Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3D**

Further Mathematics

Advanced

PAPER 3D: Decision Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

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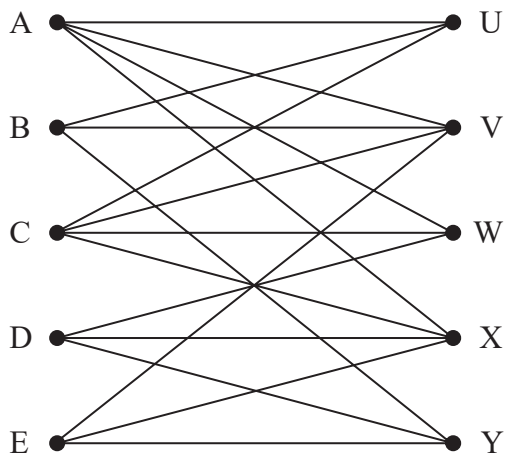


Figure 1

A Hamiltonian cycle for the graph in Figure 1 begins C, V, E, X, A, W, ….

(a) Complete the Hamiltonian cycle.

(1)

(b) Hence use the planarity algorithm to determine whether the graph shown in Figure 1 is planar. You must make your working clear and justify your answer.

(3)

(Total for Question 1 is 4 marks)

2.

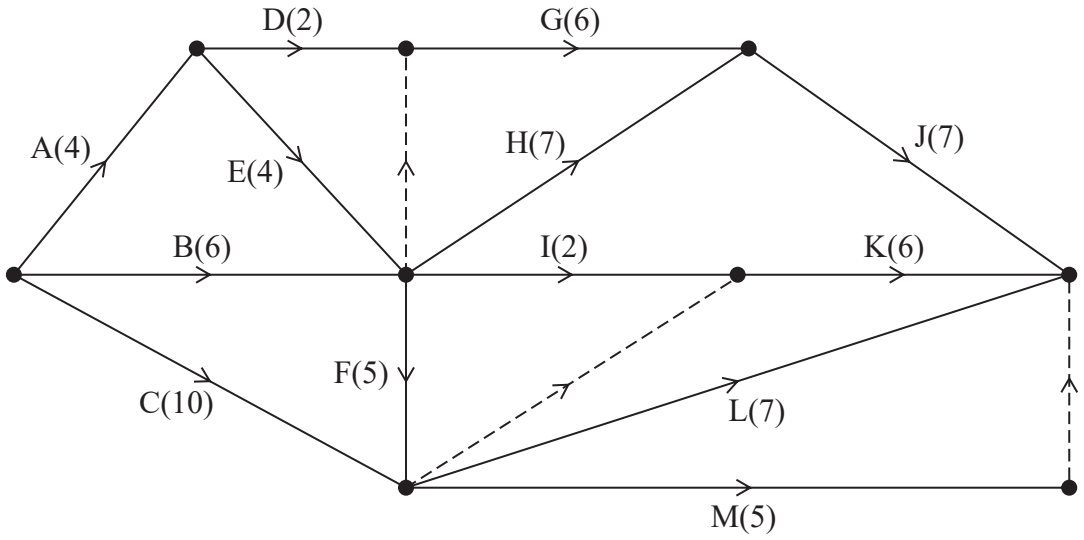


Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

Each activity requires one worker and the project must be completed in the shortest possible time using as few workers as possible.

(b) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.

(2)

(c) Schedule the activities using Grid 1 in the answer book.

(3)

(Total for Question 2 is 9 marks)

3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

Table 1

Table 1 shows the shortest distances, in miles, between eight towns, A, B, C, D, E, F, G and H.

(a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this table of distances. You must clearly state the order in which you select the edges of your tree.

(3)

(b) State the weight of the minimum spanning tree.

(1)

	A	B	C	D	E	F	G	H
J	31	27	50	29	43	25	49	35

Table 2

Table 2 shows the distances, in miles, between town J and towns A, B, C, D, E, F, G and H.

Pranav needs to visit all of the towns, starting and finishing at J, and wishes to minimise the total distance he travels.

(c) Starting at J, use the nearest neighbour algorithm to obtain an upper bound for the length of Pranav's route. You must state your route and its length.

(2)

(d) Starting by deleting J, and all of its edges, find a lower bound for the length of Pranav's route.

(2)

(Total for Question 3 is 8 marks)

4.

