Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 22 May 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/02**

Further Mathematics

Advanced

PAPER 2: Core Pure Mathematics 2

You must have:
Mathematical Formulae and Statistics Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Given that

$$z = 2 - 2\sqrt{3}\,\mathrm{i} \text{ and } w = -1 + \sqrt{3}\,\mathrm{i}$$

show that

(a) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$ **(3)**

(b) $\arg(zw) = \arg(z) + \arg(w)$ **(3)**

(a) $|z| = \sqrt{(2^2 + (-2\sqrt{3})^2)} = \sqrt{(4 + 12)} = 4$ $\quad |w| = \sqrt{((-1)^2 + (\sqrt{3})^2)} = \sqrt{(1 + 3)} = 2$

$\frac{|z|}{|w|} = \frac{4}{2} = 2$

$\frac{z}{w} = \frac{2 - 2\sqrt{3}i}{-1 + \sqrt{3}i} \times \frac{-1 - \sqrt{3}i}{-1 - \sqrt{3}i} = \frac{-2 - 2\sqrt{3}i + 2\sqrt{3}i + 6i^2}{1 + 3} = \frac{-8}{4} = -2$

$\left|\frac{z}{w}\right| = |-2| = 2 = \frac{|z|}{|w|} \quad \therefore \left|\frac{z}{w}\right| = \frac{|z|}{|w|}$

(b) z is in the 4th quadrant: $\arg(z) = -\arctan\left(\frac{2\sqrt{3}}{2}\right) = -\arctan\sqrt{3} = -\frac{\pi}{3}$

w is in the 2nd quadrant: $\arg(w) = \pi - \arctan\left(\frac{\sqrt{3}}{1}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$zw = (2 - 2\sqrt{3}i)(-1 + \sqrt{3}i) = -2 + 2\sqrt{3}i + 2\sqrt{3}i - 6i^2 = 4 + 4\sqrt{3}i$

zw is in the 1st quadrant: $\arg(zw) = \arctan\left(\frac{4\sqrt{3}}{4}\right) = \arctan\sqrt{3} = \frac{\pi}{3}$

$\arg(z) + \arg(w) = -\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3} = \arg(zw)$

$\therefore \arg(zw) = \arg(z) + \arg(w)$

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Question 1 continued

(Total for Question 1 is 6 marks)

2. An archer shoots an arrow towards a target.

In a model

- the arrow is a particle
- the flight path of the arrow is a straight line
- the target is part of a plane

Relative to a fixed origin O

- the arrow is fired from the point with position vector $3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$
- the plane containing the target has equation $2x + 4y - z = 3$

Use the model to answer parts (a) to (d).

(a) Determine the shortest distance that the arrow must travel to reach the plane.

(2)

The arrow hits the target at the point with position vector $6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

(b) Determine a vector equation of the flight path of the arrow.

(2)

(c) Determine the acute angle that the flight path of the arrow makes with the target. Give your answer to the nearest degree.

(2)

(d) Determine the distance travelled by the arrow.

(1)

(e) Comment on whether the actual distance travelled by the arrow is likely to match the answer to part (d), giving a reason for your answer.

(1)

(a) Shortest distance $= \dfrac{|2(3) + 4(-5) - 1(2) - 3|}{\sqrt{(2^2 + 4^2 + (-1)^2)}} = \dfrac{|-19|}{\sqrt{21}} = \dfrac{19}{\sqrt{21}} = 4.146\ldots$

Shortest distance = 4.15 (3 s.f.)

