(c) Solve the linear congruence

$$87x \equiv 16 \pmod{125}$$

(2)

(a) $125 = 1 \times 87 + 38$

$87 = 2 \times 38 + 11$

$38 = 3 \times 11 + 5$

$11 = 2 \times 5 + 1$

$1 = 11 - 2 \times 5$

$= 11 - 2(38 - 3 \times 11) = 7 \times 11 - 2 \times 38$

$= 7(87 - 2 \times 38) - 2 \times 38 = 7 \times 87 - 16 \times 38$

$= 7 \times 87 - 16(125 - 1 \times 87)$

$1 = -16 \times 125 + 23 \times 87$

$a = -16, \quad b = 23$

(b) From (a), $23 \times 87 \equiv 1 \pmod{125}$

so a multiplicative inverse of 87 modulo 125 is 23

(c) $87x \equiv 16 \pmod{125}$

Multiply by 23: $x \equiv 23 \times 16 \pmod{125}$

$x \equiv 368 \equiv 368 - 2 \times 125 = 118 \pmod{125}$

$x \equiv 118 \pmod{125}$

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4. Let G be a group of order $46^{46} + 47^{47}$

Using Fermat's Little Theorem and explaining your reasoning, determine which of the following are possible orders for a subgroup of G

(i) 11

(ii) 21

(7)

By Lagrange's theorem, the order of a subgroup must divide $|G| = 46^{46} + 47^{47}$

(i) 11 is prime, so by Fermat's Little Theorem $a^{10} \equiv 1 \pmod{11}$ for $11 \nmid a$

$46 \equiv 2 \pmod{11}$, $47 \equiv 3 \pmod{11}$

$46^{46} \equiv 2^{46} = (2^{10})^4 \times 2^6 \equiv 2^6 = 64 \equiv 9 \pmod{11}$

$47^{47} \equiv 3^{47} = (3^{10})^4 \times 3^7 \equiv 3^7 = 3^5 \times 3^2 = 243 \times 9$

$\equiv 1 \times 9 = 9 \pmod{11}$ $\quad (243 = 22 \times 11 + 1)$

$46^{46} + 47^{47} \equiv 9 + 9 = 18 \equiv 7 \pmod{11}$, which is not 0

So 11 does not divide $|G|$, hence 11 is not a possible order of a subgroup

(ii) $21 = 3 \times 7$, so check whether 3 and 7 both divide $|G|$

mod 3: FLT gives $a^2 \equiv 1 \pmod{3}$; $46 \equiv 1$, $47 \equiv 2 \pmod{3}$

$46^{46} + 47^{47} \equiv 1^{46} + 2^{47} = 1 + (2^2)^{23} \times 2 \equiv 1 + 2 = 3 \equiv 0 \pmod{3}$

mod 7: FLT gives $a^6 \equiv 1 \pmod{7}$; $46 \equiv 4$, $47 \equiv 5 \equiv -2 \pmod{7}$

$46^{46} \equiv 4^{46} = (4^6)^7 \times 4^4 \equiv 4^4 = 256 \equiv 4 \pmod{7}$

$47^{47} \equiv (-2)^{47} = ((-2)^6)^7 \times (-2)^5 \equiv (-2)^5 = -32 \equiv 3 \pmod{7}$

$46^{46} + 47^{47} \equiv 4 + 3 = 7 \equiv 0 \pmod{7}$

$|G|$ is divisible by both 3 and 7 (coprime), so it is divisible by 21

hence 21 is a possible order for a subgroup of G

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Question 4 continued

(Total for Question 4 is 7 marks)

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5. The point P in the complex plane represents a complex number z such that

$$|z+9| = 4|z-12\mathrm{i}|$$

Given that, as z varies, the locus of P is a circle,

(a) determine the centre and radius of this circle.

(6)

(b) Shade on an Argand diagram the region defined by the set

$$\{z \in \mathbb{C}: |z+9| < 4|z-12\mathrm{i}|\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{4} < \arg\left(z - \frac{3+44\mathrm{i}}{5}\right) < \frac{\pi}{4}\right\}$$

(4)

(a) $z = x + iy$: $|(x+9) + iy| = 4|x + (y-12)i|$

$(x+9)^2 + y^2 = 16[x^2 + (y-12)^2]$

$x^2 + 18x + 81 + y^2 = 16x^2 + 16y^2 - 384y + 2304$

$15x^2 + 15y^2 - 18x - 384y + 2223 = 0$

$x^2 + y^2 - \frac{6}{5}x - \frac{128}{5}y + \frac{741}{5} = 0$

$(x - \frac{3}{5})^2 - \frac{9}{25} + (y - \frac{64}{5})^2 - \frac{4096}{25} + \frac{741}{5} = 0$

$(x - \frac{3}{5})^2 + (y - \frac{64}{5})^2 = \frac{9 + 4096 - 3705}{25} = \frac{400}{25} = 16$

Centre $\frac{3}{5} + \frac{64}{5}i$ i.e. (0.6, 12.8), radius = 4

(b) $|z + 9| < 4|z - 12i|$ is the region outside the circle from (a).