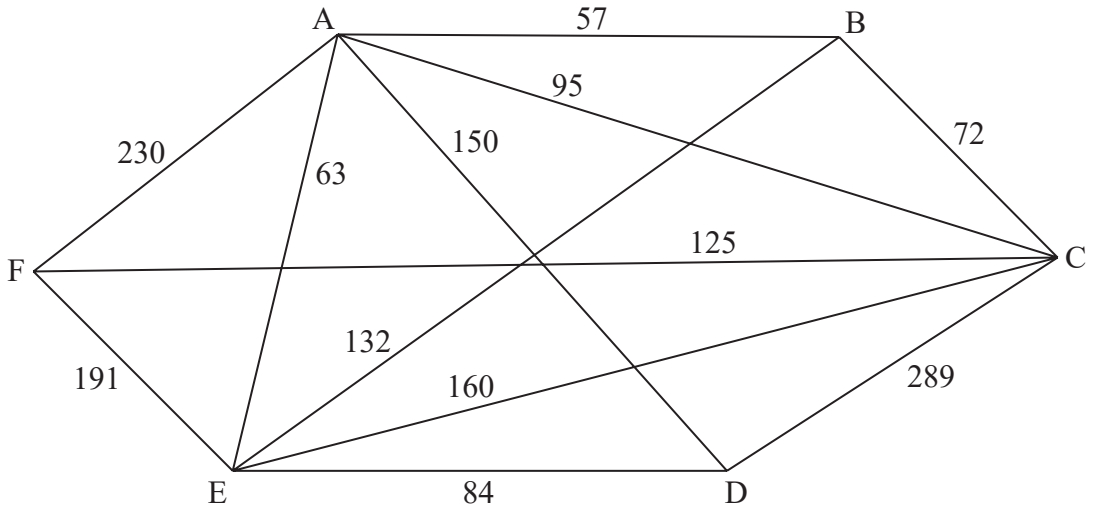


Figure 3

[*The total weight of the network is* 1648]

Direct roads between six cities, A, B, C, D, E and F, are represented in Figure 3. The weight on each arc is the time, in minutes, required to travel along the corresponding road.

Floyd's algorithm is to be used to find the complete network of shortest times between the six cities.

An initial route matrix is given in the answer book.

(a) Set up the initial time matrix.

(1)

(b) Perform the first iteration of Floyd's algorithm. You should show the time and route matrices after this iteration.

(2)

The final time matrix after completion of Floyd's algorithm is shown below.

	A	**B**	**C**	**D**	**E**	**F**
A	–	57	95	147	63	220
B	57	–	72	204	120	197
C	95	72	–	242	158	125
D	147	204	242	–	84	275
E	63	120	158	84	–	191
F	220	197	125	275	191	–

A route is needed that minimises the total time taken to traverse each road at least once.

The route must start at B and finish at E.

(c) Use an appropriate algorithm to find the roads that will need to be traversed twice. You should make your method and working clear.

(4)

(d) Write down the length of the route.

(1)

(Total for Question 4 is 8 marks)

5. 30 12 5 2 23 18 36 10 15 24

(a) The list of ten numbers above is to be sorted into descending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

The ten numbers are to be packed into bins of size n, where n is a positive integer.

When the first-fit bin packing algorithm is applied to the original list of ten numbers, the following allocation is obtained.

Bin 1:	30	12	2
Bin 2:	5	23	10
Bin 3:	18	15	
Bin 4:	36		
Bin 5:	24		

(b) Explain why the value of the integer n must be either 44 or 45

(3)

(c) Use the first-fit decreasing bin packing algorithm to determine how the numbers can be packed into bins of size 45

(3)

(Total for Question 5 is 10 marks)

6.

