Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Wednesday 18 June 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference 9FM0/3A

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. (a) Given that

$$y = e^{\text{cosec}^2 x}$$

complete the table below with the value of y corresponding to $x = 2$, giving your answer to 2 decimal places.

x	1.5	1.75	2	2.25	2.5
y	2.73	2.81	3.35	5.22	16.31

(1)

(b) Use Simpson's rule, with all the values of y in the completed table, to estimate, to one decimal place, the value of

$$\int_{1.5}^{2.5} e^{\text{cosec}^2 x}\, dx$$

(3)

(c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_{1.5}^{2.5} e^{\cot^2 x}\, dx$$

(2)

(a) $x = 2$: $\text{cosec}^2 2 = 1 \div \sin^2 2 = 1.20945\ldots$ $y = e^{1.20945\ldots} = 3.3517\ldots = 3.35$

(b) $h = \frac{2.5 - 1.5}{4} = 0.25$

$\int_{1.5}^{2.5} e^{\text{cosec}^2 x}\, dx \approx \frac{1}{3} \times 0.25 \times [\, 2.73 + 16.31 + 2(3.35) + 4(2.81 + 5.22)\,]$

$= \frac{0.25}{3} \times [\, 19.04 + 6.70 + 32.12\,] = \frac{0.25}{3} \times 57.86$

$= 4.8216\ldots = 4.8$ (1 d.p.)

(c) $\cot^2 x = \text{cosec}^2 x - 1$

$\int_{1.5}^{2.5} e^{\cot^2 x}\, dx = \int_{1.5}^{2.5} e^{\text{cosec}^2 x - 1}\, dx = \frac{1}{e} \int_{1.5}^{2.5} e^{\text{cosec}^2 x}\, dx$

$\approx \frac{1}{e} \times 4.8 = 1.7658\ldots = 1.8$ (1 d.p.)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. [*The Taylor series expansion of* $f(x)$ *about* $x = a$ *is given by*

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \ldots + \frac{(x-a)^r}{r!}f^{(r)}(a) + \ldots \quad]$$

Given that

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2xy = 4 \qquad \text{(I)}$$

(a) show that

$$\frac{d^4y}{dx^4} = a\frac{dy}{dx} + bx\frac{d^2y}{dx^2} + c\frac{d^3y}{dx^3}$$

where a, b and c are integers to be determined.

(4)

Hence, given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 2$

(b) determine a Taylor series solution, in ascending powers of $(x-2)$, up to and including the term in $(x-2)^4$, of the differential equation (I), giving each coefficient in simplest form.

(4)

(a) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 2xy = 4$

Differentiate: $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 2y - 2x\frac{dy}{dx} = 0$

Differentiate again: $\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - 2\frac{dy}{dx} - 2\frac{dy}{dx} - 2x\frac{d^2y}{dx^2} = 0$

$\frac{d^4y}{dx^4} = 4\frac{dy}{dx} + 2x\frac{d^2y}{dx^2} - 3\frac{d^3y}{dx^3}$ $\quad a = 4,\ b = 2,\ c = -3$

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Question 2 continued

(b) At $x = 2$: $y = 1$, $\frac{dy}{dx} = 1$

$$\frac{d^2y}{dx^2} = 4 - 3\frac{dy}{dx} + 2xy = 4 - 3(1) + 2(2)(1) = 5$$

$$\frac{d^3y}{dx^3} = 2y + 2x\frac{dy}{dx} - 3\frac{d^2y}{dx^2} = 2(1) + 2(2)(1) - 3(5) = -9$$

$$\frac{d^4y}{dx^4} = 4\frac{dy}{dx} + 2x\frac{d^2y}{dx^2} - 3\frac{d^3y}{dx^3} = 4(1) + 2(2)(5) - 3(-9) = 51$$

$$y = y(2) + (x-2)y'(2) + \frac{(x-2)^2}{2!}y''(2) + \frac{(x-2)^3}{3!}y'''(2) + \frac{(x-2)^4}{4!}y^{(4)}(2) + \ldots$$

$$y = 1 + (x-2) + \frac{5}{2}(x-2)^2 + \frac{-9}{6}(x-2)^3 + \frac{51}{24}(x-2)^4 + \ldots$$

$$y = 1 + (x-2) + \frac{5}{2}(x-2)^2 - \frac{3}{2}(x-2)^3 + \frac{17}{8}(x-2)^4 + \ldots$$

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the values of x for which

$$\frac{x^2 - 4}{|x - 5|} > 4x + 8$$

(6)