(b) Direction: $\begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$

$\mathbf{r} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$

(c) $\cos\theta = \dfrac{(3)(2) + (3)(4) + (-1)(-1)}{\sqrt{(3^2 + 3^2 + (-1)^2)} \times \sqrt{(2^2 + 4^2 + (-1)^2)}} = \dfrac{19}{\sqrt{19} \times \sqrt{21}} = 0.9511\ldots$

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Question 2 continued

$\theta = 17.97...°$ (angle between the flight path and the normal to the plane)

Angle with the target $= 90° - 17.97...° = 72.02...° \approx 72°$

(d) Distance $= \left| \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} \right| = \sqrt{(3^2 + 3^2 + (-1)^2)} = \sqrt{19} = 4.358... \approx 4.36$

(e) Not likely to match: the arrow is unlikely to travel in a straight line (it will follow a curved path under gravity), so the distance will be different.

Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

3. Given

$$\mathbf{A} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$$

(a) prove by mathematical induction that, for $n \in \mathbb{N}$

$$\mathbf{A}^n = \begin{pmatrix} 1 & 5(2^n - 1) \\ 0 & 2^n \end{pmatrix}$$

(6)

Given

$$\mathbf{B} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the single geometrical transformation P represented by the matrix $\mathbf{B}$

(2)

The transformation Q is represented by the matrix $\mathbf{A}^n$

The transformation P followed by the transformation Q is the transformation R, which is represented by the matrix $\mathbf{C}$

(c) Determine $\mathbf{C}$ in terms of n

(1)

Given that, for a particular value of n, the transformation R maps the point with coordinates (27, 1) to the point with coordinates (a, a), where a is a constant,

(d) determine the matrix that represents the transformation Q

(3)

(a) $n = 1$: LHS $= A^1 = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$, RHS $= \begin{pmatrix} 1 & 5(2^1 - 1) \\ 0 & 2^1 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$

So the result is true for $n = 1$

Assume true for $n = k$: $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^k = \begin{pmatrix} 1 & 5(2^k - 1) \\ 0 & 2^k \end{pmatrix}$

Then $\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 5(2^k - 1) \\ 0 & 2^k \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 5 + 2 \times 5(2^k - 1) \\ 0 & 2 \times 2^k \end{pmatrix}$

$= \begin{pmatrix} 1 & 10(2^k) - 5 \\ 0 & 2^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 5(2 \times 2^k - 1) \\ 0 & 2^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 5(2^{k+1} - 1) \\ 0 & 2^{k+1} \end{pmatrix}$

which is the result for $n = k + 1$

If the result is true for $n = k$ then it is true for $n = k + 1$, and as it is true for $n = 1$, the result is true for all $n \in \mathbb{N}$

(b) P is a reflection in the y-axis (the line $x = 0$)

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Question 3 continued

(c) $C = A^nB = \begin{pmatrix} 1 & 5(2^n - 1) \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 5(2^n - 1) \\ 0 & 2^n \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 5(2^n - 1) \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} 27 \\ 1 \end{pmatrix} = \begin{pmatrix} -27 + 5(2^n - 1) \\ 2^n \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix}$

$\Rightarrow -27 + 5(2^n) - 5 = 2^n \Rightarrow 4(2^n) = 32 \Rightarrow 2^n = 8$

Q: $A^n = \begin{pmatrix} 1 & 5(8 - 1) \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 35 \\ 0 & 8 \end{pmatrix}$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 12 marks)

4. (i) $z_1 = a + b\text{i}$ and $z_2 = c + d\text{i}$

where a, b, c and d are real constants.

Given that

- $b > d$
- $z_1 + z_2$ is real
- $|z_1| = \sqrt{13}$
- $|z_2| = 5$
- $\text{Re}(z_2 - z_1) = 2$

show that $a = 2$ and determine the value of each of b, c and d

(5)

(ii) (a) On the same Argand diagram

- sketch the locus of points z which satisfy $|z - 12| = 7$
- sketch the locus of points w which satisfy $|w - 5\text{i}| = 4$

showing the coordinates of any points of intersection with the axes.