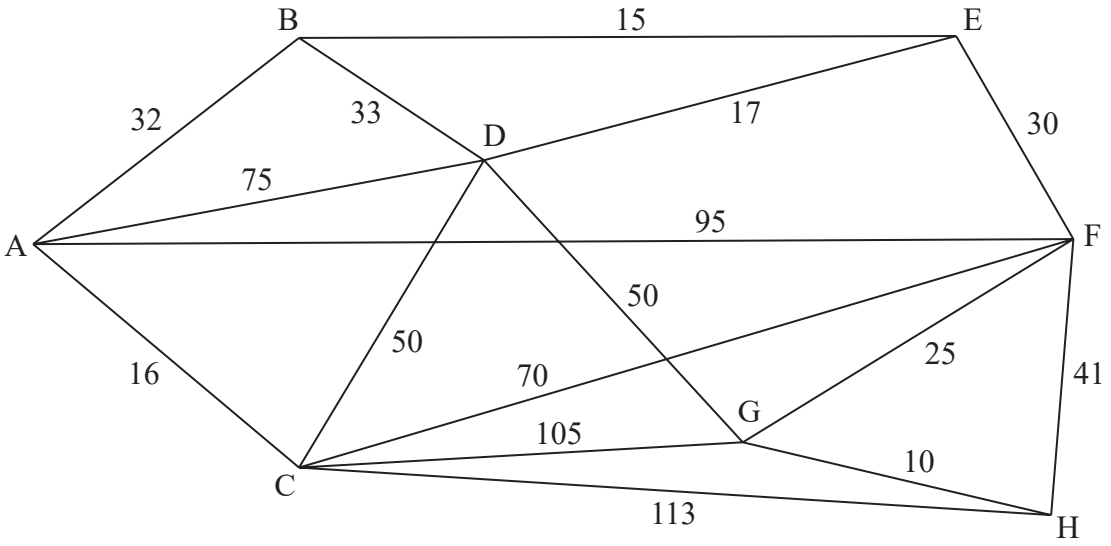


Figure 4

In Figure 4 the weights on the arcs represent distances.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to H.

(ii) State the length of the shortest path from A to H.

(6)

One application of Dijkstra's algorithm has order n^2, where n is the number of nodes in the network. A computer produces a table of shortest distances between any two different nodes by repeatedly applying Dijkstra's algorithm from each node of the network.

It takes the computer 0.082 seconds to produce a table of shortest distances for a network of 10 nodes.

(b) Calculate approximately how long it will take, in seconds, for the computer to produce a table of shortest distances for a network with 200 nodes. You must give a reason for your answer.

(3)

(c) Explain why your answer to part (b) can only be an approximation.

(1)

(Total for Question 6 is 10 marks)

7.

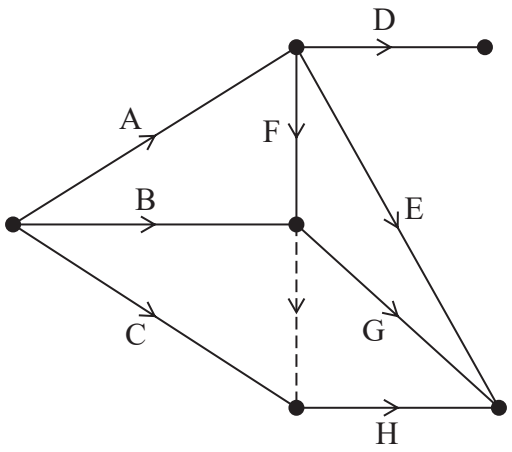


Figure 5

Figure 5 shows a partially completed activity network for a project that consists of 14 activities.

(a) Complete the precedence table in the answer book for the 8 activities in Figure 5.

(2)

The precedence table for the remaining 6 activities is given below.

Activity	Immediately preceding activities
I	D, E, G, H
J	D, E, G, H
K	E, G, H
L	I, J, K
M	J, K
N	J, K

(b) Complete the activity network in the answer book for the project. Your completed activity network must contain only the minimum number of dummies.

(4)

Given that all 14 activities have the same duration,

(c) explain why activity D cannot be critical.

(2)

(Total for Question 7 is 8 marks)

8. Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- x represent the number of hours swimming
- y represent the number of hours cycling
- z represent the number of hours running

(a) Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(5)

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage Simplex method.

As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6)

The following tableau T is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	−2	1	−5	62
x	1	0	1	0	0	−1	28
P	0	0	−1	1	0	1	11

(c) Obtain a suitable pivot for a second iteration. You must give reasons for your answer.

(2)

(d) Starting from tableau T, solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use.

(5)

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

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Time 1 hour 30 minutes

Paper reference **9FM0/3D**

Further Mathematics

Advanced

PAPER 3D: Decision Mathematics 1

Answer book

Do not return the question paper with the answer book.

Total Marks

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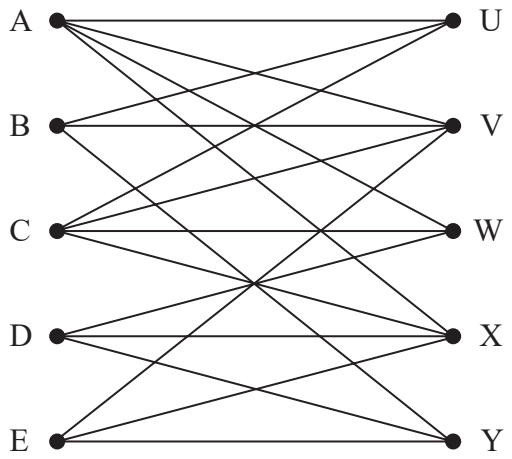


Figure 1

(a) C, V, E, X, A, W, D, Y, B, U, C

(b) Draw the Hamiltonian cycle as a polygon: C V E X A W D Y B U (C)

Arcs not in the Hamiltonian cycle: AU, AV, BV, CW, CX, DX, EY

Put AU inside (I).

