Pearson Edexcel Level 3 GCE

Tuesday 23 June 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4D**

Further Mathematics

Advanced

Paper 4D: Decision Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read **each** question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P62682RA

©2020 Pearson Education Ltd.

1/1/1/1/1/1/

P 6 2 6 8 2 R A

Pearson

1. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to exactly one task and each task must be done by exactly one worker.

Worker A cannot do task 3 and worker B cannot do task 4

The table below shows the profit, in pounds, that each worker would earn if assigned to each of the tasks.

	1	2	3	4
A	29	20	–	23
B	32	30	28	–
C	35	32	34	25
D	29	31	27	30

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total profit. You must make your method clear and show the table after each stage.

(7)

(b) Determine the resulting total profit.

(1)

(Total for Question 1 is 8 marks)

2. Jenny can choose one of three options, A, B or C, when playing a game. The profit, in pounds, associated with each outcome and their corresponding probabilities are shown on the decision tree in Figure 1.

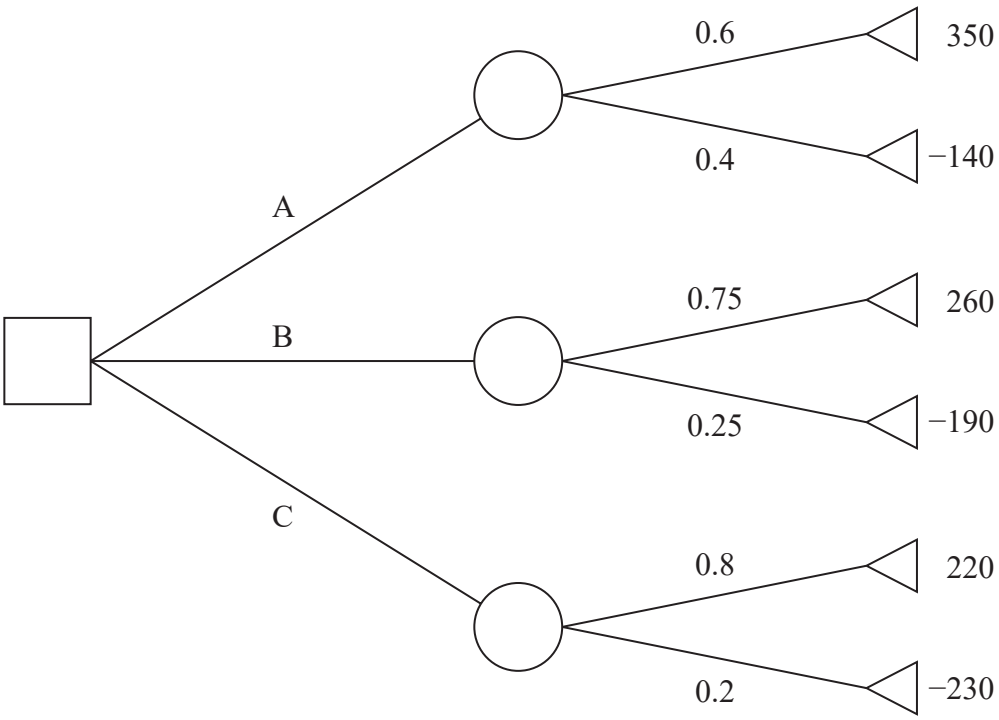


Figure 1

(a) Calculate the optimal EMV to determine Jenny's best course of action. You must make your working clear.

(3)

For a profit of £x, Jenny's utility is given by $1 - e^{-\frac{x}{400}}$

(b) Using expected utility as the criterion for the best course of action, determine what Jenny should do now to maximise her profit. You must make your working clear.

(4)

(Total for Question 2 is 7 marks)

3. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to three sales points, P, Q and R. It also shows the number of units held at each supply point and the number of units required at each sales point. A minimum cost solution is required.