(2)

(b) Determine the range of possible values of $|z - w|$

(3)

(i) $z_1 + z_2 = (a + c) + (b + d)i$ is real $\Rightarrow$ $b + d = 0$ $\Rightarrow$ $d = -b$

$\text{Re}(z_2 - z_1) = c - a = 2$ $\Rightarrow$ $c = a + 2$

$|z_1| = \sqrt{13}$ $\Rightarrow$ $a^2 + b^2 = 13$ $\quad |z_2| = 5$ $\Rightarrow$ $c^2 + d^2 = 25$

$\Rightarrow$ $(a + 2)^2 + (-b)^2 = 25$ $\Rightarrow$ $a^2 + 4a + 4 + b^2 = 25$

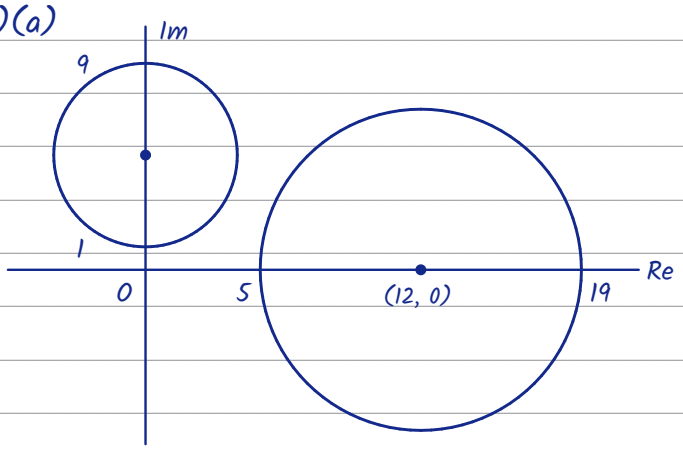
$\Rightarrow$ $13 + 4a + 4 = 25$ $\Rightarrow$ $4a = 8$ $\Rightarrow$ $a = 2$

$b^2 = 13 - 2^2 = 9$ $\Rightarrow$ $b = \pm 3$; $\quad d = -b$ and $b > d$ $\Rightarrow$ $b = 3$, $\quad d = -3$

$c = a + 2 = 4$

$a = 2$, $\quad b = 3$, $\quad c = 4$, $\quad d = -3$

(ii)(a)



$|z - 12| = 7$:
circle, centre (12, 0),
radius 7

$|w - 5i| = 4$:
circle, centre (0, 5),
radius 4

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Question 4 continued

(b) Distance between centres (12, 0) and (0, 5) = $\sqrt{(12^2 + 5^2)} = 13$

$13 > 7 + 4 = 11$, so the circles do not meet

Minimum $|z - w| = 13 - (7 + 4) = 2$

Maximum $|z - w| = 13 + (7 + 4) = 24$

$2 \leqslant |z - w| \leqslant 24$

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 10 marks)

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5. Three planes are defined by the following equations

$$2x - y + z = 3$$
$$x + py - 3z = q$$
$$3x + y - 2z = 4$$

where p and q are constants.

Given that the planes form a sheaf, determine

(i) the value of p

(ii) the value of q

(4)

(i) Planes form a sheaf ⇒ no unique solution ⇒ determinant = 0

$$\det\begin{pmatrix} 2 & -1 & 1 \\ 1 & p & -3 \\ 3 & 1 & -2 \end{pmatrix} = 2(-2p - (-3)) - (-1)(-2 - (-9)) + 1(1 - 3p)$$

$$= -4p + 6 + 7 + 1 - 3p = 14 - 7p$$

$14 - 7p = 0 \Rightarrow p = 2$

(ii) With $p = 2$: $(2x - y + z = 3) + (x + 2y - 3z = q)$

$\Rightarrow 3x + y - 2z = 3 + q$

The planes share a common line, so this must be the plane $3x + y - 2z = 4$

$\Rightarrow 3 + q = 4 \Rightarrow q = 1$

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Question 5 continued

(Total for Question 5 is 4 marks)

6. The quartic equation

$$2x^4 + Ax^3 - Ax^2 - 5x + 6 = 0$$

where A is a real constant, has roots α, β, γ and δ

(a) Determine the value of

$$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta}$$

(3)