AU intersects BV, CW, DX and EY, so these must go outside (O).

AV and CX do not intersect AU, so they can go inside (I).

AU (I), AV (I), BV (O), CW (O), CX (I), DX (O), EY (O)

But BV and CW are both outside and BV intersects CW,

(and BV, CW also both intersect AU so neither can go inside),

so the arcs cannot be drawn without crossing.

∴ the graph is not planar.

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Question 1 continued

(Total for Question 1 is 4 marks)

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2.

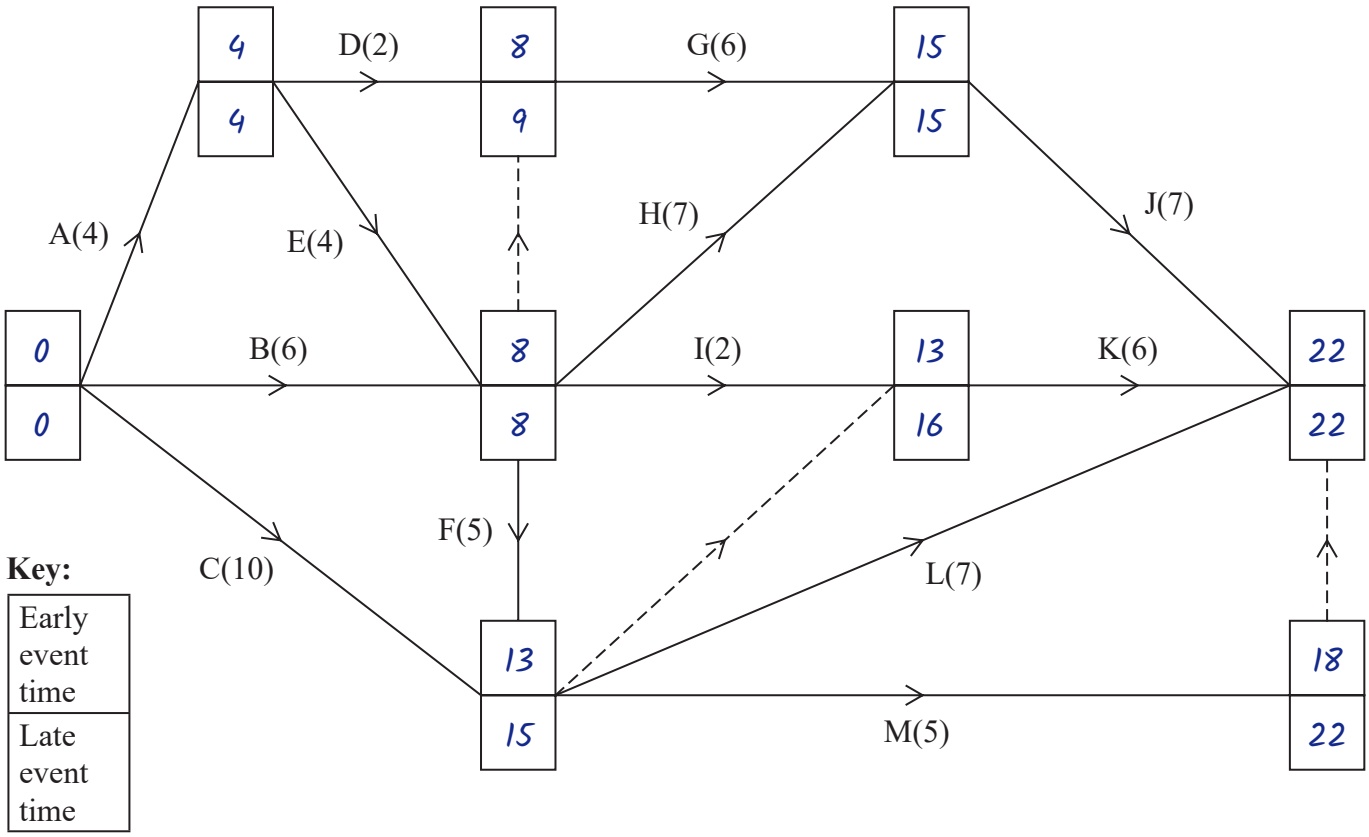


Diagram 1

(b) Sum of activity times = 4 + 6 + 10 + 2 + 4 + 5 + 6 + 7 + 2 + 7 + 6 + 7 + 5 = 71