	P	Q	R	Supply
A	25	24	17	42
B	7	12	14	68
C	13	11	20	25
D	16	15	13	40
Demand	59	72	44	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	P	Q	R
A	42		
B	17	51	
C		21	4
D			40

Table 2

(a) Taking AR as the entering cell, use the stepping-stone method to find an improved solution. Make your method clear.

(2)

(b) Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by stating

- shadow costs
- improvement indices
- route
- entering cell and exiting cell.

(4)

(c) Determine whether the solution obtained from this second iteration is optimal, giving the reason for your answer.

(3)

(d) Formulate this situation as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.

(6)

(e) Explain why the Simplex algorithm cannot be used to solve transportation linear programming problems such as that formulated in (d).

(1)

(Total for Question 3 is 16 marks)

4. The complementary function for the second order recurrence relation

$$u_{n+2} + \alpha u_{n+1} + \beta u_n = 20(-3)^n \quad n \geqslant 0$$

is given by

$$u_n = A(2)^n + B(-1)^n$$

where A and B are arbitrary non-zero constants.

(a) Find the value of α and the value of β.

(2)

Given that $2u_0 = u_1$ and $u_4 = 164$

(b) find the solution of this second order recurrence relation to obtain an expression for u_n in terms of n.

(6)

(Total for Question 4 is 8 marks)

5.

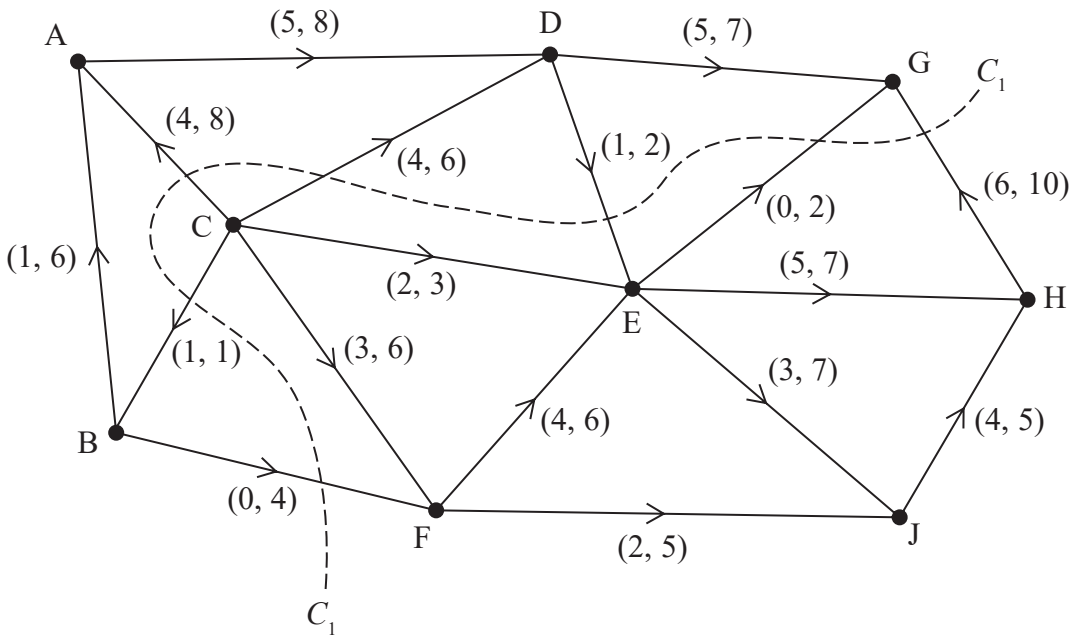


Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower capacities and upper capacities for the corresponding pipes, in litres per second.

(a) State the source node.

(1)

(b) Explain why the sink node must be G.

(1)

(c) Calculate the capacity of the cut C_1

(1)

(d) Assuming that a feasible flow exists,

(i) explain why arc JH must be at its upper capacity,
(ii) explain why arcs AD and CD must be at their lower capacities.

