Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 6 June 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. A particle A has mass $4m$ and a particle B has mass $3m$. The particles are moving along the same straight line on a smooth horizontal table. The particles are moving in opposite directions towards each other when they collide directly.

As a result of the collision, the direction of motion of each particle is reversed.

Immediately **before** the collision, the speed of A is u and the speed of B is ku, where k is a constant.

Immediately **after** the collision, the speed of A is $2v$ and the speed of B is $3v$.

The magnitude of the impulse received by A in the collision is $20mv$.

(a) Find u in terms of v only.

(3)

(b) Find the exact value of k.

(3)

(a) Take the original direction of A as positive.

Impulse on A = change in momentum of A:

$-20mv = 4m(-2v - u)$

$-20mv = -8mv - 4mu$

$4mu = 12mv$

$u = 3v$

(b) Impulse on B = 20mv (opposite direction to that on A):

$20mv = 3m(3v - (-ku))$

$20v = 9v + 3ku$

$3ku = 11v$

$3k(3v) = 11v$

$k = 11/9$

Check by CLM: $4m(u) - 3m(ku) = 3m(3v) - 4m(2v)$

$12mv - 11mv = 9mv - 8mv = mv$ ✓

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Question 1 continued

(Total for Question 1 is 6 marks)

2. A van of mass $900\,\text{kg}$ is moving along a straight horizontal road.

When the speed of the van is $v\,\text{ms}^{-1}$, the resistance to the motion of the van is modelled as a force of magnitude $50v$ N.

The engine of the van is working at a constant rate of 12 kW.

At the instant when the speed of the van is $V\,\text{ms}^{-1}$, the acceleration of the van is $0.2\,\text{ms}^{-2}$

Using the model,

(a) find the value of V.

(4)

Later on, the van is moving up a straight road that is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{12}$. When the speed of the van is $v\,\text{ms}^{-1}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $50v$ N.

When the van has constant speed $U\,\text{ms}^{-1}$, the engine of the van is working at a constant rate of 15 kW.

Using the model,

(b) find the value of U.

(4)

(a) Driving force $F = 12000/V$

N2L for the van: $F - 50V = 900 \times 0.2$

$12000/V - 50V = 180$

$50V^2 + 180V - 12000 = 0$

$V = (-180 + \sqrt{(180^2 + 4 \times 50 \times 12000)}) / 100$

$V = 13.796...$ (reject the negative root)

$V = 14$

(b) Constant speed, so resultant force is zero ($a = 0$).

Driving force $F = 15000/U$

$F - 50U - 900g \sin\alpha = 0$

$15000/U - 50U - 900 \times 9.8 \times 1/12 = 0$

$15000/U - 50U - 735 = 0$

$50U^2 + 735U - 15000 = 0$

$U = (-735 + \sqrt{(735^2 + 4 \times 50 \times 15000)}) / 100$

$U = 11.465...$ (reject the negative root)

$U = 11.5$

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Question 2 continued

(Total for Question 2 is 8 marks)

3. A particle P of mass 0.4 kg is moving with velocity $7\mathbf{i}\ \text{m s}^{-1}$ when it receives an impulse of magnitude $\sqrt{1.6}\ \text{N s}$.

The velocity of P immediately after it receives the impulse is $\lambda(2\mathbf{i} + \mathbf{j})\ \text{m s}^{-1}$, where λ is a constant.

Find the two possible values of λ

(6)

Impulse = change in momentum

$I = 0.4\lambda(2i + j) - 0.4 \times 7i$

$I = (0.8\lambda - 2.8)i + 0.4\lambda j$

Magnitude of impulse is $\sqrt{1.6}$:

$(\sqrt{1.6})^2 = (0.8\lambda - 2.8)^2 + (0.4\lambda)^2$

$1.6 = 0.64\lambda^2 - 4.48\lambda + 7.84 + 0.16\lambda^2$

$0.8\lambda^2 - 4.48\lambda + 6.24 = 0$

$5\lambda^2 - 28\lambda + 39 = 0$

$(5\lambda - 13)(\lambda - 3) = 0$

$\lambda = 13/5 = 2.6$ or $\lambda = 3$

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Question 3 continued

(Total for Question 3 is 6 marks)