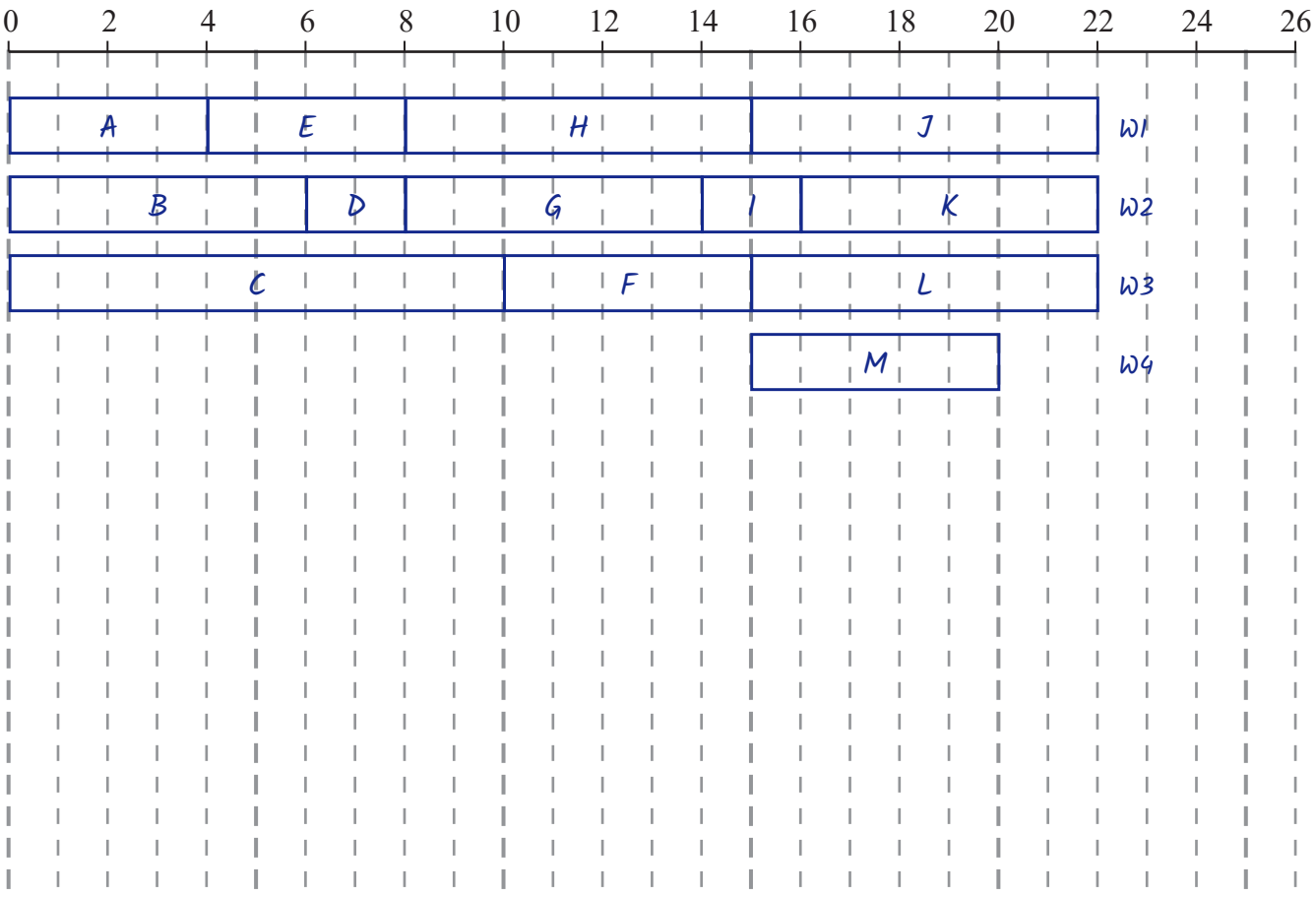
Lower bound = 71 ÷ 22 = 3.227... ⇒ 4 workers

(c) See Grid 1 below (4 workers)

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Question 2 continued



Grid 1

(Total for Question 2 is 9 marks)

3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

(a) Prim's algorithm starting at A, order of selecting edges:

AH (22), AB (24), DH (27), CD (21), CE (26), DG (29), EF (34)