(2)

(e) Use Diagram 1 in the answer book to show a flow of 18 litres per second through the system.

(2)

(f) Prove that the answer to (e) is the maximum flow through the system.

(3)

(Total for Question 5 is 10 marks)

6.

		Player B		
		Option X	Option Y	Option Z
Player A	Option Q	1	5	3
	Option R	4	−3	1
	Option S	2	−4	−2
	Option T	3	−2	0

A two person zero-sum game is represented by the pay-off matrix for player A, shown above.

(a) Explain, with justification, why this matrix may be reduced to a 3×3 matrix by removing option S from player A's choices.

(2)

(b) Verify that there is no stable solution to the reduced game.

(3)

Player A intends to make a random choice between options Q, R and T, choosing option Q with probability p_1, option R with probability p_2 and option T with probability p_3

Player A wants to find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm. Player A formulates the following linear programme, writing the constraints as inequalities.

Maximise $P = V$, where V = the value of original game + 3

subject to $V \leqslant 4p_1 + 7p_2 + 6p_3$

$V \leqslant 8p_1 + p_3$

$V \leqslant 6p_1 + 4p_2 + 3p_3$

$p_1 + p_2 + p_3 \leqslant 1$

$p_1 \geqslant 0,\ p_2 \geqslant 0,\ p_3 \geqslant 0,\ V \geqslant 0$

(c) Explain why V cannot exceed any of the following expressions

$4p_1 + 7p_2 + 6p_3 \qquad 8p_1 + p_3 \qquad 6p_1 + 4p_2 + 3p_3$

(1)

(d) Explain why it is necessary to use the constraint $p_1 + p_2 + p_3 \leqslant 1$

(1)

The Simplex algorithm is used to solve the linear programming problem.

Given that the optimal value of $p_1 = \frac{7}{11}$ and the optimal value of $p_3 = 0$

(e) calculate the value of the game to player A.

(3)

Player B intends to make a random choice between options X, Y and Z, choosing option X with probability q_1, option Y with probability q_2 and option Z with probability q_3

(f) Determine the optimal strategy for player B, making your working clear.

(4)

(Total for Question 6 is 14 marks)

7. A manufacturer can export five batches of footwear each year. Each exported batch contains just one type of footwear. The types of footwear are trainers, sandals or high heels.

The table below shows the profit, in £1000s, for the number of batches of each type of footwear.

Number of batches	**0**	**1**	**2**	**3**	**4**	**5**
Trainers	0	50	90	170	225	295
Sandals	0	70	110	165	245	300
High heels	0	75	115	x	235	305

The total annual profit is to be maximised.

(a) Use dynamic programming to determine the two possible values of the maximum total annual profit, giving one of these values in terms of x.

(10)

Given that the maximum total annual profit is £320 000

(b) advise the manufacturer on the possible ways in which the five batches should be allocated.

(2)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Tuesday 23 June 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4D**

Further Mathematics

Advanced

Paper 4D: Decision Mathematics 2

Answer Book
Do not return the question paper with the answer book

Total Marks

Turn over ▶

P62682RA

©2020 Pearson Education Ltd.

1/1/1/1/1/1/

P 6 2 6 8 2 R A 0 1 2 0

Pearson

1.