Given that $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -\frac{3}{4}$

(b) determine the possible values of A

(5)

(a) $\alpha\beta\gamma\delta = \frac{6}{2} = 3$ $\quad \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{-5}{2} = \frac{5}{2}$

$$\frac{3}{\alpha} + \frac{3}{\beta} + \frac{3}{\gamma} + \frac{3}{\delta} = \frac{3(\beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta + \alpha\beta\gamma)}{\alpha\beta\gamma\delta} = \frac{3 \times 5/2}{3} = \frac{5}{2}$$

(b) $\alpha + \beta + \gamma + \delta = -\frac{A}{2}$ $\quad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{-A}{2}$

$$(\alpha + \beta + \gamma + \delta)^2 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$$

$$\left(\frac{-A}{2}\right)^2 = -\frac{3}{4} + 2\left(\frac{-A}{2}\right)$$

$$\frac{A^2}{4} = -\frac{3}{4} - A \Rightarrow A^2 + 4A + 3 = 0 \Rightarrow (A+1)(A+3) = 0$$

$A = -1$ or $A = -3$

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

7.

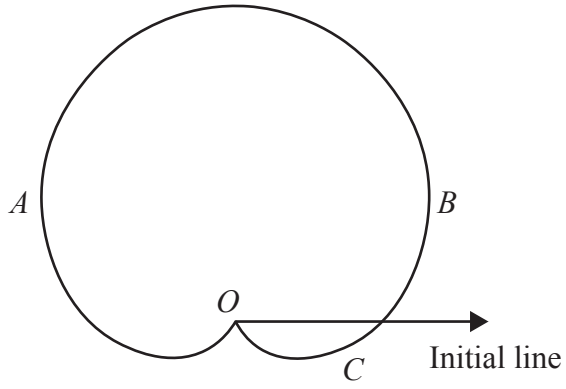


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = a(1 + \sin\theta) \qquad -\pi < \theta \leqslant \pi$$

where a is a constant.

The tangents to C at the points A and B are perpendicular to the initial line.

(a) Use calculus to determine the polar coordinates of A and B

(6)

The curve C models the perimeter of the surface of a swimming pool.

Given that, according to the model, the distance across the pool from A to B is 10 m,

(b) show that $a = \dfrac{20\sqrt{3}}{9}$

(2)

(c) Use algebraic integration to determine the surface area of the swimming pool, according to the model.

(4)

(a) $x = r\cos\theta = a(1 + \sin\theta)\cos\theta = a(\cos\theta + \frac{1}{2}\sin 2\theta)$

$\dfrac{dx}{d\theta} = a(-\sin\theta + \cos 2\theta)$

Tangent perpendicular to the initial line $\Rightarrow \dfrac{dx}{d\theta} = 0$

$-\sin\theta + 1 - 2\sin^2\theta = 0 \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0 \Rightarrow \sin\theta = \dfrac{1}{2}$ ($\sin\theta = -1$ gives $r = 0$, the pole O)

$\theta = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$, $r = a\left(1 + \dfrac{1}{2}\right) = \dfrac{3a}{2}$

A $\left(\dfrac{3a}{2}, \dfrac{5\pi}{6}\right)$ B $\left(\dfrac{3a}{2}, \dfrac{\pi}{6}\right)$

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Question 7 continued

(b) $AB = 2 \times r \cos\frac{\pi}{6} = 2 \times \frac{3a}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}a}{2} = 10$

$\Rightarrow a = \frac{20}{3\sqrt{3}} = \frac{20\sqrt{3}}{9}$

(c) Area $= \frac{1}{2}\int_{-\pi}^{\pi} r^2\, d\theta = \frac{1}{2}a^2\int_{-\pi}^{\pi} (1 + 2\sin\theta + \sin^2\theta)\, d\theta$ $\quad \sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$= \frac{1}{2}a^2\int_{-\pi}^{\pi} \left(\frac{3}{2} + 2\sin\theta - \frac{1}{2}\cos 2\theta\right) d\theta$