Critical value from the modulus: $x = 5$ ($x \neq 5$, the expression is undefined there)

Case $x > 5$: $|x - 5| = x - 5$

$x^2 - 4 = (4x + 8)(x - 5) \Rightarrow x^2 - 4 = 4x^2 - 12x - 40$

$\Rightarrow 3x^2 - 12x - 36 = 0 \Rightarrow x^2 - 4x - 12 = 0 \Rightarrow (x - 6)(x + 2) = 0 \Rightarrow x = 6, -2$

For $x > 5$: $x^2 - 4 > (4x + 8)(x - 5) \Leftrightarrow 3x^2 - 12x - 36 < 0 \Leftrightarrow -2 < x < 6$

so $5 < x < 6$

Case $x < 5$: $|x - 5| = 5 - x$

$x^2 - 4 = (4x + 8)(5 - x) \Rightarrow x^2 - 4 = -4x^2 + 12x + 40$

$\Rightarrow 5x^2 - 12x - 44 = 0 \Rightarrow (5x - 22)(x + 2) = 0 \Rightarrow x = \frac{22}{5}, -2$

For $x < 5$: $5x^2 - 12x - 44 > 0 \Leftrightarrow x < -2$ or $x > \frac{22}{5}$

so $x < -2$ or $\frac{22}{5} < x < 5$

Critical values: $x = -2,\ 22/5,\ 5,\ 6$

$\therefore\ x < -2, \quad \frac{22}{5} < x < 5, \quad 5 < x < 6$

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 6 marks)

4. $$y = \sqrt{\left(15 + e^{2x}\right)}$$

(a) Show that

$$\frac{d^2y}{dx^2} = \frac{e^{2x}\left(e^{2x} + 30\right)}{\left(15 + e^{2x}\right)^{\frac{3}{2}}}$$

(3)

(b) Hence determine the Maclaurin series expansion for y, in ascending powers of x, up to and including the term in x^2, giving each term in simplest form.

(2)

(c) Use the series expansion for $\sin x$ in ascending powers of x to determine, in simplest form, the first 2 non-zero terms in ascending powers of x of the series for $\sin 3x$.

(1)

(d) Use the answers to parts (b) and (c) to show that

$$\lim_{x \to 0} \frac{\sqrt{\left(15 + e^{2x}\right)} - 4}{\sin 3x} = \frac{1}{12}$$

(2)

(a) $y = (15 + e^{2x})^{½} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(15 + e^{2x})^{-½} \times 2e^{2x} = e^{2x}(15 + e^{2x})^{-½}$

$\frac{d^2y}{dx^2} = 2e^{2x}(15 + e^{2x})^{-½} + e^{2x} \times \left(-\frac{1}{2}\right)(15 + e^{2x})^{-3/2} \times 2e^{2x}$

$= 2e^{2x}(15 + e^{2x})^{-½} - e^{4x}(15 + e^{2x})^{-3/2}$

$= \frac{2e^{2x}(15 + e^{2x}) - e^{4x}}{(15 + e^{2x})^{3/2}} = \frac{30e^{2x} + 2e^{4x} - e^{4x}}{(15 + e^{2x})^{3/2}} = \frac{e^{2x}(e^{2x} + 30)}{(15 + e^{2x})^{3/2}}$ as required

(b) $x = 0$: $y = \sqrt{16} = 4$, $\frac{dy}{dx} = \frac{1}{\sqrt{16}} = \frac{1}{4}$, $\frac{d^2y}{dx^2} = \frac{1 \times 31}{16^{3/2}} = \frac{31}{64}$

$y = 4 + \frac{1}{4}x + \frac{31}{64} \times \frac{x^2}{2!} + \ldots \Rightarrow y = 4 + \frac{1}{4}x + \frac{31}{128}x^2 + \ldots$

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Question 4 continued

(c) $\sin x = x - \frac{x^3}{3!} + \ldots \Rightarrow \sin 3x = 3x - \frac{(3x)^3}{6} + \ldots = 3x - \frac{9}{2}x^3 + \ldots$

(d) $\lim_{x\to 0} \frac{\sqrt{(15 + e^{2x})} - 4}{\sin 3x} = \lim_{x\to 0} \frac{4 + x/4 + (31/128)x^2 + \ldots - 4}{3x - (9/2)x^3 + \ldots}$

$= \lim_{x\to 0} \frac{x/4 + (31/128)x^2 + \ldots}{3x - (9/2)x^3 + \ldots} = \lim_{x\to 0} \frac{1/4 + (31/128)x + \ldots}{3 - (9/2)x^2 + \ldots}$

