



Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further
Mathematics

Paper 9FM0/4B

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - $\square$ The second mark is dependent on gaining the first mark
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4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Qu	Scheme	Marks	AO
1	$H_0: \sigma_a^2 = \sigma_b^2 \quad H_1: \sigma_a^2 > \sigma_b^2$	B1	2.5
	$s_a^2 = \frac{145\,496}{6} = [24\,249.33\dots] \quad s_b^2 = \frac{56\,364.4}{9} = [6262.711\dots]$	M1	1.1b
	$F_{6,9} = \frac{"24\,249.33\dots"}{"6262.711\dots"}$	M1	2.1
	$= 3.872\dots$	A1	1.1b
	$F_{6,9}(5\% \text{ one-tail}) \text{ c.v.} = 3.37$	B1	1.1b
	Significant, there is evidence to support Gina's belief	A1	2.2b
		(6)	
(6 marks)			
Notes			
	1 st B1 for both hypotheses in terms of σ . 1 st M1 for at least one of the s^2 calculations correctly attempted NB $s_a = 155.72\dots$ accept awrt 156 and $s_b = 79.137\dots$ accept awrt 79.1 2 nd M1 for a correct calculation of the test statistic (ft their s^2) 1 st A1 for awrt 3.87 2 nd B1 for correct cv awrt 3.37 2 nd A1 for a correct conclusion mentioning Gina's belief (o.e.)		

Qu	Scheme	Marks	Grade	AO
2(a)	$\bar{x} = 444$	M1	Low	2.1
	$s_x^2 = \frac{1\,577\,314 - 8 \times 444^2}{7} = \frac{226}{7} = 32.2857..$	A1	Low	1.1b
	$t_7(5\%)$ 2-tail cv = 2.365	B1	Low	1.1b
	95% CI for μ is: $444 \pm 2.365 \times \sqrt{\frac{32.2857...}{8}}$	M1	Med	2.1
	$= (439.248..., 448.75...) = \text{awrt } \underline{(439, 449)}$	A1	Med	1.1b
(b)	440 is in CI so the average contents statement is OK	(5) B1 (1) (6 marks)	High	2.2b
Notes				
(a)	1 st M1 for finding mean and attempting s^2 1 st A1 for correct mean and s^2 (accept awrt 3sf) B1 for a correct cv of 2.365 or better 2 nd M1 for use of correct formula, ft their mean, s_x and cv for t (use of 1.96 is M0) 2 nd A1 for awrt (439,449)			
(b)	1 st B1 for correct statement about 440 and interval and conclusion			

Qu	Scheme	Marks	AO
3	I (Is feasible as a residual plot but) probably a non-linear relationship Since the residuals are not randomly scattered about zero	B1 B1	2.2b 2.4
	II Impossible as a residual plot Since the residuals do not sum to zero	B1 B1	2.2a 2.4
	III (Is feasible as a residual plot) and probably a linear relationship Since the points are randomly scattered about zero	B1 B1 (6)	2.2b 2.4
		(6 marks)	
Notes			
I	1 st B1 for stating possibly non-linear (allow a suitable sketch) 2 nd B1 for a suitable comment (e.g. follow a systematic pattern)		
II	1 st B1 for stating not feasible as a residual plot 2 nd B1 for a correct reason		
III	1 st B1 for stating probably a linear relationship 2 nd B1 for a suitable supporting reason		

Qu	Scheme	Marks	AO
4 (a)	$X \sim B(n, p)$ so $E(X) = np$ and $Y \sim B(m, p)$ so $E(Y) = mp$	M1	3.3
	$E(S) = \frac{E(X+Y)}{n+m} = \frac{np+mp}{n+m} = p$ so S is unbiased	M1	3.4
	$E(T) = \frac{1}{2} \left[\frac{E(X)}{n} + \frac{E(Y)}{m} \right] = \frac{1}{2} \left[\frac{np}{n} + \frac{mp}{m} \right] = \frac{1}{2} \times 2p = p$ so T is unbiased	A1cso	1.1b
		(3)	
(b)	$\text{Var}(S) = \frac{np(1-p) + mp(1-p)}{(n+m)^2} = \frac{p(1-p)}{n+m}$	M1	2.1
	$\text{Var}(T) = \frac{1}{4} \left[\frac{np(1-p)}{n^2} + \frac{mp(1-p)}{m^2} \right] = \frac{p(1-p)(m+n)}{4nm}$	A1	1.1b
	$\text{Var}(S) < \text{Var}(T) \Rightarrow \frac{p(1-p)}{n+m} < \frac{p(1-p)(m+n)}{4mn} \Leftrightarrow 4mn < (n+m)^2$	M1	1.1b
	$\Leftrightarrow 0 < m^2 + 2mn + n^2 - 4mn \Leftrightarrow 0 < (m-n)^2$ So S always has the smaller variance and is the better estimator	A1cso	2.2a
		(4)	(7 marks)
Notes			
(a)	1 st M1 for selecting correct models for X and Y 2 nd M1 for using these models to show that either S or T is unbiased A1cso for correctly showing that both are unbiased.		
(b)	1 st M1 for a correct attempt at $\text{Var}(S)$ or $\text{Var}(T)$ (need not be simplified) 1 st A1 for both correct variances 2 nd M1 for a correct inequality in m and n and a first step to clear denominators 2 nd A1cso for a correct proof and conclusion		

Qu	Scheme	Marks	AO
5(a)	$\int (1 - \cos x) \, dx = [x - \sin x]$	M1	1.1b
	Use of correct limits and $\int f(x) \, dx = 1 \Rightarrow 2\pi - 0 - 0 = 1$	M1	1.1b
	so $k = \frac{1}{2\pi}$ (*)	A1*cs	1.1b
		(3)	
	(b) $E(X) = \pi$ (symmetry) so $\mu = \pi$ so $f(\mu) = \frac{1}{2\pi}(1 - \cos \pi) = \frac{1}{\pi}$	B1	2.2a
	$\frac{1}{\sigma\sqrt{2\pi}} = \frac{1}{\pi}$; so $\sigma = \sqrt{\frac{\pi}{2}}$	M1; A1	1.1b 1.1b
		(3)	
	(c) $P\left(\frac{\pi}{2} < X < \frac{3\pi}{2}\right) = \frac{1}{2\pi} [x - \sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \frac{1}{2\pi} \left[\left(\frac{3\pi}{2} - -1\right) - \left(\frac{\pi}{2} - 1\right) \right]$	M1	3.4
	$= \frac{2 + \pi}{2\pi} (= 0.81830\dots)$	A1	1.1b
	$P\left(\frac{\pi}{2} < Y < \frac{3\pi}{2}\right) = 0.7899\dots$	B1	1.1b
	So error is $0.81830\dots - 0.7899\dots = 0.0284$	A1	1.1b
	(4)		
		(10 marks)	
Notes			
(a)	1 st M1 attempt to integrate $(1 - \cos x)$ – one correct term 2 nd M1 for use of correct limits and correct method for k A1* cs use of $\int f(x) \, dx = 1$ seen and no incorrect working seen		
(b)	B1 for correctly deducing the value of $f(\mu)$ M1 for a correct equation for σ – ft their value for $f(\mu)$ [condone for sight of correct $g(\mu)$] A1 for $\sqrt{\frac{\pi}{2}}$ or exact equivalent		
(c)	M1 for a correct attempt to find prob – some correct integration and use of limits 1 st A1 for a correct answer (exact or 0.818.. or better) B1 for a correct probability from their calculator i.e. 0.7899 or better accept 0.79 2 nd A1 for 0.0284 or better		

Qu	Scheme	Marks	AO
6(a)	From CI $\bar{x} = \frac{1.193+1.367}{2} = 1.28$	B1	1.1b
	$\text{or width} = 1.367 - 1.193 = 0.174$		
	$1.367 = "1.28" \pm 2.064 \times \frac{s}{\sqrt{25}} \text{ or } "0.174" = 2 \times 2.064 \times \frac{s}{\sqrt{25}}$	M1;A1	3.4 1.1b
	$\Rightarrow s = 0.210755...$	A1	1.1b
	$H_0 : \sigma = 0.175 \quad H_1 : \sigma \neq 0.175$	B1	2.5
	$\chi_{24}^2 = \frac{24s^2}{\sigma^2} =, \quad 34.8092...$ awrt 34.8	M1, A1	3.3 1.1b
	$\chi_{24}^2 (10\%)$ 2-tail CR is $\chi_{24}^2 < \underline{13.848}$ or $\chi_{24}^2 > \underline{36.415}$	B1	2.1
	34.8 is not significant so insufficient evidence that $\sigma \neq 0.175$	A1	2.2b
		(9)	
(b)	$"1.28" \pm z \times \frac{0.175}{\sqrt{25}}$	M1	3.3
	$z = 1.96$	B1	1.1b
	$= (1.211..., 1.349...) = \text{awrt } \underline{(1.21, 1.35)}$	A1	1.1b
		(3)	
		(12 marks)	
Notes			
(a)	1 st B1 for finding mean from CI or calculation of width of CI 1 st M1 for using the given t model to form an equation in s . (Allow t for 2.064 where $2 < t < 3$) 1 st A1 for correct use of $t_{24} = 2.064$ 2 nd A1 for $s = 0.21$ or better 2 nd B1 for correct hypotheses in terms of σ . 2 nd M1 for selecting the appropriate model for this test 3 rd A1 for test statistic awrt 34.8 3 rd B1 for at least one correct critical value 4 th A1 for a correct conclusion confirming that assuming st. dev = 0.175 is OK		
(b)	M1 for use of correct formula with $1.6 < z < 2$ (ft $\bar{x}$ if found in (a)) B1 for $z = 1.96$ or better used A1 for an interval awrt (1.21, 1.35)		

Qu	Scheme	Marks	AO
7(a)	$X = L_1 + L_2 + L_3 \sim N(594, \sqrt{75}^2)$ $Y = S_1 + \dots + S_8 \sim N(592, \sqrt{72}^2)$ $P(X > Y) = P(D > 0) \text{ where } D \sim N(2, \sqrt{147}^2)$ $= 0.56551\dots \quad \text{awrt } \underline{0.566}$	M1 A1 A1 M1 A1ft A1 (6)	3.3 1.1b 1.1b 2.1 1.1b 3.4
(b)	$W = L - \frac{8}{3}S \Rightarrow W \sim N\left(\frac{2}{3}, 25 + \frac{64}{9} \times 9\right) = N\left(\frac{2}{3}, \sqrt{89}^2\right)$ $P(W > 0) = 0.528168\dots \quad \text{awrt } \underline{0.528}$	M1 M1,A1 M1A1 (5)	3.3 2.1,1.1b 3.4,1.1b
(c)	$F = L_1 + \dots + L_5 \sim N(990, \sqrt{125}^2)$ $P(F < 1000) = 0.814455\dots \text{ (o.e.)}$ $E(\text{cost of Rosa's plan}) = 430 \times "0.814\dots" + 400 \times (1 - "0.814\dots")$ $= \text{£ } 424.43$ Buying 14 small panels cost $14 \times 30 = \text{£}420$ So Rosa's plan is likely to be <u>more expensive</u>	M1 A1 A1 M1 A1 A1 (6)	3.1b 1.1b 3.4 2.1 1.1b 3.2a
(17 marks)			
Notes			
(a)	1 st M1 for an attempt at X or Y – expression or implied by one correct distribution 1 st A1 for a correct distribution for X <u>or</u> implied by $E(D) = 2$ 2 nd A1 for a correct distribution for Y <u>or</u> implied by $\text{Var}(D) = 147$ 2 nd M1 for a correct strategy – attempt $X - Y$ and $P(D > 0)$ statement 3 rd A1ft for a correct distribution for D ft their X and Y 4 th A1 for awrt 0.566		
(b)	1 st M1 for attempt at a correct model (normal and mean) 2 nd M1 for correct expression for variance of their model provided of the form $L - kS$ <u>or</u> $kL - S$ 1 st A1 for a fully correct distribution 3 rd M1 for a correct probability statement using their distribution 2 nd A1 for awrt 0.528		
(c)	1 st M1 for a correct start to solve the problem attempt at F and correct mean 1 st A1 for a correct distribution 2 nd A1 for using this model to find $P(F < 1000) = \text{awrt } 0.814$ or $P(F > 1000) = \text{awrt } 0.186$ 2 nd M1 for a correct strategy to solve the problem i.e. attempt at expected cost ft their prob 3 rd A1 for awrt £424 4 th A1 for a correct conclusion must have comparison with £420 and reject Rosa's plan [(c) is an extended problem and a 3.1, 3.2 question]		

Qu	Scheme	Marks	AO
8(a)	$P(\pi X^2 > 10) \Rightarrow P\left(X > \sqrt{\frac{10}{\pi}}\right)$	M1	3.1a
	$= \frac{\pi - \sqrt{\frac{10}{\pi}}}{\pi}$	M1	2.1
	$= 0.43209... = \text{awrt } \underline{\underline{0.432}}$	A1	1.1b
		(3)	
	(b) $P(\text{area} > \text{median}) = 0.5$; since (a) < 0.5 therefore <u>median < 10</u>	B1	2.2a
		(1)	
	(c) Area of triangle = $0.5x^2 \sin x$	M1	3.1a
	$E(\text{area}) = \int_{[0]}^{[\pi]} \frac{1}{\pi} \frac{1}{2} x^2 \sin x \, dx$	M1	1.1b
	$= \frac{1}{2\pi} \int_{[0]}^{[\pi]} x^2 \, d(-\cos x) = \frac{1}{2\pi} \left\{ \left[-x^2 \cos x \right]_{[0]}^{[\pi]} - \int_{[0]}^{[\pi]} -2x \cos x \, dx \right\}$	M1	2.1
	$= \left\{ \left[\frac{-x^2 \cos x}{2\pi} \right]_{[0]}^{[\pi]} \right\} + \left[\frac{x \sin x}{\pi} \right]_{[0]}^{[\pi]} - \frac{1}{\pi} \int_{[0]}^{[\pi]} \sin x \, dx$	M1	1.1b
	A1	1.1b	
	$= \frac{\pi^2}{2\pi} - 0 + 0 - 0 + \left(-\frac{1}{\pi} \right) - \left(\frac{1}{\pi} \right) = \frac{\pi}{2} - \frac{2}{\pi}$	M1	1.1b
	A1	1.1b	
	(7)		
	(11 marks)		
	Notes		
(a)	1 st M1	reduce the problem to a probability about X	
	2 nd M1	for use of the uniform distribution (a correct expression ft their value 1.784..)	
	A1	for awrt 0.432	
(b)	B1	for statement that median < 10 supported by argument about answer to (a) being < 0.5	
ALT		Median area is given by $\pi \times \left(\frac{\pi}{2} \right)^2 = 7.751... < 10$ so median < 10	
(c)	1 st M1	for a correct expression for area in terms of x	
	2 nd M1	for realisation that need to use $E(g(X))$ formula and a correct expression (ignore limits)	
	3 rd M1	for attempt to use integration by parts	
	4 th M1	for a 2 nd use of integration by parts	
	1 st A1	for correct integration (ignore limits)	
	5 th M1	for clear use of the correct limits	
	2 nd A1	for $\frac{\pi}{2} - \frac{2}{\pi}$	
	[(c) is an extended problem and also involves work from pure for the integration]		