$\frac{3 + 44i}{5} = 0.6 + 8.8i$ is the lowest point of the circle (12.8 − 4 = 8.8)

$-\frac{\pi}{4} < \arg(z - (0.6 + 8.8i)) < \frac{\pi}{4}$: region between the half-lines from

$0.6 + 8.8i$ at angles $\pm\pi/4$ to the horizontal. Region shaded on the next page.

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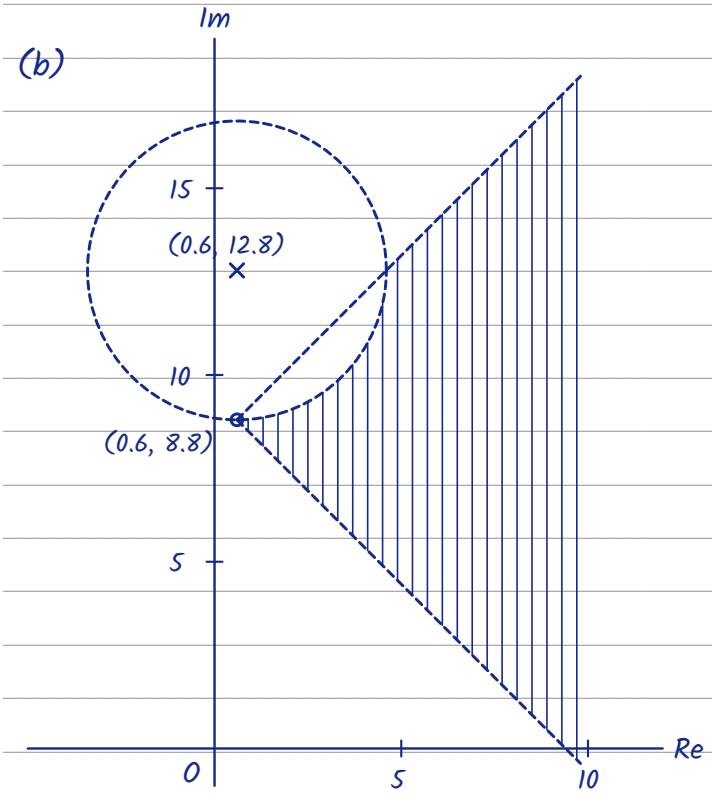
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Question 5 continued

(b)



The shaded region lies outside the circle and between the two half-lines (boundaries not included).

Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

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6. A recurrence system is defined by

$$u_{n+2} = 9(n+1)^2 u_n - 3u_{n+1} \qquad n \geqslant 1$$

$$u_1 = -3,\ u_2 = 18$$

Prove by induction that, for $n \in \mathbb{N}$,

$$u_n = (-3)^n n!$$

(6)

$n = 1$: $(-3)^1 \times 1! = -3 = u_1$ ✓

$n = 2$: $(-3)^2 \times 2! = 9 \times 2 = 18 = u_2$ ✓

So the result is true for $n = 1$ and $n = 2$

Assume the result is true for $n = k$ and $n = k + 1$, i.e.

$u_k = (-3)^k k!$ and $u_{k+1} = (-3)^{k+1} (k + 1)!$

Then $u_{k+2} = 9(k + 1)^2 u_k - 3u_{k+1}$

$= 9(k + 1)^2 (-3)^k k! - 3(-3)^{k+1} (k + 1)!$

$= (-3)^k k! [\, 9(k + 1)^2 - 3(-3)(k + 1) \,]$

$= (-3)^k k! [\, 9(k + 1)^2 + 9(k + 1) \,]$

$= (-3)^k k! \times 9(k + 1)(k + 1 + 1)$

$= (-3)^k \times (-3)^2 \times (k + 2)(k + 1)k!$

$= (-3)^{k+2} (k + 2)!$

which is the result for $n = k + 2$.

If the result is true for $n = k$ and $n = k + 1$, then it is true for $n = k + 2$.

As it is true for $n = 1$ and $n = 2$, by mathematical induction

$u_n = (-3)^n n!$ is true for all $n \in \mathbb{N}$.