	1	2	3	4
A	29	20	–	23
B	32	30	28	–
C	35	32	34	25
D	29	31	27	30

	1	2	3	4
A	6	15	100	12
B	3	5	7	100
C	0	3	1	10
D	6	4	8	5

(a) Subtract every entry from 35
(converts maximising to minimising)
Put 100 in A3 and B4 (not allowed)

	1	2	3	4
A	0	9	94	6
B	0	2	4	97
C	0	3	1	10
D	2	0	4	1

Reduce rows: subtract 6, 3, 0, 4

	1	2	3	4
A	0	9	93	5
B	0	2	3	96
C	0	3	0	9
D	2	0	3	0

Reduce columns: subtract 0, 0, 1, 1
Zeros covered by 3 lines (column 1,
row C, row D): 3 < 4 so not optimal
Smallest uncovered element = 2

	1	2	3	4
A	0	7	91	3
B	0	0	1	94
C	2	3	0	9
D	4	0	3	0

Subtract 2 from uncovered elements,
add 2 to elements covered twice
(C1 and D1), others unchanged
Now 4 lines needed, so optimal

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

	1	2	3	4
A				
B				
C				
D				

Allocation (independent zeros in A1, B2, C3, D4):

A → 1, B → 2, C → 3, D → 4

(b) Total profit = 29 + 30 + 34 + 30

= £123

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

(Total for Question 1 is 8 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

2.

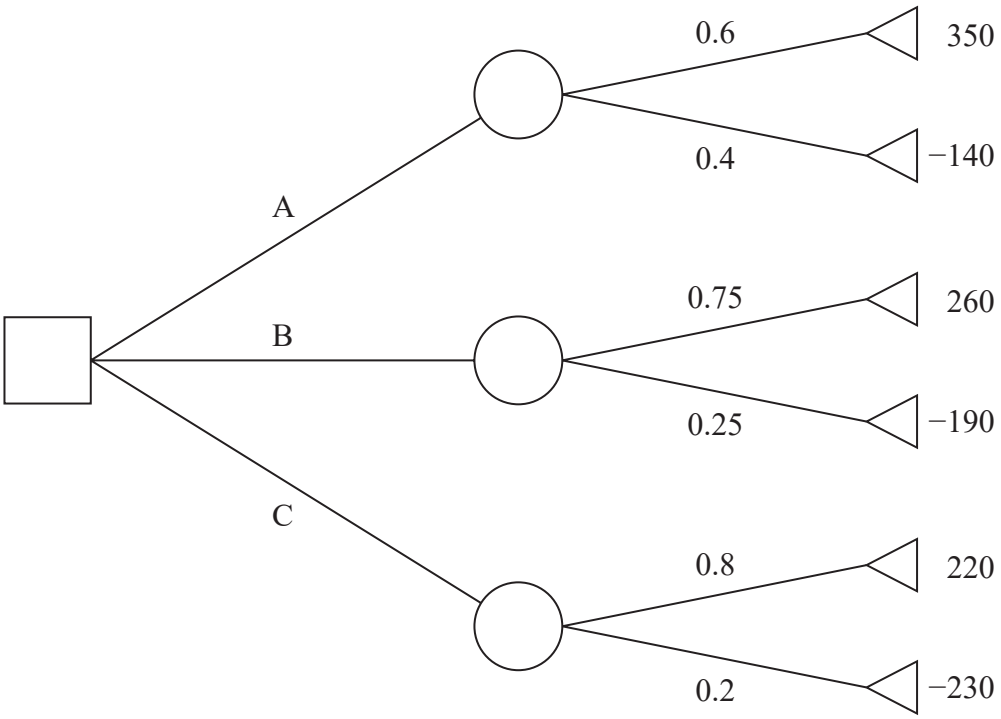


Figure 1

(a) EMV(A) = 0.6(350) + 0.4(−140) = 210 − 56 = 154

EMV(B) = 0.75(260) + 0.25(−190) = 195 − 47.5 = 147.5

EMV(C) = 0.8(220) + 0.2(−230) = 176 − 46 = 130

Optimal EMV = £154, so Jenny's best course of action is option A

(b) Replace each pay-off by its utility, $u(x) = 1 - e^{-x/400}$:

$u(350) = 1 - e^{-350/400} = 0.58313...$ $u(-140) = 1 - e^{140/400} = -0.41906...$

$u(260) = 0.47795...$ $u(-190) = -0.60801...$

$u(220) = 0.42305...$ $u(-230) = -0.77713...$

Expected utilities:

A: 0.6(0.58313...) + 0.4(−0.41906...) = 0.18225...

