Pearson Edexcel

Mark Scheme

Summer 2025

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 3C Further Mechanics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

• Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

• Omission or extra g in a resolution is an accuracy error not method error.

• Omission of mass from a resolution is a method error.

• Omission of a length from a moments equation is a method error.

• Omission of units or incorrect units is not (usually) counted as an accuracy error.

• DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

• Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

• Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
• Marks must be entered in the same order as they appear on the mark scheme.

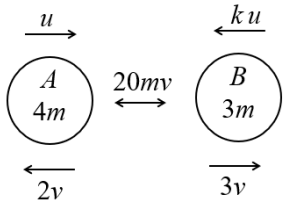
• In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

• Accept column vectors in all cases.

• Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

• Mechanics Abbreviations

M(A)	Taking moments about A.
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS, LHS	Right hand side, left hand side.

Question	Scheme	Marks	AOs
1a			
	Impulse-momentum equation for A or other complete method to form an equation in u and v only (and m)	M1	3.4
	For A: $20mv = 4m\left(2v - \left(-u\right)\right)$	A1	1.1b
	$u = 3v$	A1	1.1b
		(3)	
1b	Impulse momentum equation for B or use of CLM to form an equation in k (and u, v, m)	M1	3.4
	Either For B: $20mv = 3m\left(3v - \left(-ku\right)\right)$ Or CLM: $4m(u) - 3m(ku) = 3m(3v) - 4m(2v)$	A1ft	1.1b
	$k = \frac{11}{9}$	A1	1.1b
		(3)	
			(6 marks)

Notes: Working in parts (a) and (b) may be marked together.

(a)	
M1	Use of $I = m\left(v - u\right)$ on A or equivalent complete method to form an equation in terms of u and v (and m) only (not k). Must consider change in momenta but condone in the wrong order or velocity sign errors. Might see $(2v + u)$ rather than $(2v - -u)$. Dimensionally correct equation with correct mass and velocities pairings. If CLM equation is formed in (a), then an impulse-equation must also be used to eliminate k for this mark. Allow consistent omission of m.
A1	Correct unsimplified equation in u and v (and m)
A1	Correct only, ISW after $u = 3v$ A0 for only $v = \frac{u}{3}$
(b)	
M1	Correct use of impulse-momentum equation or CLM to form an equation in k, u, v (and m). All terms required, allow consistent omission of m. Dimensionally correct.
A1ft	Correct unsimplified equation in k, u and v. No need to replace u and v (follow through their positive u if substituted)
A1	Correct exact value. Accept $1\frac{2}{9}$ or recurring decimal $1.\dot{2}$ with correct notation.

Question	Scheme	Marks	AOs
2(a)	Equation of motion of the van	M1	3.3
	$F - 50V = 900 \times 0.2$	A1	1.1b
	Use of $P = FV$ $\left(\frac{12000}{V} - 50V = 900 \times 0.2\right)$	M1	3.4
	$\left(50V^2 + 180V - 12000 = 0\right)$ $(V =)14$ or better (correctly rounded)	A1	1.1b
		(4)	
(b)	Equation of motion of the van	M1	3.3
	$F - 50U - 900g\sin\alpha = 0$ $\frac{15000}{U} - 50U - 75g = 0$	A1 A1	1.1b 1.1b
	$\left(50U^2 + 735U - 15000 = 0\right)$ $(U =)11$ or 11.5	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)	
M1	Form an equation of motion for the van to produce an equation in V only or form an equation of motion for the van to produce an equation in V and F only where F is the driving force of the van's engine. Must be dimensionally correct with required terms. Condone sign errors.
A1	Correct unsimplified equation.
M1	Use of $P = FV$ where F is the driving force of the van's engine. Implied by use of $\frac{12000}{V}$ with their equation of motion. Condone use of 12 or a slip with the number of zeros for this M mark.
A1	Obtain only 14 or better (correctly rounded). Calculator display gives 13.79615337. If a negative value is seen, it must be rejected.
(b)	
M1	Form equation of motion for the van with $a = 0$ to produce an equation in U only or in U and F only. Must be dimensionally correct with required terms. Condone sign errors and sin/cos confusion. Condone use of another letter instead of U throughout. Condone use of 15 or a slip with the number of zeros. M0 if the answer from (a) is used in (b). M0 for use of $P = 12\ 000$
A1	Unsimplified equation in F and U (or U only) with at most one error. Trig and F do not need not be replaced.
A1	Correct unsimplified equation in U only (trig and F replaced).
A1	Obtain only 11 or 11.5 (2 or 3sf after use of 9.8). If a negative value is seen, it must be rejected. A0 if the answer follows use of $g = 9.81\text{ms}^{-2}$