(order of vertices: A, H, B, D, C, E, G, F)

(b) Weight = 22 + 24 + 27 + 21 + 26 + 29 + 34 = 183 miles

(c) Nearest neighbour from J:

J – F – E – C – D – H – A – B – G – J

25 + 34 + 26 + 21 + 27 + 22 + 24 + 39 + 49 = 267

Upper bound = 267 miles

(d) Residual MST (without J) = 183

Two least arcs from J: JF = 25 and JB = 27

Lower bound = 183 + 27 + 25 = 235 miles

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Question 3 continued

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

(Total for Question 3 is 8 marks)

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4. (a)

Initial time matrix

	A	B	C	D	E	F
A	–	57	95	150	63	230
B	57	–	72	∞	132	∞
C	95	72	–	289	160	125
D	150	∞	289	–	84	∞
E	63	132	160	84	–	191
F	230	∞	125	∞	191	–

Initial route matrix

	A	B	C	D	E	F
A	A	B	C	D	E	F
B	A	B	C	D	E	F
C	A	B	C	D	E	F
D	A	B	C	D	E	F
E	A	B	C	D	E	F
F	A	B	C	D	E	F

(b) 1st iteration

Time matrix

	A	B	C	D	E	F
A	–	57	95	150	63	230
B	57	–	72	207	120	287
C	95	72	–	245	158	125
D	150	207	245	–	84	380
E	63	120	158	84	–	191
F	230	287	125	380	191	–

Route matrix

	A	B	C	D	E	F
A	A	B	C	D	E	F
B	A	B	C	A	A	A
C	A	B	C	A	A	F
D	A	A	A	D	E	A
E	A	A	A	D	E	F
F	A	A	C	A	E	F

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Question 4 continued

(c) Odd vertices: A, B, C, D, E and F (all six have odd degree)

The route starts at B and finishes at E, so pair the other four: A, C, D, F

AC + DF = 95 + 275 = 370

AD + CF = 147 + 125 = 272 ← least

AF + CD = 220 + 242 = 462

Shortest AD route is A – E – D (63 + 84 = 147), shortest CF is CF (125)

Roads to be traversed twice: AE, DE and CF

(d) Length of route = 1648 + 272 = 1920 minutes

(Total for Question 4 is 8 marks)

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5. 30 12 5 2 23 18 36 10 15 24

(a) Pivots ringed (middle right item), fixed items underlined:

30	12	5	2	23	(18)	36	10	15	24	pivot 18
30	23	(36)	24	18	12	5	(2)	10	15	pivots 36, 2
36	30	(23)	24	18	12	5	(10)	15	2	pivots 23, 10
36	(30)	24	23	18	12	(15)	10	5	2	pivots 24, 15
36	30	24	23	18	15	12	10	5	2	

No more pivots needed (all sublists of length 1), so the sort is complete.

36 30 24 23 18 15 12 10 5 2

(b) 5 was placed in Bin 2, not Bin 1, so it did not fit in Bin 1:

$30 + 12 + 5 = 47 > n \Rightarrow n \leq 46$

2 then fitted into Bin 1, so $30 + 12 + 2 = 44 \leq n \Rightarrow n \geq 44$

18 did not fit in Bin 2 (5 + 23 = 28 there), so $5 + 23 + 18 = 46 > n \Rightarrow n \leq 45$

(18 also could not go in Bin 1: $44 + 18 > n$.)

n is an integer with $44 \leq n \leq 45$, so n must be either 44 or 45

(c) Sorted in decreasing order: 36 30 24 23 18 15 12 10 5 2

First-fit decreasing, bins of size 45:

Bin 1:	36	5	2	(43)
Bin 2:	30	15		(45)
Bin 3:	24	18		(42)
Bin 4:	23	12	10	(45)