$= \frac{1/4}{3} = \frac{1}{12}$ as required

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5. (a) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$\int \frac{1}{1+\operatorname{cosec} x}\,\mathrm{d}x = \int \frac{4t}{\left(1+t^2\right)\left(1+t\right)^2}\,\mathrm{d}t$$

(3)

(b) Hence show that

$$\int \frac{1}{1+\operatorname{cosec} x}\,\mathrm{d}x = Ax + \frac{B}{1+\tan\left(\frac{x}{2}\right)} + k$$

where A and B are constants to be determined and k is an arbitrary constant.

(5)

(a) $t = \tan\frac{x}{2}$: $\sin x = \frac{2t}{1+t^2}$, $\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2} = \frac{1+t^2}{2} \Rightarrow dx = \frac{2}{1+t^2}\,dt$

$1 + \operatorname{cosec} x = 1 + \frac{1+t^2}{2t} = \frac{t^2+2t+1}{2t} = \frac{(1+t)^2}{2t}$

$\int \frac{1}{1+\operatorname{cosec} x}\,dx = \int \frac{2t}{(1+t)^2} \times \frac{2}{1+t^2}\,dt = \int \frac{4t}{(1+t^2)(1+t)^2}\,dt$ as required

(b) $\frac{4t}{(1+t^2)(1+t)^2} \equiv \frac{At+B}{1+t^2} + \frac{C}{1+t} + \frac{D}{(1+t)^2}$

$4t \equiv (At+B)(1+t)^2 + C(1+t)(1+t^2) + D(1+t^2)$

$t = -1$: $-4 = 2D \Rightarrow D = -2$

$t = 0$: $0 = B + C + D \Rightarrow B + C = 2$; t^3: $0 = A + C$

$t = 1$: $4 = 4(A+B) + 4C + 2D \Rightarrow A + B + C = 2 \Rightarrow A = 0,\ C = 0,\ B = 2$

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Question 5 continued

$$\frac{4t}{(1+t^2)(1+t)^2} \equiv \frac{2}{1+t^2} - \frac{2}{(1+t)^2}$$

$$\int \frac{2}{1+t^2} - \frac{2}{(1+t)^2}\,dt = 2\arctan t + \frac{2}{1+t} + k$$

$$= 2\arctan\left(\tan\frac{x}{2}\right) + \frac{2}{1+\tan(x/2)} + k = x + \frac{2}{1+\tan(x/2)} + k$$

$A = 1, \quad B = 2$



Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

6. (i) Use L'Hospital's Rule to show that

$$\lim_{x \to 0} \frac{1 - \cos 7x}{x \sin 9x} = \frac{49}{18}$$

(4)

(ii) $$y = x^2 e^{3x}$$

(a) Use Leibnitz's theorem to show that, for $k \in \mathbb{N}$

$$\frac{\mathrm{d}^k y}{\mathrm{d}x^k} = 3^{k-2} e^{3x} \left(Ax^2 + Bkx + Ck(k-1) \right)$$

where A, B and C are integers to be determined.

(5)

(b) Hence determine the values of x for which

$$\frac{\mathrm{d}^9 y}{\mathrm{d}x^9} = 0$$

(2)

(i) $\frac{1 - \cos 7x}{x \sin 9x} \to \frac{0}{0}$ as $x \to 0$. L'Hospital: $\lim_{x \to 0} \frac{1 - \cos 7x}{x \sin 9x} = \lim_{x \to 0} \frac{7 \sin 7x}{\sin 9x + 9x \cos 9x}$

Still $\frac{0}{0}$ at $x = 0$, so again: $= \lim_{x \to 0} \frac{49 \cos 7x}{9 \cos 9x + 9 \cos 9x - 81x \sin 9x}$

$= \frac{49 \cos 0}{9 \cos 0 + 9 \cos 0 - 0} = \frac{49}{9 + 9} = \frac{49}{18}$ as required

(ii)(a) $u = x^2$: $\frac{du}{dx} = 2x$, $\frac{d^2u}{dx^2} = 2$, $\frac{d^3u}{dx^3} = 0$

$v = e^{3x}$: $\frac{dv}{dx} = 3e^{3x}$, $\frac{d^2v}{dx^2} = 9e^{3x}$, $\frac{d^3v}{dx^3} = 27e^{3x}$, ..., $\frac{d^kv}{dx^k} = 3^k e^{3x}$