Question	Scheme	Marks	AOs
3	Use of change in momentum	M1	3.1a
	$(\mathbf{I}) = 0.4\lambda(2\mathbf{i}+\mathbf{j}) - 0.4\times 7\mathbf{i}$	A1	1.1b
	Use of Pythagoras to form an equation for magnitude of impulse	M1	1.1b
	E.g. $1.6 = (0.8\lambda - 2.8)^2 + (0.4\lambda)^2$ $1.6 = 0.4^2\left((2\lambda-7)^2 + \lambda^2\right)$	A1	1.1b
	$5\lambda^2 - 28\lambda + 39 = 0 \quad \Rightarrow \quad \lambda = ...$	dM1	2.1
	$\lambda = 2.6, \quad \lambda = 3$	A1	2.2a
		(6)	
			(6 marks)

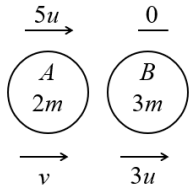
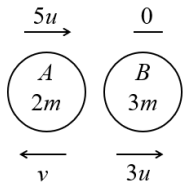
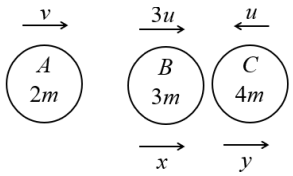
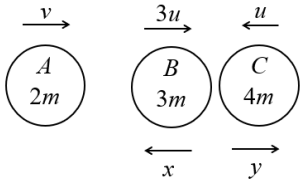
Notes: Accept column vectors throughout

M1	Form an expression for change in momentum in λ, correct number of terms and dimensionally correct. Must use velocities and both components. Subtraction may be either way round. If present, ignore LHS. For the M mark, condone poor expanding of the velocity i.e. $(0.8\lambda\mathbf{i} + 0.4\mathbf{j})$ or $(0.8\mathbf{i} + 0.4\lambda\mathbf{j})$ M0 if speed is used
A1	Correct unsimplified expression for change in momentum, accept terms either way round. Must use conventional vector notation ie column vector form or **i-j** form. If present, ignore LHS. A0 for unconventional vector notation, unless recovered.
M1	Correct use of Pythagoras (squaring and adding) to form an equation for magnitude of impulse. Must use the given magnitude, $\sqrt{1.6}$, and the change in momentum components, to form an equation in λ only. Other unknowns may be introduced to represent the impulse components. E.g. $\begin{pmatrix} a \\ b \end{pmatrix} = 0.4\begin{pmatrix} 2\lambda \\ \lambda \end{pmatrix} - 0.4\begin{pmatrix} 7 \\ 0 \end{pmatrix}$ However, the M mark is only awarded when Pythagoras is used correctly to form an equation in λ only. E.g. $\left(\sqrt{1.6}\right)^2 = a^2 + b^2 \quad \Rightarrow \quad 1.6 = (0.8\lambda - 2.8)^2 + (0.4\lambda)^2$
A1	Correct unsimplified equation in λ only.
dM1	Dependent on previous two M's. Complete method using the change in momentum and magnitude of impulse to form a 3TQ in λ only and solve to find two λ values. No need to see the method for solving 3TQ. Must reach $\lambda =$…
A1	Both correct values o.e. eg $\lambda = \frac{13}{5}, \quad \lambda = 3$

Question	Scheme	Marks	AOs
4(a)	Work-energy equation:	M1	3.4
	$\frac{1}{2}\times 0.5\times 8^2 = W + 0.5g\times 18\sin\theta$	A1	1.1b
	$W = 16 - 9g\times\frac{1}{14} = 9.7$ *	A1*	2.2a
		(3)	
(b)	Use of $F = \mu R = \mu\times 0.5g\cos\theta$	M1	3.1b
	Complete method to form a dimensionally correct equation in μ (and θ) using • Either work done = $F\times 18$ • Or *suvat* and N2L	M1	3.4
	• Either $9.7 = \mu\times 0.5g\cos\theta\times 18$ • Or relevant *suvat* to find a $\left(=-\frac{16}{9}\right)$ and N2L to form $-\mu\times 0.5g\cos\theta\times 18 - 0.5g\sin\theta = 0.5\left(-\frac{16}{9}\right)$	A1	1.1b
	$\mu = 0.11$ (0.110)	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)	
M1	Form work-energy equation in terms of W (and g and θ) only. All required terms present and no extras. All terms dimensionally correct (of correct structure). Condone ± sign errors on terms and sin/cos confusion on vertical height. M0 if a term is missing or for incorrect trig use eg $18\tan\theta, \frac{18}{\sin\theta}, \frac{18}{\cos\theta}$. M0 for use of *suvat*
A1	Correct unsimplified equation, no need to replace trig.
A1*	Obtain given answer from complete and correct working. Must see a line of working between the initial equation and the given answer. Condone missing W during working but must see 'W = …' for the final mark.
(b)	
M1	Correct use of $F = \mu R$ and $R = 0.5g\cos\theta$ to form an expression for Friction. Dimensionally correct. Condone sin/cos confusion. Missing g is an accuracy error not a method error.
M1	Complete method to form a dimensionally correct equation in μ (θ and g). Trig does not need to be replaced for M mark. M0 for $W = \mu R$. May use 9.7 and work done = $F\times 18$ to form a dimensionally correct equation in μ . May see relevant *suvat* to find acceleration, followed by N2L to form dimensionally correct equation μ. N2L must contain all relevant terms and no extras. Condone sin/cos confusion on the weight components.
A1	Correct unsimplified equation in μ (and g) with trig replaced correctly.
A1	2 sf or 3 sf only. A0 for use of $g = 9.81\text{ms}^{-2}$

