Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Monday 23 June 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/4C**

Further Mathematics

Advanced

PAPER 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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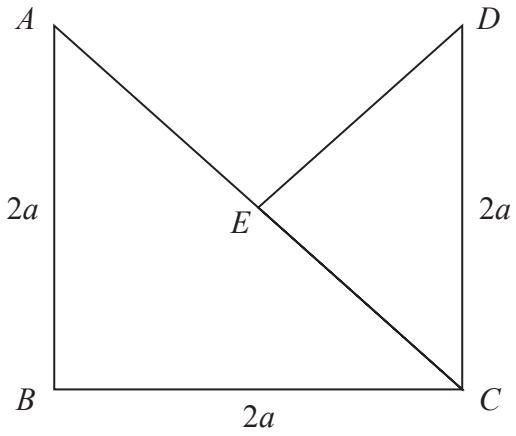


Figure 1

A uniform rod is cut into six pieces. The pieces are used to form the framework $ABCDE$ shown in Figure 1.

- All the pieces of the framework lie in the same plane.
- $AB = BC = CD = 2a$
- $AE = EC = ED = \sqrt{2}\,a$
- Point E is the midpoint of AC
- Angle ABC = angle $BCD = 90°$

The distance of the centre of mass of the framework from AB is d

(a) Show that $d = \dfrac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}}a$

(4)

The mass of the framework is M. A particle of mass kM is attached to the framework at B

The distance of the centre of mass of the loaded framework from AB is a

(b) Find the value of k

(3)

(a) Mass of each rod ∝ its length. Distances of midpoints from AB:

AB: 0, BC: a, CD: 2a, AC (= AE + EC = 2√2a): a, ED: 1.5a

Moments about AB:

$2a \times 0 + 2a \times a + 2a \times 2a + 2\sqrt{2}a \times a + \sqrt{2}a \times \frac{3a}{2} = d(6a + 3\sqrt{2}a)$

$6a^2 + \frac{7\sqrt{2}}{2}a^2 = d(6 + 3\sqrt{2})a$

$d = \frac{6 + 3.5\sqrt{2}}{6 + 3\sqrt{2}}a \Rightarrow d = \frac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}}a$ as required

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Question 1 continued

(b) Moments about AB for the loaded framework (particle at B is on AB):

$$Mg \times \frac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}}\, a + kMg \times 0 = (M + kM)g \times a$$

$$1 + k = \frac{12 + 7\sqrt{2}}{12 + 6\sqrt{2}} \quad \Rightarrow \quad k = \frac{12 + 7\sqrt{2} - 12 - 6\sqrt{2}}{12 + 6\sqrt{2}}$$

$$k = \frac{\sqrt{2}}{12 + 6\sqrt{2}} = \frac{\sqrt{2} - 1}{6} \quad (= 0.069036\ldots)$$

$$k = 0.0690$$

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 7 marks)

2.

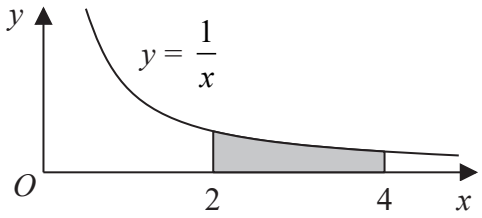


Figure 2

The shaded region shown in Figure 2 is bounded by the curve with equation $y = \frac{1}{x}$,

the line with equation $x = 2$, the line with equation $x = 4$, and the x-axis.
This region is rotated through 360° about the x-axis to form a solid.

This solid is used to model a uniform solid pedestal of height 2 m, upper radius 0.5 m and base radius 0.25 m.

Given that the volume of the pedestal is $\frac{\pi}{4}\text{m}^3$

(a) show that the centre of mass of the pedestal is $(4 - 4\ln 2)$ m from its base.

(4)

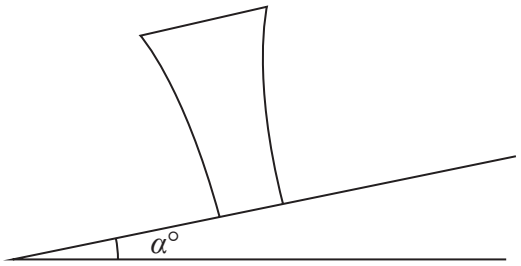


Diagram not drawn to scale

Figure 3

The pedestal is placed on a rough plane that is inclined at an angle $\alpha°$ to the horizontal.
The plane base of the pedestal is in contact with the inclined plane, as shown in Figure 3.
The inclined plane is sufficiently rough to prevent the pedestal from sliding.

Given that the pedestal is on the point of toppling,

(b) find the value of α

(3)

(a) Moments about O (axis along the x-axis):

$\frac{\pi}{4}\ \bar{x} = \int_2^4 \pi y^2 x\ dx = \pi \int_2^4 \frac{1}{x^2} \times x\ dx = \pi \int_2^4 \frac{1}{x}\ dx$

$= \pi [\ln x]_2^4 = \pi(\ln 4 - \ln 2) = \pi \ln 2$

$\bar{x} = \frac{\pi \ln 2}{\pi/4} = 4 \ln 2$ (distance from O)

The base (radius 0.25 m) is at $x = 4$, so distance from the base

$= 4 - 4 \ln 2$ m as required

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Question 2 continued

(b) On the point of toppling, G is vertically above the lowest point of the edge of the base (base radius 0.25 m):

$$\tan \alpha^\circ = \frac{0.25}{4 - 4 \ln 2} = 0.20368...$$

$\alpha = 11.5126...$

$\alpha = 11.5$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

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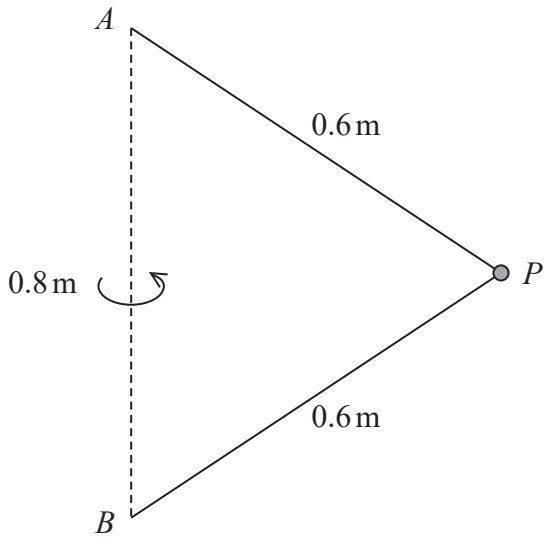


Figure 4

A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to the fixed point A.
A second light inextensible string of length 0.6 m has one end attached to P and the other end attached to the fixed point B.
The point B is vertically below A such that $AB = 0.8$ m, as shown in Figure 4.

The particle moves in a horizontal circle with constant angular speed ω radians per second, with both strings taut.

Given that the tension in the string from P to A is twice the tension in the string from P to B, find the value of ω.

(7)

Let θ be the angle each string makes with the horizontal; $T_B = T$, $T_A = 2T$

$\sin\theta = \frac{0.4}{0.6} = \frac{2}{3}$, $\cos\theta = \frac{\sqrt{5}}{3}$, radius $r = 0.6\cos\theta = \frac{\sqrt{5}}{5}$ m

Resolve vertically: $2T\sin\theta = 0.2g + T\sin\theta$

$T\sin\theta = 0.2g \Rightarrow T = \frac{0.2g}{2/3} = 0.3g$ (= 2.94 N)

Equation of motion horizontally (towards the centre):

$0.2r\omega^2 = 2T\cos\theta + T\cos\theta = 3T\cos\theta$

$0.2 \times \frac{\sqrt{5}}{5}\omega^2 = 3T \times \frac{\sqrt{5}}{3} \Rightarrow \omega^2 = 25T$

$\omega^2 = 25 \times 0.3g = 7.5g = 73.5$

$\omega = 8.5732...$

$\omega = 8.57$ rad s^{-1}

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Question 3 continued

(Total for Question 3 is 7 marks)

4.

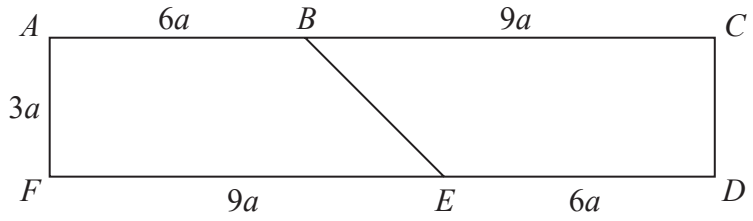


Figure 5

The uniform rectangular lamina $ACDF$ shown in Figure 5 has $AC = FD = 15a$ and $AF = CD = 3a$. The point B on AC is such that $AB = 6a$. The point E on FD is such that $ED = 6a$.

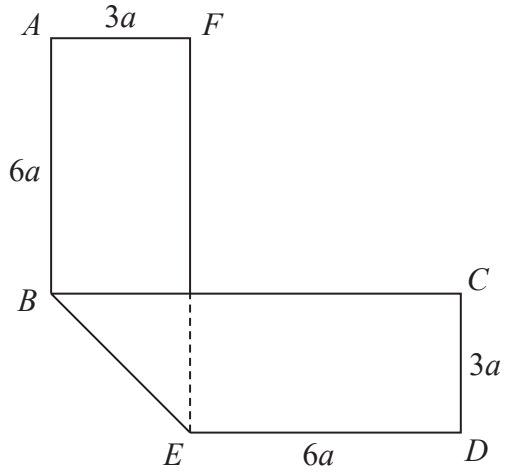


Figure 6

The rectangular lamina is folded along BE to form the folded lamina shown in Figure 6. The folded lamina has AB perpendicular to BC and the two sections, $ABEF$ and $BEDC$, of the lamina lie in the same plane.

(a) Show that the distance of the centre of mass of the folded lamina from EF is $\frac{2}{5}a$.

(5)

(b) Explain why the centre of mass of the folded lamina lies on the perpendicular bisector of BE.

(1)

The folded lamina is freely suspended from F and hangs in equilibrium.

(c) Find the size of the angle between FE and the downward vertical.

(3)

(a) Pieces (mass ∝ area) and distances from EF (BC side positive):

Left rectangle (3a × 6a): mass 18, distance −1.5a

2 triangles (each $\frac{1}{2}$ × 3a × 3a): mass 2 × 4.5 = 9, distance −a

Right rectangle (6a × 3a): mass 18, distance 3a; Total: mass 45, distance d

Moments about EF:

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Question 4 continued

$18 \times (-1.5a) + 9 \times (-a) + 18 \times 3a = 45d$

$-27a - 9a + 54a = 45d \quad \Rightarrow \quad 18a = 45d$

$d = \frac{18a}{45} = \frac{2}{5}a$ as required

(b) The folded lamina is symmetrical about the perpendicular bisector of BE (reflecting in it swaps ABEF and CDEB), so the centre of mass lies on this line of symmetry.

(c) Axes as in Figure 6: F origin, x towards C, y upwards, EF on $x = 0$

perpendicular bisector of BE: B$(-3a, -6a)$, E$(0, -9a)$, midpoint $(-1.5a, -7.5a)$

line $y = x - 6a$; G is at $x = 0.4a$, so vertical distance of G below F

$= 6a - \frac{2}{5}a = \frac{28a}{5}$

G hangs vertically below F, so FG is vertical:

$\tan\theta = \frac{2a/5}{28a/5} = \frac{2}{28} = \frac{1}{14}$

$\theta = 4.0856...°$

Angle between FE and the downward vertical = 4.09°

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5. A car of mass 1000 kg is moving along a straight horizontal road.
The engine of the car produces a constant driving force of 800 N.
At time t seconds, $t \geqslant 0$, the speed of the car is $v\,\text{m s}^{-1}$ and the resistance to the motion of the car has magnitude $2v^2$ N.

The time taken for the speed of the car to increase from $5\,\text{m s}^{-1}$ to $15\,\text{m s}^{-1}$ is T seconds.

(a) Show that $T = \frac{25}{2}\ln\left(\frac{21}{5}\right)$

(7)

When $t = 0$, $v = 2$

(b) Show that $\frac{20+v}{20-v} = pe^{qt}$ where p and q are rational numbers to be found.

(2)

(c) Hence explain why $v < 20$ for all values of $t \geqslant 0$

(1)

(a) Equation of motion: $800 - 2v^2 = 1000\frac{dv}{dt}$

$\int \frac{1000}{800-2v^2}\,dv = \int 1\,dt \Rightarrow \int \frac{500}{400-v^2}\,dv = \int 1\,dt$

$\frac{500}{(20-v)(20+v)} = \frac{25}{2}\left(\frac{1}{20-v} + \frac{1}{20+v}\right)$

$\frac{25}{2}\left(\ln(20+v) - \ln(20-v)\right) = \frac{25}{2}\ln\frac{20+v}{20-v} = t + C$

$T = \left[\frac{25}{2}\ln\frac{20+v}{20-v}\right]_5^{15} = \frac{25}{2}\left(\ln\frac{35}{5} - \ln\frac{25}{15}\right)$

$T = \frac{25}{2}\ln\left(\frac{35}{5}\times\frac{15}{25}\right) = \frac{25}{2}\ln\left(\frac{21}{5}\right)$ as required

(b) $t = 0$, $v = 2$: $\frac{25}{2}\ln\frac{22}{18} = C \Rightarrow C = \frac{25}{2}\ln\frac{11}{9}$

$\ln\frac{20+v}{20-v} = \frac{2t}{25} + \ln\frac{11}{9}$

$\frac{20+v}{20-v} = \frac{11}{9}e^{2t/25}$ so $p = \frac{11}{9}$, $q = \frac{2}{25}$

(c) $\frac{11}{9}e^{2t/25} > 0$ for all t (exponential is always positive),

and $20 + v > 0$ $(v > 0)$, so $20 - v > 0$, i.e. $v < 20$ for all $t \geq 0$

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)

6. A uniform semicircular lamina has radius r and the midpoint of the diameter of the lamina is at O. The distance of the centre of mass of the lamina from O is d.

(a) Use algebraic integration to show that $d = \dfrac{4r}{3\pi}$

(4)

The uniform plane template T, shown shaded in Figure 7, is formed from a rectangle and a semicircular ring.
The rectangle has $AD = BC = 2a$ and $AB = DC = 8a$.
The midpoint of CD is O.
The semicircular ring has centre O, outer radius $OD = 4a$ and inner radius $OE = 2a$.
The template is modelled as a uniform lamina.

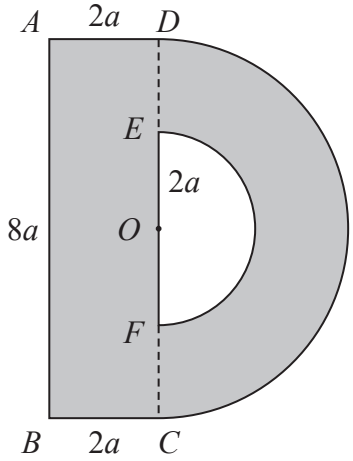


Figure 7

(b) Show that the distance of the centre of mass of T from DC is $\dfrac{32a}{3(8+3\pi)}$

(5)

The template is freely suspended from A. The weight of T is W newtons.
A horizontal force is applied to T at B so that T is held in equilibrium with AB vertical.
The force acts in the same vertical plane as T and has magnitude λW newtons.

(c) Find the value of λ

(3)

(a) Diameter along the y-axis, O at the origin: $y = \sqrt{(r^2 - x^2)}$,

strip of width δx has area $2y\,\delta x$ at distance x from O.

$\frac{1}{2}\pi r^2 \, d = \int_0^r 2xy\,dx = \int_0^r 2x\sqrt{(r^2 - x^2)}\,dx$

$= \left[-\frac{2}{3}(r^2 - x^2)^{3/2}\right]_0^r = 0 + \frac{2}{3}r^3 = \frac{2}{3}r^3$

$d = \frac{2r^3/3}{\pi r^2/2} = \frac{4r}{3\pi}$ as required

(b) Mass ratio ($\propto$ area) and distance from DC (on the side of the ring +):

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Question 6 continued

Rectangle: $16a^2$, $-a$; large semicircle: $8\pi a^2$, $\frac{4(4a)}{3\pi} = \frac{16a}{3\pi}$;

small semicircle: $2\pi a^2$, $\frac{8a}{3\pi}$; template T: $(16 + 6\pi)a^2$, $\bar{x}$

Moments about DC:

$$8\pi \times \frac{16a}{3\pi} - 2\pi \times \frac{8a}{3\pi} - 16 \times a = (16 + 6\pi)\bar{x}$$

$$\frac{128a}{3} - \frac{16a}{3} - 16a = \frac{64a}{3} = (16 + 6\pi)\bar{x}$$

$$\bar{x} = \frac{64a}{3(16 + 6\pi)} = \frac{32a}{3(8 + 3\pi)} \quad \text{as required}$$

(c) AB vertical; horizontal distance of G from AB = $2a + \bar{x}$

Moments about A (force λW at B, AB = $8a$):

$$W\left(2a + \frac{32a}{3(8 + 3\pi)}\right) = \lambda W \times 8a$$

$$\lambda = \frac{1}{8}\left(2 + \frac{32}{3(8 + 3\pi)}\right) = \frac{9\pi + 40}{12(8 + 3\pi)} = 0.32651\ldots$$

$\lambda = 0.327$

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

7. A particle P of mass 0.4 kg is moving in a straight line with simple harmonic motion. The maximum speed of P is $5\,\text{m s}^{-1}$ and the maximum magnitude of acceleration of P is $12.5\,\text{m s}^{-2}$
The time taken for one complete oscillation is T seconds.

(a) Show that $T = \dfrac{4\pi}{5}$

(4)

(b) Find the time within each complete oscillation for which P is more than 0.5 m from the centre of the oscillation.

(4)

(c) Find the kinetic energy of P at the instant when P is 0.2 m from the centre of the oscillation.

(3)

(a) Maximum speed: $a\omega = 5$

Maximum acceleration: $a\omega^2 = 12.5$

$\omega = \dfrac{12.5}{5} = 2.5, \quad a = \dfrac{5}{2.5} = 2$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2.5} = \dfrac{4\pi}{5}$ as required

(b) $x = 2\cos\dfrac{5}{2}t$ ($t = 0$ at an end of the oscillation)

$x = 0.5$: $2\cos\dfrac{5}{2}t = 0.5 \Rightarrow t = \dfrac{2}{5}\cos^{-1}\dfrac{1}{4} = 0.52724\ldots$

P is more than 0.5 m from the centre for this time either side of each end, twice per oscillation:

Total time $= 4 \times \dfrac{2}{5}\cos^{-1}\dfrac{1}{4} = 2.1089\ldots$

$= 2.11$ s

(c) $v^2 = \omega^2(a^2 - x^2) = 2.5^2(2^2 - 0.2^2) = 6.25 \times 3.96 = 24.75$

$KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 0.4 \times 24.75 = \dfrac{99}{20}$

$KE = 4.95$ J

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

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8. **In this question use $g = 10\,\text{m s}^{-2}$**

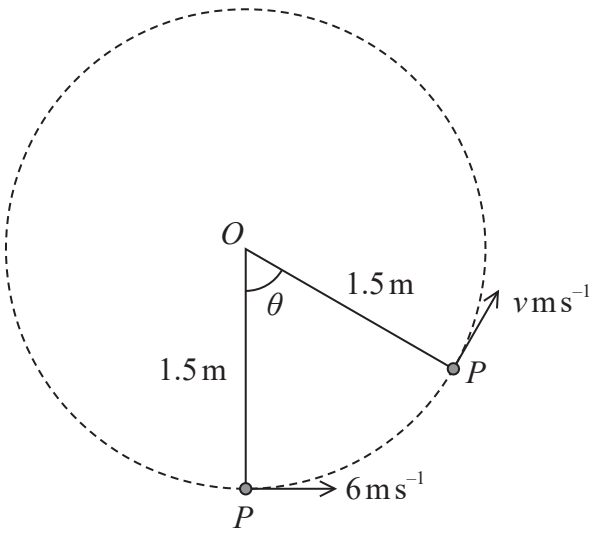


Figure 8

A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 1.5 m. The other end of the string is attached to a fixed point O. The particle is at rest vertically below O with the string taut.

The particle is then projected horizontally with speed $6\,\text{m s}^{-1}$

When the string has turned through an angle θ, the string is still taut and the speed of P is $v\,\text{m s}^{-1}$, as shown in Figure 8.

(a) Show that when $v = 3\sqrt{2}$, the tension in the string is 6.4 N.

(7)

(b) Find the magnitude of the acceleration of P when $v = 3\sqrt{2}$

(5)

(a) Conservation of energy (P rises $1.5(1 - \cos\theta)$):

$\frac{1}{2}(0.4)(6^2) = \frac{1}{2}(0.4)v^2 + 0.4 \times 10 \times 1.5(1 - \cos\theta)$

$v = 3\sqrt{2}$, $v^2 = 18$: $7.2 = 3.6 + 6(1 - \cos\theta)$

$1 - \cos\theta = 0.6 \Rightarrow \cos\theta = 0.4$

Equation of motion along PO (towards O):

$T - mg\cos\theta = \frac{mv^2}{r} \Rightarrow T - 0.4 \times 10 \times 0.4 = \frac{0.4 \times 18}{1.5}$

$T = 1.6 + 4.8 = 6.4$ N as required

(b) Radial (towards O) component of acceleration:

$\frac{v^2}{r} = \frac{18}{1.5} = 12\ \text{m s}^{-2}$

$\sin\theta = \sqrt{(1 - 0.4^2)} = \frac{\sqrt{21}}{5}$

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Question 8 continued

Transverse component: $mg \sin\theta = ma \Rightarrow a = g \sin\theta = 2\sqrt{21}\ m\ s^{-2}$

$|a| = \sqrt{(12^2 + 100 \times \frac{21}{25})} = \sqrt{(144 + 84)} = \sqrt{228} = 2\sqrt{57}$

$|a| = 15.099...$

$|a| = 15.1\ m\ s^{-2}$

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Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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