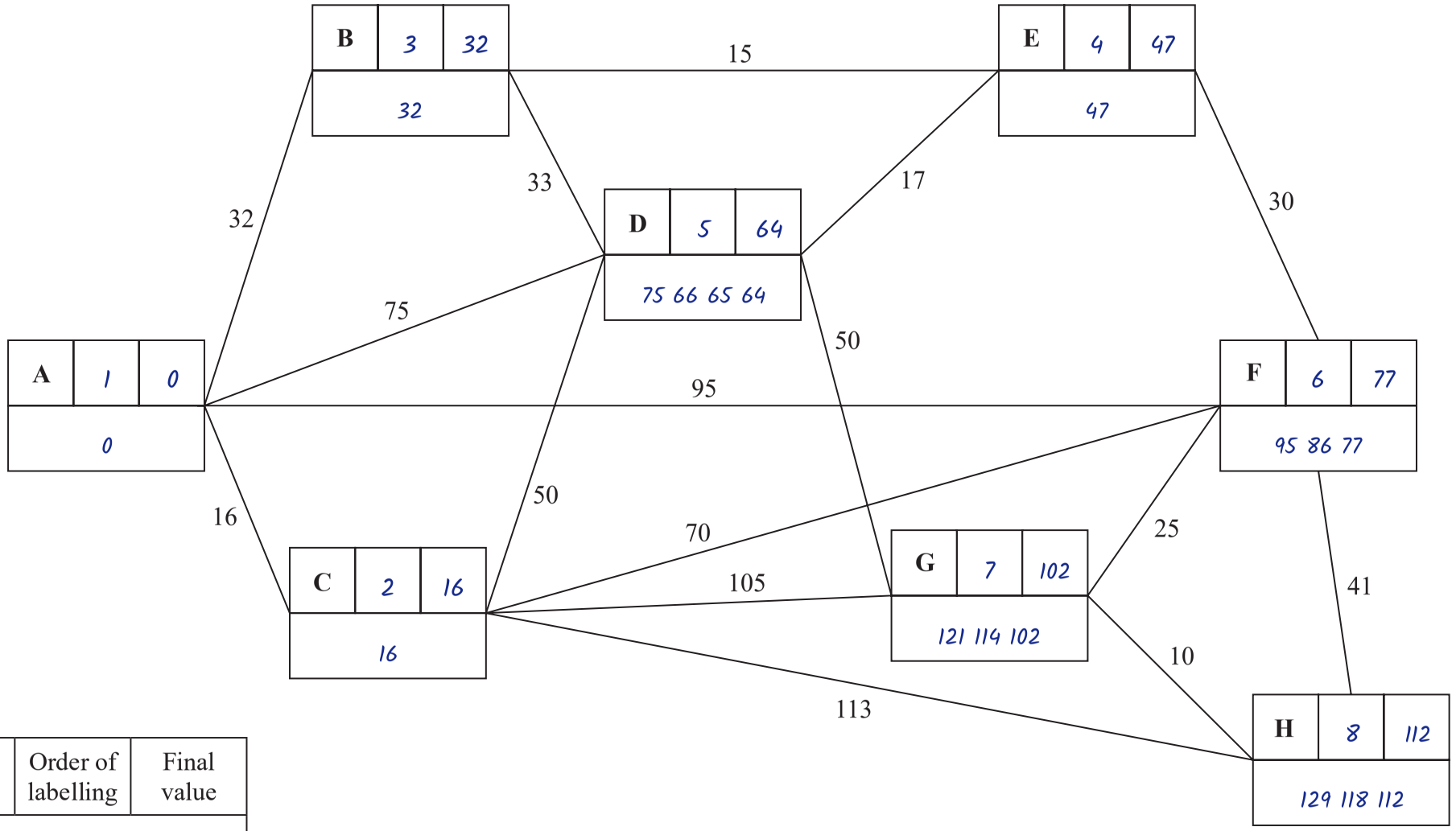
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Question 5 continued

(Total for Question 5 is 10 marks)

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6.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to H: A B E F G H

Length of shortest path from A to H: 112

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P 6 6 8 0 1 A 0 1 2 2 0

Question 6 continued

(a)(ii) Length of shortest path from A to H = 112

(b) One application of Dijkstra's algorithm has order n^2 and it is applied n times (once from each node), so producing the table has order $n \times n^2 = n^3$.

$t = 0.082 \times (200 \div 10)^3$

$= 0.082 \times 8000$

$= 656$ seconds

(c) Order n^3 only means that n^3 is the dominant term; the time is not exactly proportional to n^3 (there are lower order terms, e.g. $an^3 + bn^2 + \ldots$), so the answer to (b) can only be an approximation.