B: 0.75(0.47795...) + 0.25(−0.60801...) = 0.20646...

C: 0.8(0.42305...) + 0.2(−0.77713...) = 0.18301...

Optimal expected utility = 0.206 (option B), so using expected utility as the criterion Jenny should choose option B.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

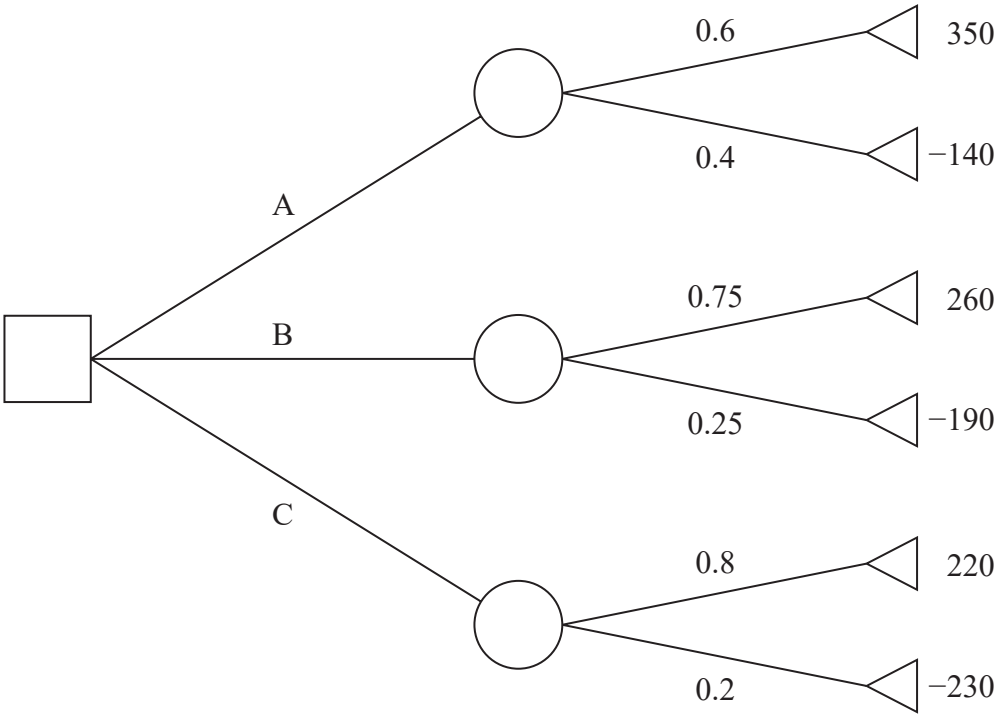


Figure 1

(Total for Question 2 is 7 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

3.

	P	Q	R	Supply
A	25	24	17	42
B	7	12	14	68
C	13	11	20	25
D	16	15	13	40
Demand	59	72	44	

Table 1

	P	Q	R
A	42 − θ		θ
B	17 + θ	51 − θ	
C		21 + θ	4 − θ
D			40

(a) Entering cell AR. Route:

AR(+) CR(−) CQ(+) BQ(−) BP(+) AP(−)

θ = min(4, 51, 42) = 4, so CR exits

	P	Q	R
A	38		4
B	21	47	
C		25	
D			40

Improved solution:

AP 38, AR 4, BP 21, BQ 47,

CQ 25, DR 40

	P	Q	R
A	X	−6	X
B	X	X	15
C	7	X	22
D	−5	−11	X

(b) Shadow costs (cost = $R_i + K_j$):

$R_A = 0$, $R_B = -18$, $R_C = -19$, $R_D = -4$

$K_P = 25$, $K_Q = 30$, $K_R = 17$

Improvement indices $I_{ij} = c_{ij} - R_i - K_j$:

AQ −6, BR 15, CP 7, CR 22, DP −5, DQ −11

Most negative is DQ (−11): DQ enters

	P	Q	R
A	38 − θ		4 + θ
B	21 + θ	47 − θ	
C		25	
D		θ	40 − θ

Route: DQ(+) DR(−) AR(+) AP(−)

BP(+) BQ(−)

θ = min(40, 38, 47) = 38

Entering cell DQ, exiting cell AP

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

	P	Q	R
A			42
B	59	9	
C		25	
D		38	2

	P	Q	R
A	11	5	X
B	X	X	4
C	7	X	11
D	6	X	X

	P	Q	R
A			
B			
C			
D			

	P	Q	R
A			
B			
C			
D			

	P	Q	R
A			
B			
C			
D			

Improved solution after (b):

AR 42, BP 59, BQ 9, CQ 25,

DQ 38, DR 2

(c) Shadow costs:

$R_A = 0$, $R_B = -7$, $R_C = -8$, $R_D = -4$

$K_P = 14$, $K_Q = 19$, $K_R = 17$

Improvement indices (table above):

AP 11, AQ 5, BR 4, CP 7, CR 11, DP 6

There are no negative improvement indices, so the solution is optimal.

Question 3 continued

(d) Let x_{ij} be the number of units transported from supply point i to sales point j, where $i \in \{A, B, C, D\}$, $j \in \{P, Q, R\}$ and $x_{ij} \geq 0$

Minimise $C = 25x_{AP} + 24x_{AQ} + 17x_{AR} + 7x_{BP} + 12x_{BQ} + 14x_{BR} + 13x_{CP} + 11x_{CQ} + 20x_{CR} + 16x_{DP} + 15x_{DQ} + 13x_{DR}$

Subject to:

$x_{AP} + x_{AQ} + x_{AR} \leq 42$

$x_{BP} + x_{BQ} + x_{BR} \leq 68$

$x_{CP} + x_{CQ} + x_{CR} \leq 25$

$x_{DP} + x_{DQ} + x_{DR} \leq 40$

$x_{AP} + x_{BP} + x_{CP} + x_{DP} \geq 59$

$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} \geq 72$

$x_{AR} + x_{BR} + x_{CR} + x_{DR} \geq 44$

$x_{ij} \geq 0$ for all i, j

(e) The Simplex algorithm cannot be used because not all of the constraints are in the form $\Sigma x \leq k$ (k a positive constant): the demand constraints are of the form $\Sigma x \geq k$.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 16 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

4.

(a) CF $u_n = A(2)^n + B(-1)^n$, so the auxiliary equation has roots 2, −1:

$(m - 2)(m + 1) = 0 \Rightarrow m^2 - m - 2 = 0$

Comparing with $m^2 + \alpha m + \beta = 0$: $\alpha = -1$, $\beta = -2$

(b) Particular solution: try $u_n = \lambda(-3)^n$

$\lambda(-3)^{n+2} - \lambda(-3)^{n+1} - 2\lambda(-3)^n = 20(-3)^n$

Divide by $(-3)^n$: $9\lambda + 3\lambda - 2\lambda = 20 \Rightarrow 10\lambda = 20 \Rightarrow \lambda = 2$

General solution: $u_n = A(2)^n + B(-1)^n + 2(-3)^n$

$u_0 = A + B + 2$, $u_1 = 2A - B - 6$

$2u_0 = u_1$: $2A + 2B + 4 = 2A - B - 6 \Rightarrow 3B = -10 \Rightarrow B = -10/3$

$u_4 = 164$: $16A + B + 2(81) = 164 \Rightarrow 16A + B = 2$

$16A - 10/3 = 2 \Rightarrow 16A = 16/3 \Rightarrow A = 1/3$

$\therefore u_n = \frac{1}{3}(2)^n - \frac{10}{3}(-1)^n + 2(-3)^n$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 8 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

5.