$= \frac{1}{2}a^2\left[\frac{3\theta}{2} - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{-\pi}^{\pi}$

$= \frac{1}{2}a^2\left[\left(\frac{3\pi}{2} + 2 - 0\right) - \left(-\frac{3\pi}{2} + 2 - 0\right)\right] = \frac{1}{2}a^2 \times 3\pi = \frac{3\pi}{2}a^2$

$a^2 = \frac{400 \times 3}{81} = \frac{400}{27}$ $\quad$ Area $= \frac{3\pi}{2} \times \frac{400}{27} = \frac{200\pi}{9} = 69.81\ldots$

Surface area $\approx 69.8\ m^2$

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8. Given that

$$y = \cos x \sinh x \qquad x \in \mathbb{R}$$

(a) show that

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = ky$$

where k is a constant to be determined.

(5)

(b) Hence determine the first three non-zero terms of the Maclaurin series for y, giving each coefficient in simplest form.

(3)

(a) $y = \cos x \sinh x$

$\frac{dy}{dx} = -\sin x \sinh x + \cos x \cosh x$

$\frac{d^2y}{dx^2} = -\cos x \sinh x - \sin x \cosh x - \sin x \cosh x + \cos x \sinh x = -2\sin x \cosh x$

$\frac{d^3y}{dx^3} = -2\cos x \cosh x - 2\sin x \sinh x$

$\frac{d^4y}{dx^4} = 2\sin x \cosh x - 2\cos x \sinh x - 2\cos x \sinh x - 2\sin x \cosh x = -4\cos x \sinh x$

$\therefore \frac{d^4y}{dx^4} = -4y, \quad k = -4$

(b) $\frac{d^5y}{dx^5} = -4\frac{dy}{dx}$

At $x = 0$: $y = 0$, $y' = 1$, $y'' = 0$, $y''' = -2$, $y^{(4)} = 0$, $y^{(5)} = -4$

$y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{(4)}(0) + \frac{x^5}{5!}y^{(5)}(0) + \ldots$

$= x + \frac{-2}{6}x^3 + \frac{-4}{120}x^5 + \ldots$

$y = x - \frac{1}{3}x^3 - \frac{1}{30}x^5 + \ldots$

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 8 marks)

9. Given that

$$y = \arcsin 3x \qquad -\frac{1}{3} \leqslant x \leqslant \frac{1}{3}$$

(a) determine $\frac{dy}{dx}$ in terms of x

(2)

The curve C has equation

$$y = \cos(\arcsin 3x) \qquad -\frac{1}{3} \leqslant x \leqslant \frac{1}{3}$$

(b) Determine $\frac{dy}{dx}$ giving your answer in simplest form.

(2)

(c) Hence determine the x coordinate of the point on C at which the gradient is 4

(3)

(a) $y = \arcsin 3x \Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{(1-(3x)^2)}} = \frac{3}{\sqrt{(1-9x^2)}}$

(b) $y = \cos(\arcsin 3x) \Rightarrow \frac{dy}{dx} = -\sin(\arcsin 3x) \times \frac{3}{\sqrt{(1-9x^2)}}$

$\frac{dy}{dx} = -\frac{9x}{\sqrt{(1-9x^2)}}$

(c) $-\frac{9x}{\sqrt{(1-9x^2)}} = 4 \Rightarrow -9x = 4\sqrt{(1-9x^2)}$

$\Rightarrow 81x^2 = 16(1-9x^2) \Rightarrow 225x^2 = 16 \Rightarrow x^2 = \frac{16}{225}$

$x = \pm \frac{4}{15}$; but $-9x = 4\sqrt{(1-9x^2)} > 0$ so $x < 0$

$x = -\frac{4}{15}$

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Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS

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