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 6 marks)

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7. **In this question you must show all stages of your working.**

You must not use the integration facility on your calculator.

$$I_n = \int t^n \sqrt{4 + 5t^2}\, \mathrm{d}t \qquad n \geqslant 0$$

(a) Show that, for $n > 1$

$$I_n = \frac{t^{n-1}}{5(n+2)}\left(4 + 5t^2\right)^{\frac{3}{2}} - \frac{4(n-1)}{5(n+2)} I_{n-2}$$

(5)

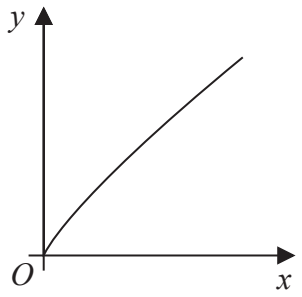


Figure 1

The curve shown in Figure 1 is defined by the parametric equations

$$x = \frac{1}{\sqrt{5}} t^5 \qquad y = \frac{1}{2} t^4 \qquad 0 \leqslant t \leqslant 1$$

This curve is rotated through 2π radians about the x-axis to form a hollow open shell.

(b) Show that the external surface area of the shell is given by

(5)

$$\pi \int_0^1 t^7 \sqrt{4 + 5t^2}\, \mathrm{d}t$$

Using the results in parts (a) and (b) and making each step of your working clear,

(c) determine the value of the external surface area of the shell, giving your answer to 3 significant figures.

(5)

(a) $I_n = \int t^{n-1} \times t(4 + 5t^2)^{1/2}\, dt$

$u = t^{n-1} \Rightarrow du/dt = (n-1)t^{n-2}$

$dv/dt = t(4 + 5t^2)^{1/2} \Rightarrow v = \frac{1}{15}(4 + 5t^2)^{3/2}$

$I_n = t^{n-1} \times \frac{1}{15}(4 + 5t^2)^{3/2} - \int (n-1)t^{n-2} \times \frac{1}{15}(4 + 5t^2)^{3/2}\, dt$

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Question 7 continued

$$= \frac{t^{n-1}}{15}(4 + 5t^2)^{3/2} - \frac{n-1}{15}\int t^{n-2}(4 + 5t^2)(4 + 5t^2)^{1/2}\,dt$$

$$= \frac{t^{n-1}}{15}(4 + 5t^2)^{3/2} - \frac{4(n-1)}{15}\int t^{n-2}\sqrt{(4 + 5t^2)}\,dt - \frac{5(n-1)}{15}\int t^{n}\sqrt{(4 + 5t^2)}\,dt$$

$$15I_n = t^{n-1}(4 + 5t^2)^{3/2} - 4(n-1)I_{n-2} - 5(n-1)I_n$$

$$(15 + 5n - 5)I_n = t^{n-1}(4 + 5t^2)^{3/2} - 4(n-1)I_{n-2}$$

$$5(n+2)I_n = t^{n-1}(4 + 5t^2)^{3/2} - 4(n-1)I_{n-2}$$

$$I_n = \frac{t^{n-1}}{5(n+2)}(4 + 5t^2)^{3/2} - \frac{4(n-1)}{5(n+2)}I_{n-2} \quad \text{as required}$$

(b) Surface area $S = 2\pi\int_0^1 y\sqrt{((dx/dt)^2 + (dy/dt)^2)}\,dt$

$$\frac{dx}{dt} = \frac{5}{\sqrt{5}}t^4 = \sqrt{5}\,t^4, \qquad \frac{dy}{dt} = 2t^3$$

$$(dx/dt)^2 + (dy/dt)^2 = 5t^8 + 4t^6 = t^6(4 + 5t^2)$$

$$S = 2\pi\int_0^1 \frac{1}{2}t^4\sqrt{(t^6(4 + 5t^2))}\,dt$$

$$= 2\pi\int_0^1 \frac{1}{2}t^4 \times t^3\sqrt{(4 + 5t^2)}\,dt \qquad (t \geqslant 0)$$

$$= \pi\int_0^1 t^7\sqrt{(4 + 5t^2)}\,dt \quad \text{as required}$$

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Question 7 continued

(c) Let I_n now denote the integral from 0 to 1. At $t = 1$: $(4 + 5)^{3/2} = 27$, and at $t = 0$: $t^{n-1} = 0$ for $n > 1$, so

$$I_n = \frac{27}{5(n+2)} - \frac{4(n-1)}{5(n+2)} I_{n-2}$$

$$I_1 = \int_0^1 t\sqrt{(4 + 5t^2)}\, dt = \left[\frac{1}{15} (4 + 5t^2)^{3/2} \right]_0^1 = \frac{27}{15} - \frac{8}{15} = \frac{19}{15}$$