(a) The source node is C

(b) All the arcs incident to G (DG, EG and HG) flow into G, so nothing can flow out of G: G must be the sink.

(c) Capacity of C_1 = 10 + 2 − 1 + 6 + 8 + 1 − 0 = 26 litres per second
(HG 10, EG 2, DE −1, CD 6, CA 8, CB 1, BF −0)

(d)(i) The only arcs into J are EJ and FJ, with total lower capacity 3 + 2 = 5. The only arc out of J is JH, with upper capacity 5, so the flow in JH must be 5: JH is at its upper capacity.

(ii) The only arcs out of D are DG and DE, with total upper capacity 7 + 2 = 9. The arcs into D are AD and CD, with total lower capacity 5 + 4 = 9. Flow in = flow out ≤ 9, so AD and CD must be at their lower capacities (5 and 4).

(e) See Diagram 1 (flow of 18 litres per second)

(f) Consider the cut through DG, DE, CE, CF, CB and BA, which separates {C, A, D} from {B, E, F, G, H, J}.
Capacity = 7 + 2 + 3 + 6 + 1 − 1 = 18
(BA flows from the sink side to the source side, so its lower capacity 1 is subtracted.)
The flow in (e) is 18 = capacity of this cut, so by the maximum flow minimum cut theorem the flow of 18 is maximal.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

(e) Flow of 18 litres per second shown on Diagram 1 below:

out of C: CA 4 + CD 4 + CB 1 + CE 3 + CF 6 = 18

into G: DG 7 + EG 1 + HG 10 = 18

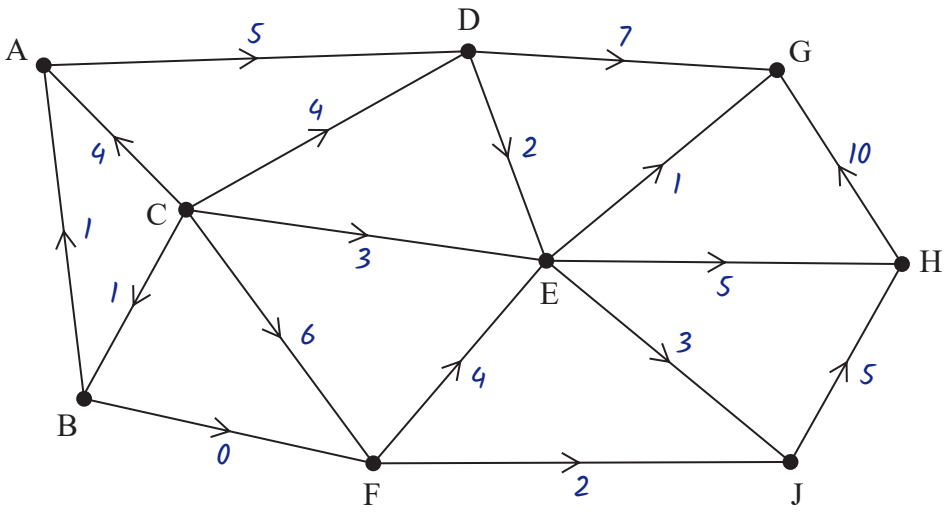


Diagram 1

(Total for Question 5 is 10 marks)

6.

		Player B		
		Option X	Option Y	Option Z
Player A	Option Q	1	5	3
	Option R	4	−3	1
	Option S	2	−4	−2
	Option T	3	−2	0

(a) Option R dominates option S, because $4 > 2$, $-3 > -4$ and $1 > -2$,
so player A would never choose S and it can be removed.

(b) Reduced game: row minima Q: 1, R: −3, T: −2; max = 1
column maxima X: 4, Y: 5, Z: 3; min = 3
Row maximin (1) ≠ column minimax (3), so there is no stable solution.