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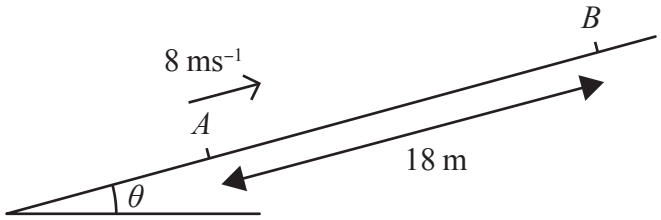


Figure 1

A rough straight ramp is fixed to horizontal ground. The ramp is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The points A and B are on a line of greatest slope of the ramp with $AB = 18$ m and B above A, as shown in Figure 1.

A package of mass 0.5 kg is projected up the ramp from A with speed 8 m s^{-1} and comes to instantaneous rest at B.

The work done against friction as the package moves from A to B is W joules.

The package is modelled as a particle and air resistance is ignored.

(a) Use the work-energy principle to show that $W = 9.7$

(3)

The coefficient of friction between the package and the ramp is μ

(b) Find the value of μ

(4)

(a) Work-energy principle from A to B:

KE lost = gain in GPE + work done against friction

$\frac{1}{2} \times 0.5 \times 8^2 = 0.5g \times 18 \sin\theta + W$

$16 = 0.5 \times 9.8 \times 18 \times 1/14 + W$

$16 = 6.3 + W$

$W = 16 - 6.3 = 9.7$ as required

(b) $\sin\theta = 1/14 \Rightarrow \cos\theta = \sqrt{(1 - 1/196)} = \sqrt{195}/14$

Perpendicular to the ramp: $R = 0.5g\cos\theta$

Friction is limiting: $F = \mu R = \mu \times 0.5g\cos\theta$

Work done against friction = $F \times 18$:

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Question 4 continued

$9.7 = \mu \times 0.5 \times 9.8 \times \sqrt{195}/14 \times 18$

$9.7 = 6.3\sqrt{195}\,\mu$

$\mu = 9.7 / (6.3\sqrt{195}) = 0.11026...$

$\mu = 0.11$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

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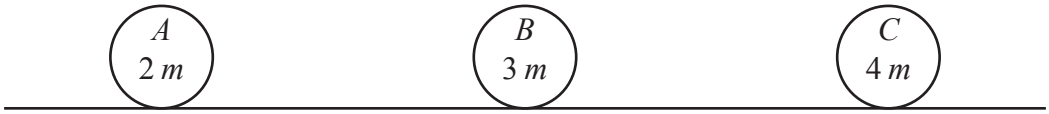


Figure 2

Three small balls, A, B and C, have masses $2m$, $3m$ and $4m$ respectively.

The balls are initially at rest in a straight line on a smooth horizontal surface, with B between A and C, as shown in Figure 2.

Ball A is projected towards B with speed $5u$ and A and B collide directly.
Immediately after the collision, the speed of B is $3u$

The balls are modelled as uniform spheres with equal radii.

(a) Find, in terms of u, the speed of A immediately after the collision with B.

(3)

(b) Find the coefficient of restitution between A and B.

(3)

After the collision between A and B, ball C is projected towards B with speed u
Balls B and C collide directly.
The coefficient of restitution between B and C is f

Given that there is a second direct collision between A and B

(c) find the complete range of possible values of f

(7)

(a) Take the direction of A's initial motion as positive.

Let A have velocity v after the collision.

CLM: $2m(5u) + 0 = 2mv + 3m(3u)$

$10u = 2v + 9u$

$v = \frac{1}{2}u$

Speed of A = $u/2$

(b) NEL: speed of separation = e × speed of approach

$3u - \frac{1}{2}u = e \times 5u$

$5u/2 = 5ue$

$e = \frac{1}{2}$

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Question 5 continued

(c) Before B and C collide: A has velocity $u/2$, B has velocity $3u$, C has velocity $-u$. After: B has velocity x, C has velocity y.