$$I_3 = \frac{27}{25} - \frac{8}{25} I_1 = \frac{27}{25} - \frac{8}{25} \times \frac{19}{15} = \frac{253}{375}$$

$$I_5 = \frac{27}{35} - \frac{16}{35} I_3 = \frac{27}{35} - \frac{16}{35} \times \frac{253}{375} = \frac{6077}{13125}$$

$$I_7 = \frac{27}{45} - \frac{24}{45} I_5 = \frac{3}{5} - \frac{8}{15} \times \frac{6077}{13125} = \frac{69509}{196875}$$

$$S = \pi I_7 = \frac{69509\pi}{196875} = 1.10917...$$

External surface area = 1.11 (3 s.f.)

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Question 7 continued

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8.

$$\mathbf{A} = \begin{pmatrix} 5 & -2 & 5 \\ 0 & 3 & p \\ -6 & 6 & -4 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ is an eigenvector for $\mathbf{A}$

(a) (i) determine the eigenvalue corresponding to this eigenvector

(1)

(ii) hence show that $p = 2$

(2)

(iii) determine the remaining eigenvalues and corresponding eigenvectors of $\mathbf{A}$

(7)

(b) Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{PDP}^{-1}$

(1)

(c) (i) Solve the differential equation $\dot{u} = ku$, where k is a constant.

(2)

With respect to a fixed origin O, the velocity of a particle moving through space is modelled by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

By considering $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ so that $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$

(ii) determine a general solution for the displacement of the particle.

(4)

(a)(i) $\begin{pmatrix} 5 & -2 & 5 \\ 0 & 3 & p \\ -6 & 6 & -4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 - 2 - 10 \\ 3 - 2p \\ -12 + 6 + 8 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 - 2p \\ 2 \end{pmatrix} = -1 \begin{pmatrix} 2 \\ 2p - 3 \\ -2 \end{pmatrix}$

Corresponding eigenvalue is -1

(ii) Middle component: $3 - 2p = -1 \times 1 = -1 \Rightarrow 2p = 4 \Rightarrow p = 2$

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Question 8 continued

(iii) $\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -2 & 5 \\ 0 & 3-\lambda & 2 \\ -6 & 6 & -4-\lambda \end{vmatrix} = 0$

$(5 - \lambda)[(3 - \lambda)(-4 - \lambda) - 12] + 2[0 \times (-4 - \lambda) + 12] + 5[0 + 6(3 - \lambda)] = 0$

$(5 - \lambda)(\lambda^2 + \lambda - 24) + 24 + 30(3 - \lambda) = 0$

$-\lambda^3 + 4\lambda^2 + 29\lambda - 120 + 24 + 90 - 30\lambda = 0$

$\lambda^3 - 4\lambda^2 + \lambda + 6 = 0$

$(\lambda + 1)(\lambda^2 - 5\lambda + 6) = 0 \Rightarrow (\lambda + 1)(\lambda - 2)(\lambda - 3) = 0$

Remaining eigenvalues: $\lambda = 2$ and $\lambda = 3$

$\lambda = 2$: $3x - 2y + 5z = 0$, $y + 2z = 0$, $-6x + 6y - 6z = 0$

$y = -2z$, $3x + 4z + 5z = 0 \Rightarrow x = -3z$; take $z = -1$

eigenvector $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ (or any non-zero multiple)

$\lambda = 3$: $2x - 2y + 5z = 0$, $2z = 0$, $-6x + 6y - 7z = 0$

$z = 0 \Rightarrow x = y$

eigenvector $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (or any non-zero multiple)

(b) $P = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ -2 & -1 & 0 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Question 8 continued

(c)(i) $\frac{du}{dt} = ku \Rightarrow \int \frac{1}{u}\, du = \int k\, dt$

$\ln|u| = kt + c$

$u = Ae^{kt}$ (A constant)

(ii) $\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = PDP^{-1}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow P^{-1}\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = DP^{-1}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\Rightarrow \frac{d}{dt}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = D\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -u \\ 2v \\ 3w \end{pmatrix}$

Using (i): $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} Ae^{-t} \\ Be^{2t} \\ Ce^{3t} \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ -2 & -1 & 0 \end{pmatrix}\begin{pmatrix} Ae^{-t} \\ Be^{2t} \\ Ce^{3t} \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2Ae^{-t}+3Be^{2t}+Ce^{3t} \\ Ae^{-t}+2Be^{2t}+Ce^{3t} \\ -2Ae^{-t}-Be^{2t} \end{pmatrix}$

where A, B and C are arbitrary constants

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

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