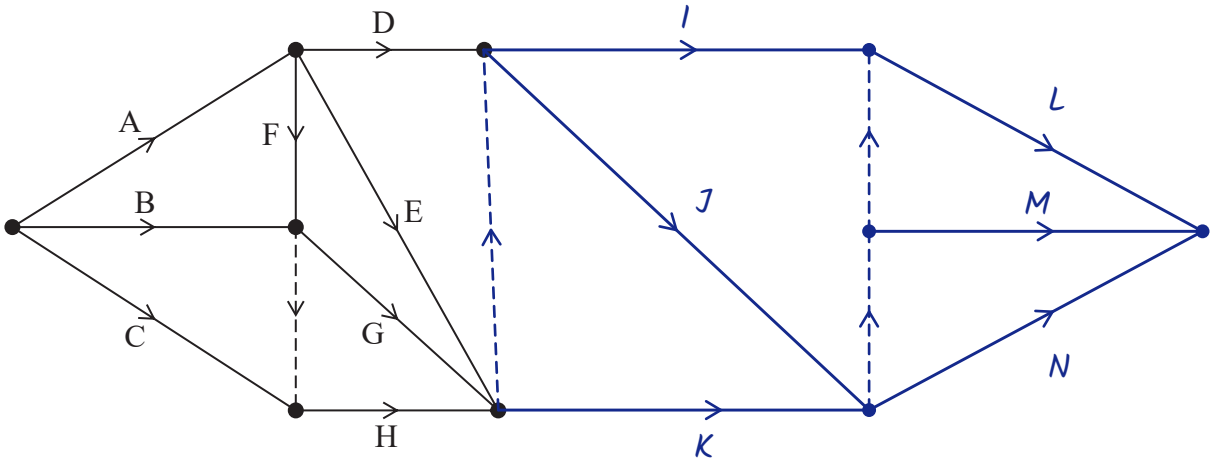
(Total for Question 6 is 10 marks)

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7. (a)

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A

Activity	Immediately preceding activities
E	A
F	A
G	B, F
H	B, C, F



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Question 7 continued

(c) As all 14 activities have the same duration, the critical path(s) are the paths with the most activities. The longest paths contain 5 activities,

e.g. A – F – G – J – M .

Every path through D contains only 4 activities,

e.g. A – D – I – L, A – D – J – M, A – D – J – N.

So D has a non-zero float and cannot be critical.

(Total for Question 7 is 8 marks)

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8.

(a) Maximise $P = y + z$

subject to:

$x + y + z \leq 39$

$x \geq 0.4(x + y + z) \Rightarrow 5x \geq 2x + 2y + 2z \Rightarrow -3x + 2y + 2z \leq 0$

$x + z \geq 28$

$x, y, z \geq 0$

(b) $x + y + z \leq 39 \Rightarrow x + y + z + s_1 = 39$ (s_1 slack)

$-3x + 2y + 2z \leq 0 \Rightarrow -3x + 2y + 2z + s_2 = 0$ (s_2 slack)

$x + z \geq 28 \Rightarrow x + z - s_3 + a_1 = 28$ (s_3 surplus, a_1 artificial)

Objective: $P = y + z \Rightarrow P - y - z = 0$

Stage 1: minimise a_1, i.e. maximise $I = -a_1$

$a_1 = 28 - x - z + s_3$

$I = -a_1 = x + z - s_3 - 28 \Rightarrow I - x - z + s_3 = -28$

Initial tableau: see the table on page 17.

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Question 8 continued

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value
s_1	1	1	1	1	0	0	0	39
s_2	−3	2	2	0	1	0	0	0
a_1	1	0	1	0	0	−1	1	28
P	0	−1	−1	0	0	0	0	0
I	−1	0	−1	0	0	1	0	−28

Question 8 continued

(c) In tableau T the only negative value in the objective (P) row is the -1 in the z column, so the pivot is in the z column.

θ values: s_2 row: $62 \div 5 = 12.4$, x row: $28 \div 1 = 28$

(y row has 0 in the z column so is not considered)

$12.4 < 28$, so the pivot is the 5 in the s_2 row (z enters, s_2 leaves).

(d) See the completed tableau on page 19.

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Question 8 continued

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	−2	1	−5	62
x	1	0	1	0	0	−1	28
P	0	0	−1	1	0	1	11

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
y	0	1	0	1	0	1	11	R1
z	0	0	1	$\frac{-2}{5}$	$\frac{1}{5}$	−1	$\frac{62}{5}$	R2 ÷ 5
x	1	0	0	$\frac{2}{5}$	$\frac{-1}{5}$	0	$\frac{78}{5}$	R3 − R2(new)
P	0	0	0	$\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{117}{5}$	R4 + R2(new)

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b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

No negative values in the objective row, so this tableau is optimal.

$P = 117/5 = 23.4$, $x = 78/5 = 15.6$, $y = 11$, $z = 62/5 = 12.4$

Susie should spend 15.6 hours swimming, 11 hours cycling and 12.4 hours running (maximum total cycling and running = 23.4 hours).

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Question 8 continued

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

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