Question	Scheme		Marks	AOs
5(a)				
	Use of CLM:		M1	3.4
	$2m(5u) = 2mv + 3m(3u)$	$2m(5u) = 2m(-v) + 3m(3u)$	A1	1.1b
	$v = \frac{1}{2}u$	$v = -\frac{1}{2}u \quad \Rightarrow \quad \lvert v \rvert = \frac{1}{2}u$	A1	1.1b
			(3)	
(b)	Use of NEL		M1	3.4
	$3u - v = e \times 5u$	$3u + v = e \times 5u$	A1	1.1b
	$\Rightarrow e = \frac{1}{2}$		A1	1.1b
			(3)	
(c)				
	Use of CLM		M1	3.4
	$3m(3u) + 4m(-u) = 3mx + 4my$	$3m(3u) + 4m(-u) = 3m(-x) + 4my$	A1	1.1b
	Use of NEL		M1	3.4
	$y - x = f \times (3u - -u)$	$y + x = f \times (3u - -u)$	A1	1.1b
	E.g. $\begin{cases} 3x + 4y = 5u \\ y - x = 4uf \end{cases}$ $\Rightarrow x = \frac{u}{7}(5 - 16f)$ o.e.	E.g. $\begin{cases} -3x + 4y = 5u \\ y + x = 4uf \end{cases}$ $\Rightarrow x = \frac{u}{7}(16f - 5)$ o.e.	M1	1.1b
	$v > x$ $\Rightarrow \frac{1}{2}u > \frac{u}{7}(5 - 16f)$	$v > -x$ $\Rightarrow \frac{1}{2}u > -\frac{u}{7}(16f - 5)$	dM1	3.1b
	$\frac{3}{32} < f \leqslant 1$		A1	2.2a
			(7)	
				(13 marks)

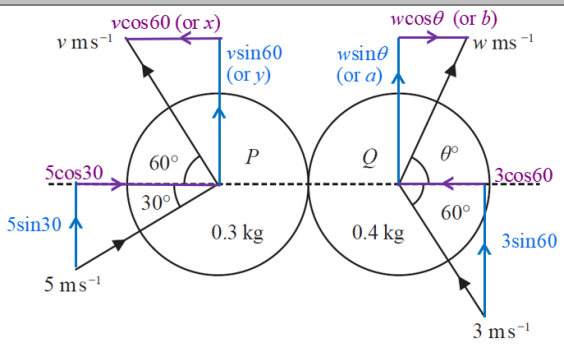
Notes:	
(a)	
M1	Use of CLM for A and B, to form an equation in v and u (and m). Dimensionally correct with correct mass and velocity pairings. Condone sign errors. May use two impulse-momentum equations: $-2m(v-5u)=3m(3u-0)$
A1	Correct unsimplified equation.
A1	Correct only, must be positive.
(b)	
M1	Correct use of Impact Law for A and B to form an equation in u (and v). Allow consistent omission of u. Dimensionally correct, condone sign errors on the velocities. M0 if separation and approach are on the wrong sides.
A1	Correct unsimplified equation.
A1	Correct only.
(c)	
M1	Use CLM, dimensionally correct, with correct mass and velocity pairings. Must have all non-zero velocities. Condone sign errors. Follow their directions for the unknown velocities of B and C after impact. (Ignore the diagram if it benefits the candidate.)
A1	Correct unsimplified equation.
M1	Correct use of NEL for B and C, dimensionally correct. Condone sign errors on velocity. Must have all non-zero velocities. Condone use of another letter for f. M0 if separation and approach are on the wrong sides.
A1	Correct unsimplified equation, the directions of B and C after impact must be consistent with their CLM equation.
M1	Solve simultaneous equations with two unknowns to find an expression for the velocity of B after the second collision in terms of f and u. Condone use of another letter for f. Must use CLM and NEL to reach $x = \ldots$ or a multiple of $x = \ldots$
dM1	Dependent on 3 previous M's. Complete method to find the values of f for a second collision. Must consider A and B moving in the same direction and the speed of A > speed of B.
A1	$0.094 < f \leqslant 1$ (0.094 or better) Both ends required with correct inequality signs and must use f as coefficient of restitution.

Question	Scheme	Marks	AOs
6(a)	Correct use of Hooke's law $T=\dfrac{\frac{8}{3}mge}{2a}$ or $T=\dfrac{\frac{8}{3}mg\left(AB-2a\right)}{2a}$	M1	3.4
	Resolve parallel to slope	M1	3.1b
	$T=mg\sin\theta\left(=\frac{1}{3}mg\right)$	A1	1.1b
	$\Rightarrow e=\frac{2a}{8}\left(=\frac{a}{4}\right)$ $AB=\frac{9}{4}a$ * ; $\Rightarrow AB-2a=\frac{2a}{8}\left(=\frac{a}{4}\right)$ $AB=\frac{9}{4}a$ *	A1*	2.2a
		(4)	
(b)	Loss in EPE	M1	3.3
	$\dfrac{\lambda\left(3a-2a\right)^2}{2(2a)}-\dfrac{\lambda\left(\frac{9a}{4}-2a\right)^2}{2(2a)}$	A1	1.1b
	$=\dfrac{5mga}{8}$	A1	1.1b
		(3)	
(c)	Energy equation	M1	3.1b
	$"\dfrac{5mga}{8}"=mg\left(3a-\dfrac{9a}{4}\right)\sin\theta+\dfrac{1}{2}mv^2$	A1ft	1.1b
	$v=\sqrt{\dfrac{3ga}{4}}$	A1	1.1b
		(3)	
			(10 marks)

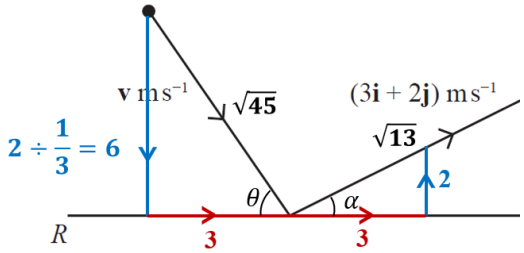
Notes:

(a)	
M1	Correct use of Hooke's law with $\frac{8mg}{3}$ and $2a$ substituted.
M1	Resolve parallel to the slope with all required terms and no extras. Dimensionally correct. Weight must be resolved but condone sin/cos confusion. T does not need to be replaced.
A1	Correct unsimplified equilibrium equation, T does not need to be replaced.
A1*	A complete and correct method using Hooke's law with the parallel equilibrium equation to obtain the given answer. There must be at least one line of working between the initial equations and the given answer. Must see '$AB=\ldots$' Accept fractions $\frac{9a}{4}$ or $\frac{9}{4}a$.

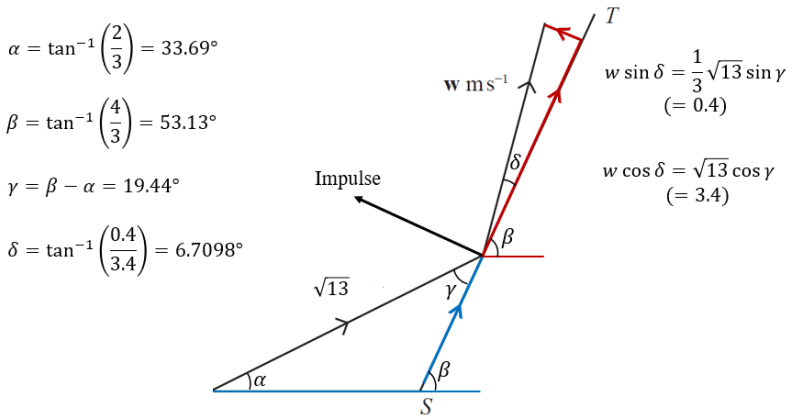
(b)	
M1	Correct method for difference in EPE at B and C, allow either way round. Dimensionally correct and of the correct structure. No need to substitute λ. For M mark, condone denominator of $2a$. Must use extensions $(3a-2a)$ and $\left(\frac{9a}{4}-2a\right)$.
A1	Correct unsimplified expression for change in EPE. Allow $\pm$
A1	Correct answer, o.e. ISW. Accept $0.625mga$ and $0.63mga$. Must be positive but allow a negative expression to change to a positive expression without justification.
(c)	
M1	Use of conservation of energy principle from C to B to form an equation with all terms of the correct structure and dimensionally correct. Condone sin/cos confusion on vertical height. For GPE, $mg\left(3a-\frac{9a}{4}\right)\sin\theta$ o.e for example $mg\frac{3a}{4}\sin\theta$, $mg\frac{3a}{4}\left(\frac{1}{3}\right)$, $mg\frac{a}{4}$ For EPE change, may use their answer from (b) if dimensionally correct (of the form kma where k is a constant) or start again with 2 EPE terms: $\frac{\frac{8mg}{3}(3a-2a)^2}{2(2a)}$ and $\frac{\frac{8mg}{3}\left(3a-\frac{9}{4}a\right)^2}{2(2a)}$. For M mark, condone denominator of $2a$.
A1ft	Correct unsimplified equation. Terms must be correct when the equation is formed but follow their answer to (b) for EPE if used.
A1	Correct answer in terms of $\sqrt{ag}$ ISW. Accept eg $\frac{1}{2}\sqrt{3ag}$, $0.87\sqrt{ag}$ or better. **N.B.** If the final mark in (b) is A0 due to substituting $g = 9.8$ ms^{-2}, do not penalise again for the same reason here.