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Question 6 continued

Leibniz: $\frac{d^k y}{dx^k} = u\frac{d^k v}{dx^k} + k\frac{du}{dx}\frac{d^{k-1}v}{dx^{k-1}} + \frac{k(k-1)}{2!}\frac{d^2u}{dx^2}\frac{d^{k-2}v}{dx^{k-2}}$

$= 3^k e^{3x}x^2 + k \times 2x \times 3^{k-1}e^{3x} + \frac{k(k-1)}{2} \times 2 \times 3^{k-2}e^{3x}$

$= 3^{k-2}e^{3x}(9x^2 + 6kx + k(k-1))$

$A = 9, \quad B = 6, \quad C = 1$

(ii)(b) $\frac{d^9 y}{dx^9} = 3^7 e^{3x}(9x^2 + 6 \times 9x + 9 \times 8) = 0$

$e^{3x} \neq 0 \Rightarrow 9x^2 + 54x + 72 = 0 \Rightarrow x^2 + 6x + 8 = 0 \Rightarrow (x+2)(x+4) = 0$

$x = -4, \quad x = -2$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 11 marks)

7. The concentration, $P\,\text{mg m}^{-3}$, of a pollutant in a reservoir, t days after the pollutant entered the reservoir, is modelled by the differential equation

$$\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}t} = 4tP^2 - 1 \qquad \text{(I)}$$

(a) Show that the transformation $x = \dfrac{1}{P^2}$ transforms equation (I) into the equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} - 2x = -8t \qquad \text{(II)}$$

(3)

Given that $P = 0.5$ when $t = 0$

(b) solve differential equation (II) to show that, according to the model,

$$P^2 = \frac{1}{4t + 2 + k\mathrm{e}^{2t}}$$

where k is a constant to be determined.

(6)

Given that the concentration of the pollutant in the reservoir, 3 days after the pollutant entered the reservoir, was $0.034\,\text{mg m}^{-3}$

(c) comment on the reliability of the model, giving a reason for your answer.

(2)

(a) $x = P^{-2} \Rightarrow \dfrac{dx}{dt} = -2P^{-3}\dfrac{dP}{dt} \Rightarrow \dfrac{dP}{dt} = -\dfrac{P^3}{2}\dfrac{dx}{dt}$

$\dfrac{1}{P}\dfrac{dP}{dt} = -\dfrac{P^2}{2}\dfrac{dx}{dt} = -\dfrac{1}{2x}\dfrac{dx}{dt}$ and $4tP^2 - 1 = \dfrac{4t}{x} - 1$

$-\dfrac{1}{2x}\dfrac{dx}{dt} = \dfrac{4t}{x} - 1 \Rightarrow$ (multiply by $-2x$) $\dfrac{dx}{dt} = -8t + 2x$

$\Rightarrow \dfrac{dx}{dt} - 2x = -8t$ as required

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Question 7 continued

(b) Integrating factor $I = e^{\int -2\,dt} = e^{-2t}$

$\frac{d}{dt}(xe^{-2t}) = -8te^{-2t} \Rightarrow xe^{-2t} = \int -8te^{-2t}\,dt$

By parts: $\int -8te^{-2t}\,dt = 4te^{-2t} - \int 4e^{-2t}\,dt = 4te^{-2t} + 2e^{-2t} + k$

$xe^{-2t} = 4te^{-2t} + 2e^{-2t} + k \Rightarrow x = 4t + 2 + ke^{2t}$

$\frac{1}{P^2} = 4t + 2 + ke^{2t}$; $P = 0.5, t = 0$: $\frac{1}{0.5^2} = 4 = 2 + k \Rightarrow k = 2$

$\frac{1}{P^2} = 4t + 2 + 2e^{2t} \Rightarrow P^2 = \frac{1}{4t + 2 + 2e^{2t}}$

(c) $t = 3$: $P^2 = \frac{1}{12 + 2 + 2e^6} = \frac{1}{14 + 2e^6} = 0.0012182\ldots \Rightarrow P = 0.034903\ldots$

0.0349 mg m^{-3} is very close to the actual value of 0.034 mg m^{-3}, so the model is reliable (a good model) at $t = 3$.

Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

8. The ellipse E has equation

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

The line l has equation $y = mx + c$ where m and c are constants.

(a) Show that the x coordinates of the points of intersection of E and l satisfy the equation

$$\left(16m^2 + 9\right)x^2 + 32cmx + 16c^2 + k = 0$$

where k is a constant to be determined.