(c) Each expression is A's augmented expected pay-off if B plays X, Y
or Z. For each choice of p values, B will choose the option giving A
the smallest of these, so V (the worst possible augmented expected
pay-off, which A wants to maximise) is less than or equal to each one.

(d) The probabilities must sum to 1. Writing this as the inequality
$p_1 + p_2 + p_3 \leq 1$ (form $\Sigma \leq k$, with a slack variable) enables the
Simplex algorithm to pivot on a row that will increase the value of P.

(e) $p_2 = 1 - 7/11 - 0 = 4/11$
$V \leq 4(7/11) + 7(4/11) + 6(0) = 56/11$
$V \leq 8(7/11) + 0 = 56/11$
$V \leq 6(7/11) + 4(4/11) + 3(0) = 58/11$
So $V = 56/11$

Value of the game to player A $= \frac{56}{11} - 3 = \frac{23}{11}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

(f) Using the value of the game to A, 23/11 (B wants to hold A to this):

A plays Q: $q_1 + 5q_2 + 3q_3 = 23/11$

A plays R: $4q_1 - 3q_2 + q_3 = 23/11$

$q_1 + q_2 + q_3 = 1$

Substituting $q_3 = 1 - q_1 - q_2$:

$-2q_1 + 2q_2 + 3 = 23/11 \Rightarrow q_1 - q_2 = 5/11$

$3q_1 - 4q_2 + 1 = 23/11 \Rightarrow 3q_1 - 4q_2 = 12/11$

$\Rightarrow q_2 = 3/11, \; q_1 = 8/11, \; q_3 = 0$

(check A plays T: $3(8/11) - 2(3/11) + 0 = 18/11 \leq 23/11$)

Player B should play option X with probability $\frac{8}{11}$, option Y with probability $\frac{3}{11}$ and never play option Z.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 14 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

7.

Stage	State	Action	Destination	Value
Trainers	0	0	0	0
	1	1	0	50
	2	2	0	90
	3	3	0	170
	4	4	0	225
	5	5	0	295
Sandals	0	0	0	0
	1	1	0	70 + 0 = 70*
		0	1	0 + 50 = 50
	2	2	0	110 + 0 = 110
		1	1	70 + 50 = 120*
		0	2	0 + 90 = 90
	3	3	0	165 + 0 = 165
		2	1	110 + 50 = 160
		1	2	70 + 90 = 160
		0	3	0 + 170 = 170*
	4	4	0	245 + 0 = 245*
		3	1	165 + 50 = 215
		2	2	110 + 90 = 200
		1	3	70 + 170 = 240
		0	4	0 + 225 = 225
	5	5	0	300 + 0 = 300*
		4	1	245 + 50 = 295
		3	2	165 + 90 = 255
		2	3	110 + 170 = 280
		1	4	70 + 225 = 295
		0	5	0 + 295 = 295
High heels	5	5	0	305 + 0 = 305
		4	1	235 + 70 = 305
		3	2	x + 120
		2	3	115 + 170 = 285
		1	4	75 + 245 = 320*
		0	5	0 + 300 = 300

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

Stage	State	Action	Destination	Value

(a) Final stage, state S: max(305, 305, x + 120, 285, 320, 300)

The two possible values of the maximum total annual profit are

320 (i.e. £320 000), when $x \leq 200$,

or $x + 120$ (in £1000s, i.e. £1000$(x + 120)$), when $x > 200$.

(b) Given the maximum is £320 000 (so $x \leq 200$):

Trainers 0, Sandals 4, High heels 1 $(0 + 245 + 75 = 320)$

or, if $x = 200$, also:

Trainers 1, Sandals 1, High heels 3 $(50 + 70 + 200 = 320)$

So the manufacturer should export 4 batches of sandals and 1 of high heels, or (if $x = 200$) 1 of trainers, 1 of sandals and 3 of high heels.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA