Please check the examination details below before entering your candidate information

Candidate surname: StemUp Model Solution | Other names:

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Pearson Edexcel Level 3 GCE

Monday 23 June 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/4A**

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The set $S = \{1, 3, 5, 9, 11, 13\}$ forms the group G, under the operation multiplication modulo 14

(a) Complete the Cayley table below for the group G

$\times_{14}$	**1**	**3**	**5**	**9**	**11**	**13**
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11	3	13	9
9	9	13	3	11	1	5
11	11	5	13	1	9	3
13	13	11	9	5	3	1

A spare table can be found on page 5 if you need to rewrite your Cayley table.

(3)

(b) Write down a subgroup of G of order 2

(1)

The group H is defined by the Cayley table below.

*	***p***	***q***	***r***	***s***	***t***	***u***
p	*p*	*q*	*r*	*s*	*t*	*u*
q	*q*	*t*	*u*	*r*	*s*	*p*
r	*r*	*u*	*t*	*q*	*p*	*s*
s	*s*	*r*	*q*	*p*	*u*	*t*
t	*t*	*s*	*p*	*u*	*r*	*q*
u	*u*	*p*	*s*	*t*	*q*	*r*

(c) Show that G and H are isomorphic.

(3)

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Question 1 continued

(b) $\{1, 13\}$

(c) In G: $3^1 = 3$, $3^2 = 9$, $3^3 = 27 \equiv 13$, $3^4 \equiv 39 \equiv 11$,

$3^5 \equiv 33 \equiv 5$, $3^6 \equiv 15 \equiv 1 \pmod{14}$

so 3 has order 6 and generates G: G is cyclic of order 6

In H (identity p): $q^2 = t$, $q^3 = s$, $q^4 = r$, $q^5 = u$, $q^6 = p$

so q has order 6 and generates H: H is cyclic of order 6

Both G and H are cyclic groups of order 6, so G and H are isomorphic.

(e.g. $3^k \leftrightarrow q^k$: $1 \leftrightarrow p$, $3 \leftrightarrow q$, $9 \leftrightarrow t$, $13 \leftrightarrow s$, $11 \leftrightarrow r$, $5 \leftrightarrow u$)

Question 1 continued

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Question 1 continued

Only use this grid if you need to rewrite your Cayley table.

$\times_{14}$	**1**	**3**	**5**	**9**	**11**	**13**
1	1	3	5	9	11	13
3	3	9	1	13	5	11
5	5	1	11			
9	9	13		11		
11	11	5			9	
13	13	11				1

(Total for Question 1 is 7 marks)

2.

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$$

(a) Determine the eigenvalues of matrix **A**

(3)

(b) Hence determine an orthogonal matrix **P** and a diagonal matrix **D** such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{AP}$$

(5)

(a) $\det(A - \lambda I) = 0$:

$$\begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda) - (-2)(-2) = 0$$

$4 - 5\lambda + \lambda^2 - 4 = 0 \Rightarrow \lambda^2 - 5\lambda = 0 \Rightarrow \lambda(\lambda - 5) = 0$

$\lambda = 0, \ \lambda = 5$

(b) $\lambda = 0$:

$$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x - 2y = 0 \Rightarrow x = 2y$$

eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda = 5$:

$$\begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow x - 2y = 5x \Rightarrow y = -2x$$

eigenvector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Normalise: both eigenvectors have length $\sqrt{(2^2 + 1^2)} = \sqrt{5}$

$$P = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$$

(eigenvectors are orthogonal, so P is orthogonal and $P^TAP = D$)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3. The circle C has equation

$$|z-5+4\mathrm{i}|=2|z-2+\mathrm{i}|$$

where $z = x + \mathrm{i}y$

(a) Show that an equation for C is

$$x^2-2x+y^2-7=0$$

(3)

The half-line with equation

$$\arg(z-a)=-\frac{3\pi}{4}$$

where a is a real constant, is a tangent to C

(b) Determine the value of a.

(6)

(a) $|(x-5)+i(y+4)| = 2|(x-2)+i(y+1)|$

$(x-5)^2+(y+4)^2 = 4[(x-2)^2+(y+1)^2]$

$x^2-10x+25+y^2+8y+16 = 4x^2-16x+16+4y^2+8y+4$

$x^2-10x+y^2+8y+41 = 4x^2-16x+4y^2+8y+20$

$0 = 3x^2-6x+3y^2-21$

$\therefore\ x^2-2x+y^2-7=0$ as required

(b) Half-line starts at $(a, 0)$ with gradient $\tan(-\frac{3\pi}{4}) = 1$, going down-left

so it lies on $y = x - a$ (with $x < a$, $y < 0$)

Substitute into C: $x^2-2x+(x-a)^2-7=0$

$2x^2-(2+2a)x+a^2-7=0$

Tangent $\Rightarrow$ repeated root $\Rightarrow b^2-4ac=0$:

$(2+2a)^2-4(2)(a^2-7)=0$

$4+8a+4a^2-8a^2+56=0 \Rightarrow -4a^2+8a+60=0$

$a^2-2a-15=0 \Rightarrow (a-5)(a+3)=0 \Rightarrow a=5$ or $a=-3$

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Question 3 continued

Q3(b) continued

$a = -3$: $x = \frac{2 + 2a}{4} = -1$, $y = x + 3 = 2 > 0$: the point of contact is above the real axis, not on the half-line (which has $y < 0$), so reject $a = -3$

$a = 5$: $x = \frac{2 + 2a}{4} = 3$, $y = 3 - 5 = -2 < 0$, so the half-line touches C

$\therefore\ a = 5$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 9 marks)

4. A sequence is defined by

$$u_{n+2} = 6u_{n+1} - 8u_n \qquad n \geqslant 1$$

$$u_1 = 399.6 \qquad u_2 = 798.4$$

(a) Prove by induction that, for $n \in \mathbb{N}$

$$u_n = 200(2^n) - \frac{1}{10}(4^n)$$

(6)

(b) Hence determine the value of the first negative term of the sequence.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(a) $n = 1$: $200(2^1) - \frac{1}{10}(4^1) = 400 - 0.4 = 399.6 = u_1$ ✓

$n = 2$: $200(2^2) - \frac{1}{10}(4^2) = 800 - 1.6 = 798.4 = u_2$ ✓

So the result is true for $n = 1$ and $n = 2$

Assume true for $n = k$ and $n = k + 1$, i.e.

$u_k = 200(2^k) - \frac{1}{10}(4^k)$ and $u_{k+1} = 200(2^{k+1}) - \frac{1}{10}(4^{k+1})$

Then $u_{k+2} = 6u_{k+1} - 8u_k$

$= 6[200(2^{k+1}) - \frac{1}{10}(4^{k+1})] - 8[200(2^k) - \frac{1}{10}(4^k)]$

$= 1200(2^{k+1}) - 1600(2^k) - \frac{6}{10}(4^{k+1}) + \frac{8}{10}(4^k)$

$= 600(2^{k+2}) - 400(2^{k+2}) - \frac{24}{10}(4^k) + \frac{8}{10}(4^k)$

$= 200(2^{k+2}) - \frac{16}{10}(4^k) = 200(2^{k+2}) - \frac{1}{10}(4^{k+2})$

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Question 4 continued

Q4(a) continued

which is the result with $n = k + 2$.

If the result is true for $n = k$ and $n = k + 1$ then it is true for $n = k + 2$.

As it is true for $n = 1$ and $n = 2$, it is true for all $n \in \mathbb{N}$ by mathematical induction.

(b) $u_n < 0 \Leftrightarrow 200(2^n) < \frac{1}{10}(4^n) = \frac{1}{10}(2^n)^2$

$\Leftrightarrow 2^n > 2000$ (dividing by $2^n > 0$)

$2^n = 2000 \Rightarrow n = \frac{\log 2000}{\log 2} = 10.965\ldots$

so $n = 11$ is the first n with $u_n < 0$ $(2^{10} = 1024,\ 2^{11} = 2048)$

$u_{11} = 200(2^{11}) - \frac{1}{10}(4^{11}) = 409600 - 419430.4$

First negative term: $u_{11} = -9830.4$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5. (i) A password is to be created from 5 characters using the following criteria

- one capital letter
- 3 distinct single digits
- one of the four symbols @ £ $ &

Determine how many different passwords can be created if

(a) the characters are in the order of the bullet points above,

(2)

(b) the characters can be in any order.