(2)

Hence, given that the line l

- is a tangent to E
- passes through the point (10, 20)

(b) determine the possible equations of l

(5)

(a) $\frac{x^2}{64} + \frac{(mx + c)^2}{36} = 1 \Rightarrow \frac{x^2}{64} + \frac{m^2x^2 + 2cmx + c^2}{36} = 1$

$\times 2304$: $36x^2 + 64m^2x^2 + 128cmx + 64c^2 = 2304$

$\div 4$: $(16m^2 + 9)x^2 + 32cmx + 16c^2 - 576 = 0$ $\quad k = -576$

(b) Tangent $\Rightarrow$ repeated root $\Rightarrow b^2 - 4ac = 0$

$(32cm)^2 - 4(16m^2 + 9)(16c^2 - 576) = 0$

$1024c^2m^2 - 1024c^2m^2 + 36864m^2 - 576c^2 + 20736 = 0 \Rightarrow 64m^2 - c^2 + 36 = 0$

Through (10, 20): $20 = 10m + c \Rightarrow c = 20 - 10m$

$64m^2 - (20 - 10m)^2 + 36 = 0 \Rightarrow 64m^2 - 400 + 400m - 100m^2 + 36 = 0$

$\Rightarrow 36m^2 - 400m + 364 = 0 \Rightarrow 9m^2 - 100m + 91 = 0 \Rightarrow (m - 1)(9m - 91) = 0$

$m = 1 \Rightarrow c = 10$; $\quad m = \frac{91}{9} \Rightarrow c = 20 - \frac{910}{9} = -\frac{730}{9}$

$y = x + 10$ and $y = \frac{91}{9}x - \frac{730}{9}$

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 7 marks)

9. The parallelogram P has vertices E, F, G and H, with coordinates

$$E(10, -1, -6) \quad F(7, -2, 7) \quad G(10, 2, 9) \quad H(13, 3, -4)$$

(a) Determine the exact area of P, giving your answer as a simplified surd.

(3)

The line l passes through E and is perpendicular to the plane containing P

(b) Determine an equation for l giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(2)

Given that

- P is one face of a parallelepiped
- the opposite face of the parallelepiped lies in the plane Π
- the point (25, –4, 13) lies in Π

(c) determine the volume of the parallelepiped.

(5)

(a) $\overrightarrow{EF} = \begin{pmatrix} 7-10 \\ -2+1 \\ 7+6 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 13 \end{pmatrix}$, $\overrightarrow{EH} = \begin{pmatrix} 13-10 \\ 3+1 \\ -4+6 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$

$\overrightarrow{EF} \times \overrightarrow{EH} = \begin{pmatrix} (-1)(2) - (13)(4) \\ (13)(3) - (-3)(2) \\ (-3)(4) - (-1)(3) \end{pmatrix} = \begin{pmatrix} -54 \\ 45 \\ -9 \end{pmatrix}$

Area $= |\overrightarrow{EF} \times \overrightarrow{EH}| = \sqrt{(54^2 + 45^2 + 9^2)} = \sqrt{5022} = 9\sqrt{62}$

(b) Normal to plane: $-54\mathbf{i} + 45\mathbf{j} - 9\mathbf{k} = -9(6\mathbf{i} - 5\mathbf{j} + \mathbf{k})$

l: $(\mathbf{r} - (10\mathbf{i} - \mathbf{j} - 6\mathbf{k})) \times (6\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = \mathbf{0}$

(c) Π is parallel to P, so normal $6\mathbf{i} - 5\mathbf{j} + \mathbf{k}$: $6x - 5y + z = d$

(25, –4, 13): $d = 6(25) - 5(-4) + 13 = 183 \Rightarrow \Pi$: $6x - 5y + z = 183$

l: $\mathbf{r} = (10 + 6\lambda)\mathbf{i} + (-1 - 5\lambda)\mathbf{j} + (-6 + \lambda)\mathbf{k}$ meets Π:

$6(10 + 6\lambda) - 5(-1 - 5\lambda) + (\lambda - 6) = 183 \Rightarrow 59 + 62\lambda = 183 \Rightarrow \lambda = 2$

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Question 9 continued

Perpendicular height = $|2(6\mathbf{i} - 5\mathbf{j} + \mathbf{k})| = 2\sqrt{(6^2 + 5^2 + 1^2)} = 2\sqrt{62}$

Volume = area × height = $9\sqrt{62} \times 2\sqrt{62} = 18 \times 62$

Volume = 1116

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Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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