Question	Scheme	Marks	AOs
			
7(a)	Velocity of P perpendicular to the line of centres unchanged	M1	3.4
	$5\sin 30° = v\sin 60°$	A1	1.1b
	$\frac{5}{2} = \frac{v\sqrt{3}}{2}$ $\Rightarrow v = \frac{5\sqrt{3}}{3}$ *	A1*	2.2a
		(3)	
(b)	Velocity of Q perpendicular to the line of centres unchanged	M1	3.4
	Either $w\sin\theta° = 3\sin 60° \left(= \frac{3\sqrt{3}}{2}\right)$ or $a = 3\sin 60° \left(= \frac{3\sqrt{3}}{2}\right)$	A1	1.1b
	CLM parallel to the line of centres	M1	3.4
	Either $0.3\times 5\cos 30° - 0.4\times 3\cos 60° = 0.4w\cos\theta° - 0.3v\cos 60°$ Or $0.3\times 5\cos 30° - 0.4\times 3\cos 60° = 0.4b - 0.3x$	A1	1.1b
	Solve for w or θ using $\begin{cases} w\sin\theta° = \frac{3\sqrt{3}}{2} \\ w\cos\theta° = \frac{5\sqrt{3}-3}{2} \end{cases}$ or $\begin{cases} a = \frac{3\sqrt{3}}{2} \\ b = \frac{5\sqrt{3}-3}{2} \end{cases}$	dM1	3.1a
	$\theta = 43 \quad (42.552...)$ or $w = 3.8 \quad (3.84182....)$	A1	2.2a
	$\theta = 43 \quad (42.552...)$ and $w = 3.8 \quad (3.84182....)$	A1	2.2a
		(7)	
(c)	NEL parallel to the line of centres	M1	3.1b
	• $v\cos 60° + w\cos\theta° = e(5\cos 30° + 3\cos 60°)$ • $x + b = e(5\cos 30° + 3\cos 60°)$	A1ft	1.1b
	$e = 0.73 \quad (0.73300....)$	A1	1.1b
		(3)	
			(13 marks)
Notes:			

(a)	
M1	Use the model for components of P perpendicular to the line of centres. Condone sin/cos confusion if clearly working perpendicular to the line of centres.
A1	A correct unsimplified equation in v. For example • $5\sin 30° = v\sin 60°$ • $5\sin 30° = x\tan 60° \quad \Rightarrow \quad v = \sqrt{\left(\frac{5}{2}\right)^2 + x^2} = \ldots$ • $\begin{pmatrix} 5\cos 30° \\ 5\sin 30° \end{pmatrix} \bullet \begin{pmatrix} -v\cos 60° \\ 5\sin 30° \end{pmatrix} = 0$
A1*	Obtain given answer from complete and correct working. Must see 'v = …' and the exact expression. There must be at least one line of working between the initial equation and the given answer.
(b)	
M1	Use the model for components of Q perpendicular to the line of centres. Condone sin/cos confusion if clearly working perpendicular to the line of centres. M0 if speeds are used instead of components.
A1	Correct unsimplified equation.
M1	Correct use of CLM parallel to the line of centres. Dimensionally correct. Need all terms. No need to replace v. Condone sign errors. Condone sin/cos confusion consistent with perpendicular components. Must have correct mass-velocity pairings. M0 if speeds are used instead of components.
A1	Correct unsimplified equation (no need to replace v).
dM1	Dependent on both previous M marks. Complete method using two correctly formed equations to obtain w or θ. i.e. form an equation in w or θ only and solve to reach $w = \ldots$ or $\theta = \ldots$ If numerical values appear without working out, a complete method using a calculator may be implied by correct answers following correct equations.
A1	One value correct to 2 sf or better , $\theta = 43 \quad (42.552...)$ or $w = 3.8 \quad (3.84182....)$ θ must be in degrees.
A1	Both values correct to 2 sf or better, $\theta = 43 \quad (42.552...)$ and $w = 3.8 \quad (3.84182....)$ θ must be in degrees.
(c)	
M1	Correct use of NEL parallel to the line of centres. Must form an equation in e and their components parallel to the line of centres. Condone sign errors. Condone sin/cos confusion consistent with perpendicular components. If seen in earlier work, NEL must be used in (c) to earn the marks here. M0 if separation and approach are on the wrong side. M0 if speeds are used instead of velocity components.
A1ft	Correct unsimplified equation (in e and their components parallel to the line of centres).
A1	Correct answer, 2 sf or better.

Question	Scheme	Marks	AOs
8(a)			
	Component of **v** parallel to $RS = 3\mathbf{i}$	B1	3.4
	Use of NEL perpendicular to RS: $2 = \frac{1}{3} \times$ perpendicular component	M1	3.4
	$(\mathbf{v} = 3\mathbf{i}) - 6\mathbf{j}$	A1	1.1b
	KE loss	M1	3.1b
	$= \frac{1}{2} \times 0.25(3^2 + 6^2) - \frac{1}{2} \times 0.25(3^2 + 2^2)$	A1	1.1b
	$= 4$ (J) *	A1*	2.2a
		(6)	
(b) **ALT 1**	Method [1] using scalar product and components of $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$ and $(3\mathbf{i} + 2\mathbf{j})$ parallel to ST $(3\mathbf{i} + 4\mathbf{j})$. Allow without $\frac{1}{5}$.	M1	3.1b
	$\frac{1}{5}(3\mathbf{i} + 4\mathbf{j}).(a\mathbf{i} + b\mathbf{j}) = \frac{1}{5}(3\mathbf{i} + 4\mathbf{j}).(3\mathbf{i} + 2\mathbf{j})$ $\{3a + 4b = 17\}$	A1	1.1b
	Method [1] using scalar product and components of $\mathbf{w} = a\mathbf{i} + b\mathbf{j}$ and $(3\mathbf{i} + 2\mathbf{j})$ perpendicular to ST, $(-4\mathbf{i} + 3\mathbf{j})$. Any multiple of $(-4\mathbf{i} + 3\mathbf{j})$. Must have $\pm\frac{1}{3}$. Allow without $\frac{1}{5}$.	M1	3.1b
	$\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j}).(a\mathbf{i} + b\mathbf{j}) = -\frac{1}{3}\left[\frac{1}{5}(-4\mathbf{i} + 3\mathbf{j}).(3\mathbf{i} + 2\mathbf{j})\right]$ $\{-4a + 3b = 2\}$	A1	1.1b
	Complete method [1] to solve for a and b and form **w** $\begin{cases} 3a + 4b = 17 \\ -4a + 3b = 2 \end{cases} \Rightarrow \mathbf{w} = a\mathbf{i} + b\mathbf{j}$	dM1	2.1
	$= \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}$ (**i**-**j** notation required)	A1	2.2a
		(6)	
(b)	Method [2] using scalar product and component of $(3\mathbf{i} + 2\mathbf{j})$ with a unit vector parallel to ST $(3\mathbf{i} + 4\mathbf{j})$	M1	3.1b

ALT 2	$\frac{1}{5}(3\mathbf{i}+4\mathbf{j}).(3\mathbf{i}+2\mathbf{j}) \quad \left(=\frac{17}{5}\right)$	A1	1.1b
	Method [2] using scalar product and component of $(3\mathbf{i}+2\mathbf{j})$ with a unit vector perpendicular to ST, $(-4\mathbf{i}+3\mathbf{j})$.	M1	3.1b
	$\frac{1}{5}(-4\mathbf{i}+3\mathbf{j}).(3\mathbf{i}+2\mathbf{j}) \quad \left(=-\frac{6}{5}\right)$	A1	1.1b
	Complete method [2] to combine correctly with $\pm\frac{1}{3}$ and form $\mathbf{w}$ $\mathbf{w}=\frac{17}{5}\times\frac{1}{5}(3\mathbf{i}+4\mathbf{j})+-\frac{1}{3}\times-\frac{6}{5}\times\frac{1}{5}(-4\mathbf{i}+3\mathbf{j})$	dM1	2.1
	$=\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ (**i-j** notation required)	A1	2.2a
		(6)	
(b) **ALT 3**	Method [3] finding coefficients to express $(3\mathbf{i}+2\mathbf{j})$ as multiples of $(3\mathbf{i}+4\mathbf{j})$ and $(-4\mathbf{i}+3\mathbf{j})$.	M1	3.1b
	$3\mathbf{i}+2\mathbf{j}=p(3\mathbf{i}+4\mathbf{j})+q(-4\mathbf{i}+3\mathbf{j})$	A1	1.1b
	Equate components and solve for p and q: eg $\begin{cases}3=3p-4q\\2=4p+3q\end{cases}$	M1	3.1b
	$p=\frac{17}{25} \quad q=-\frac{6}{25}$	A1	1.1b
	Complete method [3] to combine correctly with $\pm\frac{1}{3}$ and form $\mathbf{w}$ $\mathbf{w}=p(3\mathbf{i}+4\mathbf{j})+-\frac{1}{3}\times q(-4\mathbf{i}+3\mathbf{j})$	dM1	2.1
	$=\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ (**i-j** notation required)	A1	2.2a
		(6)	
(b) **ALT 4**	Method [4] using rotation matrix on $3\mathbf{i}+2\mathbf{j}$	M1	
	$\begin{pmatrix}\frac{3}{5} & \frac{4}{5}\\-\frac{4}{5} & \frac{3}{5}\end{pmatrix}\begin{pmatrix}3\\2\end{pmatrix} \quad \left\{=\begin{pmatrix}\frac{17}{5}\\-\frac{6}{5}\end{pmatrix}\right\}$	A1	
	Method [4] using $\pm\frac{1}{3}$ with the correct component to find velocity relative to ST.	M1	
	$\begin{pmatrix}\frac{17}{5}\\\frac{2}{5}\end{pmatrix}$	A1	
	Complete method [4] to reverse rotation and form $\mathbf{w}$	dM1	

	$\begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} \frac{17}{5} \\ \frac{2}{5} \end{pmatrix} = \begin{pmatrix} \frac{43}{25} \\ \frac{74}{25} \end{pmatrix}$		
	$= \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}$ (**i-j** notation required)	A1	
		(6)	
(b) **ALT 5**			
	Method [5] using speed and direction parallel to ST	M1	3.1b
	$w\cos\delta = \sqrt{13}\cos\gamma \quad (= 3.4)$	A1	1.1b
	Method [5] using speed and direction perpendicular to ST with $\pm\frac{1}{3}$	M1	3.1b
	$w\sin\delta = \frac{1}{3}\sqrt{13}\sin\gamma\,(= 0.4)$	A1	1.1b
	Complete method [5] to combine modulus with angle $(\beta+\delta)$ and form **w** $\mathbf{w} = \frac{\sqrt{293}}{5}\cos(\beta+\delta)\mathbf{i} + \frac{\sqrt{293}}{5}\sin(\beta+\delta)\mathbf{j}$	dM1	2.1
	$= \frac{43}{25}\mathbf{i} + \frac{74}{25}\mathbf{j}$ (**i-j** notation required)	A1	2.2a
		(6)	
			(12 marks)

Notes:

(a)	
B1	Correct only, 3**i**. Accept a magnitude of 3, if seen with correct direction indicated. Accept trig components with 3 and direction indicated E.g. $v\cos\theta = \sqrt{13}\cos\alpha = \sqrt{13}\times\frac{3}{\sqrt{13}} = 3$
M1	Correct use of the impact law perpendicular to RS with $\frac{1}{3}$ and 2. Allow $\pm$. M0 if separation and approach are on the wrong side.

	May also use scalar product eg $\begin{pmatrix}3\\2\end{pmatrix}\bullet\begin{pmatrix}0\\1\end{pmatrix}=-\frac{1}{3}\begin{pmatrix}x\\y\end{pmatrix}\bullet\begin{pmatrix}0\\1\end{pmatrix}$ or magnitude and direction with trig components eg $v\sin\theta=\frac{\sqrt{13}\sin\alpha}{\frac{1}{3}}=3\times\sqrt{13}\times\frac{2}{\sqrt{13}}=6$
A1	Correct perpendicular component of **v**, $-6\mathbf{j}$. Accept a magnitude of 6, if seen with correct direction indicated. A0 for $+6\mathbf{j}$
M1	Correct method for change in KE. Allow ± 6
A1	Correct unsimplified equation for change in KE (allow ± 6)
A1*	Obtain given answer from complete and correct working including consideration of parallel components. **NB** • If the given answer is obtained by only considering the KE of the perpendicular components ie $\frac{1}{2}\times 0.25\left(6^2\right)-\frac{1}{2}\times 0.25\left(2^2\right)$, the method is incomplete. Max score B1M1A1M1A1A0* • If the given answer is obtained by considering the KE of the perpendicular components, $\frac{1}{2}\times 0.25\left(6^2\right)-\frac{1}{2}\times 0.25\left(2^2\right)$ **and** states that the KE parallel to *RS* is unchanged, all marks are available. • If the solution **only** consists of the energy equation, this is insufficient working for a given answer $\frac{1}{2}\times 0.25\left(3^2+6^2\right)-\frac{1}{2}\times 0.25\left(3^2+2^2\right)$. Max score B1M0A0M1A1A0*
(b) **ALT 1**	Method [1] Accept column vectors throughout working but must be in **i-j** form for final A mark.
M1	Method [1] using scalar product and components of $\mathbf{w}=a\mathbf{i}+b\mathbf{j}$ and $\left(3\mathbf{i}+2\mathbf{j}\right)$ parallel to *ST* $\left(3\mathbf{i}+4\mathbf{j}\right)$ i.e. any multiple of $\left(3\mathbf{i}+4\mathbf{j}\right)$ to form an equation in *a* and *b*. Allow without $\frac{1}{5}$.
A1	Correct equation in *a* and *b*.
M1	Method [1] using scalar product and components of $\mathbf{w}=a\mathbf{i}+b\mathbf{j}$ and $\left(3\mathbf{i}+2\mathbf{j}\right)$ perpendicular to *ST* $\left(-4\mathbf{i}+3\mathbf{j}\right)$ i.e. any multiple of $\left(-4\mathbf{i}+3\mathbf{j}\right)$ to form an equation in *a* and *b*. Must have $\pm\frac{1}{3}$. Allow without $\frac{1}{5}$. N.B. It is possible to form 2 simultaneous equations using 2 perpendicular equations instead of 1 parallel and 1 perpendicular NEL: $\frac{1}{5}\left(-4\mathbf{i}+3\mathbf{j}\right).\left(a\mathbf{i}+b\mathbf{j}\right)=-\frac{1}{3}\left[\frac{1}{5}\left(-4\mathbf{i}+3\mathbf{j}\right).\left(3\mathbf{i}+2\mathbf{j}\right)\right] \Rightarrow -4a+3b=2$ Perp.: $\mathbf{I}=0.25\begin{pmatrix}a-3\\b-2\end{pmatrix}$ and $\mathbf{I}\bullet\begin{pmatrix}3\\4\end{pmatrix}=0 \Rightarrow 3a+4b=17$
A1	Correct equation in *a* and *b*.

dM1	Dependent on previous 2 M's. Complete method [1] to solve simultaneous equations and obtain **w**.
A1	Correct **i-j** form. Accept $\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i}+2.96\mathbf{j})$
(b) ALT 2	Method [2] Accept column vectors throughout working but must be in **i-j** form for final A mark.
M1	Method [2] using scalar product and component of $(3\mathbf{i}+2\mathbf{j})$ and unit vector parallel to *ST*
A1	Correct expression
M1	Method [2] using scalar product and component of $(3\mathbf{i}+2\mathbf{j})$ and unit vector perpendicular to *ST*
A1	Correct expression.
dM1	Dependent on previous 2 M's. Complete method [2] to combine and form **w**. Must see $\pm\frac{1}{3}$ with $(-4\mathbf{i}+3\mathbf{j})$ component.
A1	Correct **i-j** form. Accept $\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i}+2.96\mathbf{j})$
(b) ALT 3	Method [3] Accept column vectors throughout working but must be in **i-j** form for final A mark.
M1	Method [3] finding coefficients to express $(3\mathbf{i}+2\mathbf{j})$ as multiples of $(3\mathbf{i}+4\mathbf{j})$ and $(-4\mathbf{i}+3\mathbf{j})$
A1	Correct expression
M1	Solve to obtain the coefficients.
A1	Correct only.
dM1	Dependent on previous 2 M's. Complete method [3] to **w**. Must see $\pm\frac{1}{3}$ with $(-4\mathbf{i}+3\mathbf{j})$.
A1	Correct **i-j** form. Accept $\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i}+2.96\mathbf{j})$
(b) ALT 4	Method [4] Accept column vectors throughout working but must be in **i-j** form for final A mark.
M1	Method [4] using rotation matrix on 3**i** + 2**j**
A1	Correct matrix expression
M1	Method [4] using $\pm\frac{1}{3}$ with the correct component to find velocity relative to *ST*.
A1	Correct only.
dM1	Dependent on previous 2 M's. Complete method [4] by reversing rotation to form **w**

A1	Correct **i-j** form. Accept $\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i}+2.96\mathbf{j})$
(b) ALT 5	Method [5] Answer must be in **i-j** form for final A mark.
M1	Method [5] using speed and direction parallel to *ST*. Use $\sqrt{3^2+2^2}$ with γ where $\gamma=\tan^{-1}\left(\frac{4}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)$
A1	$w\cos\delta=\sqrt{3^2+2^2}\cos\gamma \quad (=3.4)$
M1	Method [5] using speed and direction perpendicular to *ST*. Use $\pm\frac{1}{3}$ and $\sqrt{3^2+2^2}$ with γ where $\gamma=\tan^{-1}\left(\frac{4}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)$
A1	$w\sin\delta=\frac{1}{3}\sqrt{3^2+2^2}\sin\gamma \quad (=0.4)$
dM1	Dependent on previous 2 M's. Complete method [5] to combine the modulus with the relevant angle and form **w** $$\mathbf{w}=\frac{\sqrt{293}}{5}\cos(\beta+\delta)\mathbf{i}+\frac{\sqrt{293}}{5}\sin(\beta+\delta)\mathbf{j}$$
A1	Correct **i-j** form. Accept $\frac{43}{25}\mathbf{i}+\frac{74}{25}\mathbf{j}$ or $(1.72\mathbf{i}+2.96\mathbf{j})$

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