(1)

(ii) Use Fermat's Little Theorem to determine the least positive residue of 4^{50} modulo 13

(3)

(iii) Determine the total number of different, positive **odd** numbers, less than 1000, that contain the digit 2

(4)

(i)(a) $26 \times 10 \times 9 \times 8 \times 4 = 74880$

(i)(b) positions for the letter and the symbol: $5 \times 4 = 20$

(digits fill the other 3 places), i.e. $\frac{5!}{3!} = 20$

$74880 \times 20 = 1497600$

(ii) 13 is prime and 13 does not divide 4, so by Fermat's Little Theorem

$4^{12} \equiv 1 \pmod{13}$

$50 = 4 \times 12 + 2$

$4^{50} = (4^{12})^4 \times 4^2 \equiv 1^4 \times 16 = 16 \equiv 3 \pmod{13}$

Least positive residue = 3

(iii) Positive odd numbers less than 1000: 1, 3, 5, ..., 999 $\Rightarrow$ 500

Odd numbers with no digit 2:

1-digit: 1, 3, 5, 7, 9 $\Rightarrow$ 5

2-digit: 8 choices (not 0, not 2) $\times$ 5 = 40

3-digit: $8 \times 9 \times 5$ = 360

Total with no 2 = 5 + 40 + 360 = 405

Number containing the digit 2 = 500 − 405 = 95

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Question 5 continued

(Total for Question 5 is 10 marks)

6.

$$I_n = \int_0^2 \left(4 - x^2\right)^n \mathrm{d}x$$

(a) Prove that, for $n \geqslant 1$

$$I_n = \frac{8n}{2n+1} I_{n-1}$$

(4)

(b) Using the result from part (a) and showing all stages of your working, determine the value of n for which

$$I_n = \frac{65536}{315}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

(a) $I_n = \int_0^2 1 \times (4 - x^2)^n \, dx$

$u = (4 - x^2)^n \Rightarrow du/dx = -2nx(4 - x^2)^{n-1}$; $dv/dx = 1 \Rightarrow v = x$

$I_n = [\, x(4 - x^2)^n \,]_0^2 + \int_0^2 2nx^2(4 - x^2)^{n-1} \, dx$

$= 0 + 2n \int_0^2 [4 - (4 - x^2)](4 - x^2)^{n-1} \, dx$

$= 8n \int_0^2 (4 - x^2)^{n-1} \, dx - 2n \int_0^2 (4 - x^2)^n \, dx$

$I_n = 8nI_{n-1} - 2nI_n \Rightarrow (2n + 1)I_n = 8nI_{n-1}$

$\therefore I_n = \frac{8n}{2n + 1} I_{n-1}$ as required

(b) $I_0 = \int_0^2 1 \, dx = [\, x \,]_0^2 = 2$

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Question 6 continued

Q6(b) continued

$I_1 = \frac{8}{3} \times 2 = \frac{16}{3}$

$I_2 = \frac{16}{5} \times \frac{16}{3} = \frac{256}{15}$

$I_3 = \frac{24}{7} \times \frac{256}{15} = \frac{6144}{105} = \frac{2048}{35}$

$I_4 = \frac{32}{9} \times \frac{2048}{35} = \frac{65536}{315}$

$\therefore \; n = 4$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

7. (i) Explain why the congruence equation

$$21x \equiv 11 (\text{mod } 36)$$

has no solutions.

(2)

(ii) Use modular arithmetic to solve the congruence equation

$$36y \equiv 192 (\text{mod } 40)$$

(4)

(i) hcf(21, 36) = 3

21x ≡ 11 (mod 36) ⇒ 21x − 36k = 11 for some integer k,

but the left side is a multiple of 3 and 3 does not divide 11,

so there are no solutions.

(ii) 192 = 4 × 40 + 32 ⇒ 36y ≡ 32 (mod 40)

hcf(36, 40) = 4 and 4 divides 32, so divide through by 4:

9y ≡ 8 (mod 10)

9 × 9 = 81 ≡ 1 (mod 10), so $9^{-1} \equiv 9$ (mod 10)

y ≡ 9 × 8 = 72 ≡ 2 (mod 10)

y ≡ 2 (mod 10), i.e. y ≡ 2, 12, 22, 32 (mod 40)

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Question 7 continued

(Total for Question 7 is 6 marks)

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8.