CLM: $3m(3u) + 4m(-u) = 3mx + 4my$

$3x + 4y = 5u$ ... (1)

NEL: $y - x = f(3u - (-u))$

$y - x = 4uf$ ... (2)

Substitute $y = x + 4uf$ into (1):

$3x + 4x + 16uf = 5u$

$7x = 5u - 16uf$

$x = u(5 - 16f)/7$

A is still moving in the positive direction with speed $u/2$, and B does not collide with A again unless B's velocity is less than $u/2$.

For a second collision between A and B we need $x < u/2$:

$u(5 - 16f)/7 < u/2$

$10 - 32f < 7$

$32f > 3$

$f > 3/32$

(If $x \leqslant 0$, B moves back towards A, so a collision certainly occurs.)

Also $f \leqslant 1$ as f is a coefficient of restitution.

Range: $3/32 < f \leqslant 1$

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 13 marks)

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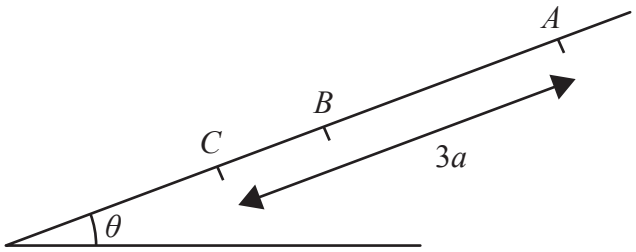


Figure 3

A smooth plane is inclined at an angle θ to horizontal ground, where $\sin\theta = \frac{1}{3}$

The points A, B and C lie on a line of greatest slope of the plane, with B between A and C, with A above B and $AC = 3a$, as shown in Figure 3.

A light elastic string of natural length $2a$ has one end attached to the fixed point A. The other end of the string is attached to a parcel P of mass m.

The modulus of elasticity of the string is $\frac{8}{3}mg$

The parcel rests in equilibrium on the plane at the point B.

The parcel is modelled as a particle.

(a) Show that $AB = \frac{9}{4}a$

(4)

The parcel is now held at C and released from rest.

(b) Find the elastic potential energy lost by the string when P moves from C to B.

(3)

(c) Use the principle of conservation of mechanical energy to find the speed of P at the instant it passes B.

(3)

(a) Let the extension at B be e, so AB = 2a + e.

Hooke's law: T = λe/l = (8mg/3)e / (2a) = 4mge/(3a)

Resolve parallel to the slope: T = mg sin θ = mg/3

4mge/(3a) = mg/3

e = a/4

AB = 2a + a/4 = 9a/4 as required

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Question 6 continued

(b) At C the extension is $AC - 2a = 3a - 2a = a$

At B the extension is $9a/4 - 2a = a/4$

EPE lost $= \lambda(3a - 2a)^2/(2 \times 2a) - \lambda(9a/4 - 2a)^2/(2 \times 2a)$

$= (8mg/3)(a^2)/(4a) - (8mg/3)(a^2/16)/(4a)$

$= 2mga/3 - mga/24$

$= 15mga/24$

$= 5mga/8$

(c) Conservation of mechanical energy from C to B.

P rises a distance $CB = 3a - 9a/4 = 3a/4$ along the slope,

i.e. a vertical height of $3a/4 \times \sin\theta = a/4$.

EPE lost = GPE gained + KE gained

$5mga/8 = mg(3a - 9a/4)\sin\theta + \frac{1}{2}mv^2$

$5mga/8 = mga/4 + \frac{1}{2}mv^2$

$\frac{1}{2}v^2 = 5ga/8 - 2ga/8 = 3ga/8$

$v^2 = 3ga/4$

$v = \sqrt{(3ga/4)} = \frac{1}{2}\sqrt{(3ga)}$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

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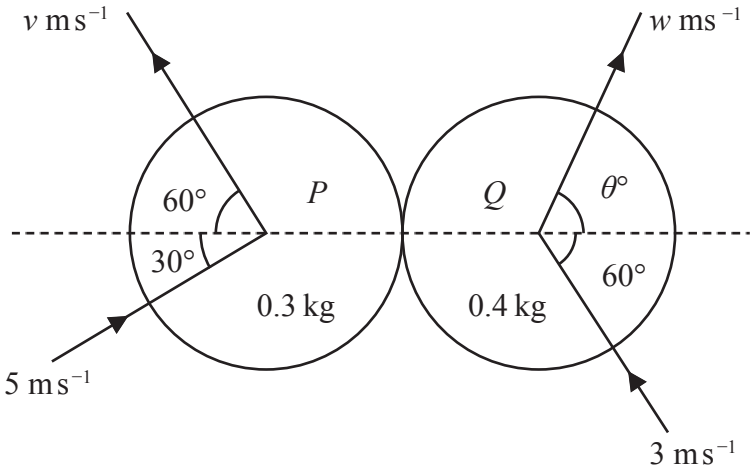


Figure 4

Two uniform spheres P and Q have equal radii. The mass of P is 0.3 kg and the mass of Q is 0.4 kg. The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately **before** the collision,

- P is moving with speed $5\,\text{m s}^{-1}$ at 30° to the line of centres of the spheres
- Q is moving with speed $3\,\text{m s}^{-1}$ at 60° to the line of centres of the spheres

Immediately **after** the collision,

- P is moving with speed $v\,\text{m s}^{-1}$ at 60° to the line of centres of the spheres
- Q is moving with speed $w\,\text{m s}^{-1}$ at θ° to the line of centres of the spheres

as in the plan view shown in Figure 4.

(a) Show that $v = \dfrac{5\sqrt{3}}{3}$

(3)

(b) Find (i) the value of w

(ii) the value of θ

(7)

(c) Find the coefficient of restitution between P and Q.

(3)

(a) The component of P's velocity perpendicular to the line of centres is unchanged: $5 \sin 30° = v \sin 60°$

$5/2 = v\sqrt{3}/2$

$v = 5/\sqrt{3} = 5\sqrt{3}/3$ as required

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Question 7 continued

(b)(i) The component of Q's velocity perpendicular to the line of centres is unchanged:

$w \sin \theta° = 3 \sin 60° = 3\sqrt{3}/2$

CLM parallel to the line of centres (P to Q positive):

$0.3 \times 5 \cos 30° - 0.4 \times 3 \cos 60° = 0.4w \cos \theta° - 0.3v \cos 60°$

$0.3 \times 5\sqrt{3}/2 - 0.6 = 0.4w \cos \theta° - 0.3 \times 5\sqrt{3}/3 \times ½$

$3\sqrt{3}/4 - 0.6 = 0.4w \cos \theta° - \sqrt{3}/4$

$0.4w \cos \theta° = \sqrt{3} - 0.6$

$w \cos \theta° = (5\sqrt{3} - 3)/2 \quad (= 2.830...)$

$w^2 = (3\sqrt{3}/2)^2 + ((5\sqrt{3} - 3)/2)^2$

$w^2 = 6.75 + 8.0096... = 14.759...$

$w = 3.84182...$

$w = 3.8$

(ii) $\tan \theta° = (3\sqrt{3}/2) \div ((5\sqrt{3} - 3)/2) = 3\sqrt{3}/(5\sqrt{3} - 3)$

$\theta = 42.552...$

$\theta = 43$

(c) NEL parallel to the line of centres:

speed of separation = e × speed of approach

$v \cos 60° + w \cos \theta° = e(5 \cos 30° + 3 \cos 60°)$

$5\sqrt{3}/6 + (5\sqrt{3} - 3)/2 = e(5\sqrt{3}/2 + 3/2)$

$4.2735... = 5.8301... e$

$e = 0.73300...$

$e = 0.73$

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 13 marks)

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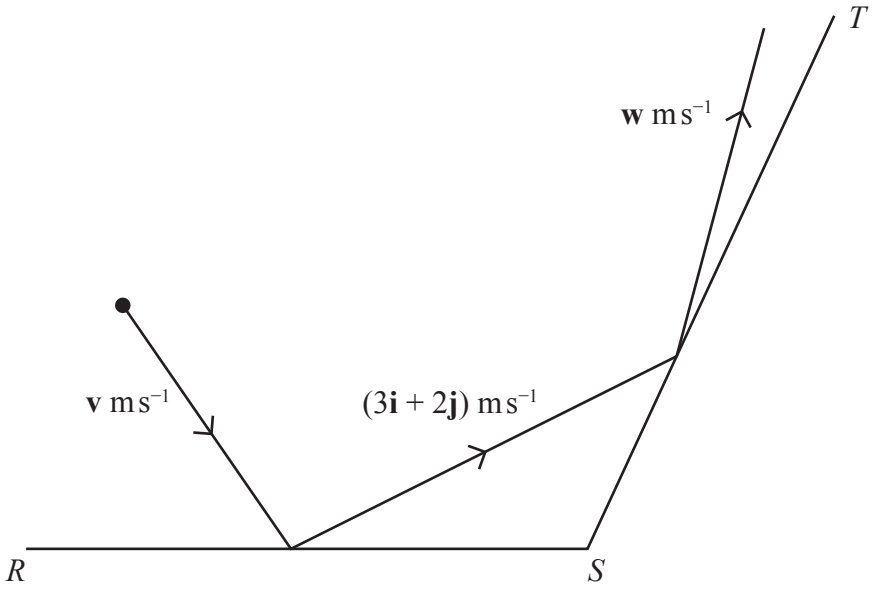


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where RS and ST are smooth fixed vertical walls.

The vector $\overrightarrow{RS}$ is in the direction of the vector $\mathbf{i}$.

The vector $\overrightarrow{ST}$ is in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$.

A small ball B of mass $0.25\,\text{kg}$ is projected across the floor towards RS.

Immediately before the impact with RS, the velocity of B is $\mathbf{v}\,\text{m s}^{-1}$

Immediately after the impact with RS, the velocity of B is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

The coefficient of restitution between B and RS is $\frac{1}{3}$

The ball is modelled as a particle.

(a) Show that the kinetic energy lost by B in the impact with RS is 4 J.

(6)

Immediately after the impact with ST, the velocity of B is $\mathbf{w}\,\text{m s}^{-1}$

Given that the coefficient of restitution between B and ST is $\frac{1}{3}$

(b) find $\mathbf{w}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(6)

(a) Let v = ai + bj. Component parallel to RS is unchanged: a = 3

NEL perpendicular to RS: 2 = 1/3 × (−b)

b = −6

v = 3i − 6j

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Question 8 continued

KE lost = ½ × 0.25 × $(3^2 + 6^2)$ − ½ × 0.25 × $(3^2 + 2^2)$

= ½ × 0.25 × 45 − ½ × 0.25 × 13

= 5.625 − 1.625

= 4 J as required

(b) Unit vector parallel to ST: 1/5 (3i + 4j)

Unit vector perpendicular to ST: 1/5 (−4i + 3j)

Before impact with ST, velocity 3i + 2j:

component parallel to ST = 1/5 (3i + 4j)·(3i + 2j) = (9 + 8)/5 = 17/5

component perp. to ST = 1/5 (−4i + 3j)·(3i + 2j) = (−12 + 6)/5 = −6/5

Let w = ai + bj. Parallel component unchanged:

1/5 (3i + 4j)·(ai + bj) = 17/5 ⇒ 3a + 4b = 17 ... (1)

Perpendicular component reversed and multiplied by 1/3:

1/5 (−4i + 3j)·(ai + bj) = −1/3 × (−6/5)

−4a + 3b = 2 ... (2)

4 × (1) + 3 × (2): 16b + 9b = 68 + 6 ⇒ 25b = 74 ⇒ b = 74/25

3 × (1) − 4 × (2): 9a + 16a = 51 − 8 ⇒ 25a = 43 ⇒ a = 43/25

w = 43/25 i + 74/25 j (= 1.72i + 2.96j)

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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