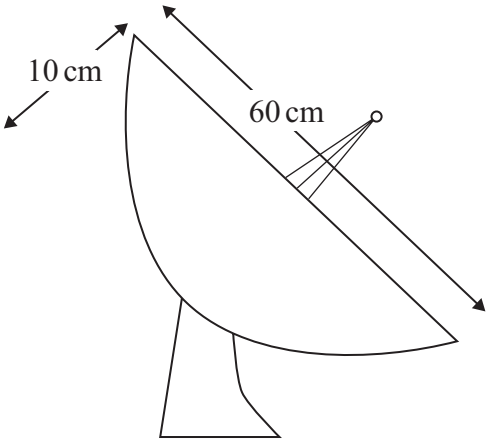


Figure 1

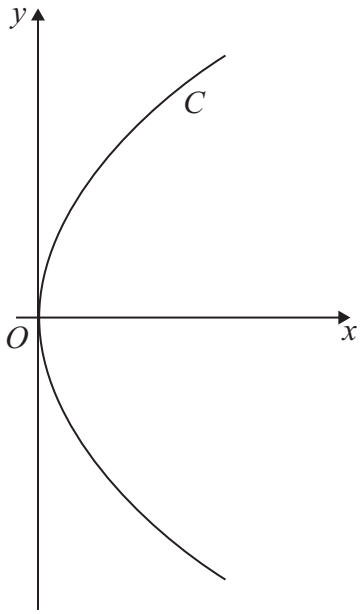


Figure 2

Figure 1 shows a satellite dish.

Figure 2 shows a sketch of the curve C with equation

$$y^2 = Ax \qquad 0 \leqslant x \leqslant 10$$

where A is a positive constant.

The curved inner surface of the satellite dish is modelled by the surface of revolution formed by rotating curve C through π radians about the x-axis.

The inner surface of the satellite dish has

- a largest diameter of 60 cm
- a depth of 10 cm

as shown in Figure 1.

(a) Determine the value of A.

(2)

(b) Using algebraic integration, determine, in cm^2, the area of the curved inner surface of the satellite dish, according to the model. Give your answer to 2 significant figures.

(7)

(c) State a limitation of the model.

(1)

(a) At $x = 10$ (depth), $y = 30$ (half of the 60 cm diameter):

$30^2 = A \times 10 \Rightarrow A = 90$

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Question 8 continued

(b) $y^2 = 90x \Rightarrow 2y\,dy/dx = 90 \Rightarrow dy/dx = 45/y$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{2025}{y^2} = \frac{y^2 + 2025}{y^2}$$

$$S = 2\pi \int_0^{10} y\sqrt{(1 + (dy/dx)^2)}\,dx = 2\pi \int_0^{10} \sqrt{(y^2 + 2025)}\,dx$$

$$= 2\pi \int_0^{10} (90x + 2025)^{1/2}\,dx$$

$$= 2\pi \left[\frac{(90x + 2025)^{3/2}}{90 \times 3/2}\right]_0^{10}$$

$$= \frac{2\pi}{135}\left[(2925)^{3/2} - (2025)^{3/2}\right]$$

$$= \frac{2\pi}{135}(158193.56\ldots - 91125)$$

$$= 3121.5\ldots$$

Surface area $\approx 3100\ cm^2$ (2 s.f.)

(c) The real dish will not be perfectly smooth or exactly parabolic in shape, and the measurements 60 cm and 10 cm may not be exact.

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9. The transformation T from the z-plane to the w-plane, where $z = x + iy$ and $w = u + iv$ is given by

$$w = \frac{z}{z + 2i} \qquad z \neq -2i$$

The circle with equation $|z| = 4$ is mapped by T onto the circle C.

Determine

(i) the coordinates of the centre of C

(ii) the radius of C

(8)

$w(z + 2i) = z \Rightarrow wz + 2iw = z \Rightarrow 2iw = z(1 - w)$

$\Rightarrow z = \frac{2iw}{1 - w}$

$|z| = 4 \Rightarrow \frac{|2iw|}{|1 - w|} = 4 \Rightarrow 2|w| = 4|1 - w| \Rightarrow |w| = 2|1 - w|$

$w = u + iv$: $|u + iv| = 2|(1 - u) - iv|$

$u^2 + v^2 = 4[(1 - u)^2 + v^2]$

$u^2 + v^2 = 4 - 8u + 4u^2 + 4v^2$

$3u^2 - 8u + 3v^2 + 4 = 0$

$u^2 - \frac{8}{3}u + v^2 + \frac{4}{3} = 0$

$(u - \frac{4}{3})^2 - \frac{16}{9} + v^2 + \frac{4}{3} = 0$

$(u - \frac{4}{3})^2 + v^2 = \frac{4}{9}$

(i) Centre of C: $(\frac{4}{3}, 0)$

(ii) Radius of C = $\frac{2}{3}$

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Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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