Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 20 May 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference 9FM0/01

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistics Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1.

$$\mathbf{M} = \begin{pmatrix} 3 & 6 & 0 \\ a & 3 & 1 \\ 2 & a & a \end{pmatrix} \quad \text{where } a \text{ is a constant}$$

(a) Determine the values of a for which the matrix $\mathbf{M}$ is singular.

(3)

Given that matrix $\mathbf{M}$ is non-singular and that

- $\det(\mathbf{M}) = -108$
- $a < 0$

(b) determine the matrix $\mathbf{M}^{-1}$

(3)

(a) $\det M = 3(3a - a) - 6(a^2 - 2) + 0 = 0$

$6a - 6a^2 + 12 = 0 \Rightarrow 6a^2 - 6a - 12 = 0 \Rightarrow a^2 - a - 2 = 0$

$(a - 2)(a + 1) = 0 \Rightarrow a = -1 \text{ or } a = 2$

(b) $\det M = -6a^2 + 6a + 12 = -108$

$6a^2 - 6a - 120 = 0 \Rightarrow a^2 - a - 20 = 0$

$(a - 5)(a + 4) = 0, \quad a < 0 \Rightarrow a = -4$

$$M = \begin{pmatrix} 3 & 6 & 0 \\ -4 & 3 & 1 \\ 2 & -4 & -4 \end{pmatrix} \quad \text{minors: } \begin{pmatrix} -8 & 14 & 10 \\ -24 & -12 & -24 \\ 6 & 3 & 33 \end{pmatrix}$$

$$\text{cofactors: } \begin{pmatrix} -8 & -14 & 10 \\ 24 & -12 & 24 \\ 6 & -3 & 33 \end{pmatrix} \quad \text{adj } M = \begin{pmatrix} -8 & 24 & 6 \\ -14 & -12 & -3 \\ 10 & 24 & 33 \end{pmatrix}$$

$$M^{-1} = \frac{1}{\det M} \text{adj } M = \frac{1}{-108} \begin{pmatrix} -8 & 24 & 6 \\ -14 & -12 & -3 \\ 10 & 24 & 33 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{2}{27} & -\frac{2}{9} & -\frac{1}{18} \\ \frac{7}{54} & \frac{1}{9} & \frac{1}{36} \\ -\frac{5}{54} & -\frac{2}{9} & -\frac{11}{36} \end{pmatrix}$$

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Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Determine the exact values of x for which

$$\sinh 2x = 3\sinh x$$

(5)

$\sinh 2x = 2 \sinh x \cosh x$

$2 \sinh x \cosh x = 3 \sinh x$

$2 \sinh x \cosh x - 3 \sinh x = 0$

$\sinh x (2 \cosh x - 3) = 0$

$\sinh x = 0 \Rightarrow x = 0$

or $\cosh x = \frac{3}{2} \Rightarrow \frac{e^x + e^{-x}}{2} = \frac{3}{2}$

$e^{2x} - 3e^x + 1 = 0$

$e^x = \frac{3 \pm \sqrt{(9-4)}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$x = \ln\left(\frac{3+\sqrt{5}}{2}\right)$ or $x = \ln\left(\frac{3-\sqrt{5}}{2}\right)$

(equivalently $x = \ln\left[\frac{3}{2} + \sqrt{\left(\frac{9}{4} - 1\right)}\right] = \operatorname{arcosh}\frac{3}{2}$)

Since $\frac{3-\sqrt{5}}{2} = \frac{4}{2(3+\sqrt{5})} = \frac{2}{3+\sqrt{5}}$, $\ln\left(\frac{3-\sqrt{5}}{2}\right) = -\ln\left(\frac{3+\sqrt{5}}{2}\right)$

$x = 0$ or $x = \pm \ln\left(\frac{3+\sqrt{5}}{2}\right)$

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Question 2 continued

(Total for Question 2 is 5 marks)

3. The complex number $z = a + bi$ where a and b are real constants.

Given that $\frac{z}{z^*}$ is purely imaginary,

(a) show that $a^2 = b^2$

(2)

Given also that $zz^* = 50$

(b) determine the possible complex numbers z

(3)

(a) $\frac{z}{z^*} = \frac{a + bi}{a - bi} = \frac{(a + bi)(a + bi)}{(a - bi)(a + bi)} = \frac{a^2 - b^2 + 2abi}{a^2 + b^2}$

Purely imaginary $\Rightarrow$ real part $= 0$

$\frac{a^2 - b^2}{a^2 + b^2} = 0 \Rightarrow a^2 - b^2 = 0 \Rightarrow a^2 = b^2$ as required

(b) $zz^* = (a + bi)(a - bi) = a^2 + b^2 = 50$

$a^2 = b^2 \Rightarrow 2a^2 = 50 \Rightarrow a^2 = 25 \Rightarrow a = \pm 5, \ b = \pm 5$

$z = 5 + 5i, \quad 5 - 5i, \quad -5 + 5i, \quad -5 - 5i$

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Question 3 continued

(Total for Question 3 is 5 marks)

4. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{3x}$$

giving your answer in the form $y = f(x)$

(5)

Given that $y = 5$ and $\frac{dy}{dx} = 12$ when $x = 0$

(b) determine the particular solution of the differential equation.

(3)

(a) Auxiliary equation: $m^2 - 4m + 4 = 0$

$(m - 2)^2 = 0 \Rightarrow m = 2$ (repeated)

CF: $y = Ae^{2x} + Bxe^{2x}$

PI: $y = \lambda e^{3x}$, $\frac{dy}{dx} = 3\lambda e^{3x}$, $\frac{d^2y}{dx^2} = 9\lambda e^{3x}$

$9\lambda e^{3x} - 4(3\lambda e^{3x}) + 4\lambda e^{3x} = 2e^{3x}$

$\lambda e^{3x} = 2e^{3x} \Rightarrow \lambda = 2$

General solution: $y = Ae^{2x} + Bxe^{2x} + 2e^{3x}$

(b) $x = 0, y = 5$: $5 = A + 0 + 2 \Rightarrow A = 3$

$\frac{dy}{dx} = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x} + 6e^{3x}$

$x = 0, \frac{dy}{dx} = 12$: $12 = 2(3) + B + 6 \Rightarrow B = 0$

$y = 3e^{2x} + 2e^{3x}$

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Question 4 continued

(Total for Question 4 is 8 marks)

5. Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^{n} \frac{4}{r^2-1} = \frac{(pn+q)(n-1)}{n(n+1)}$$

where p and q are constants to be determined.

(5)

$\frac{4}{r^2-1} \equiv \frac{4}{(r-1)(r+1)} \equiv \frac{A}{r-1} + \frac{B}{r+1} \Rightarrow 4 \equiv A(r+1) + B(r-1)$

$r = 1: \; 4 = 2A \Rightarrow A = 2; \qquad r = -1: \; 4 = -2B \Rightarrow B = -2$

$\sum_{r=2}^{n} \frac{4}{r^2-1} = \sum_{r=2}^{n} \left(\frac{2}{r-1} - \frac{2}{r+1} \right)$

$r = 2: \frac{2}{1} - \frac{2}{3}$

$r = 3: \frac{2}{2} - \frac{2}{4}$

$r = 4: \frac{2}{3} - \frac{2}{5}$

$\vdots$

$r = n-1: \frac{2}{n-2} - \frac{2}{n}$

$r = n: \frac{2}{n-1} - \frac{2}{n+1}$

$\text{Sum} = \frac{2}{1} + \frac{2}{2} - \frac{2}{n} - \frac{2}{n+1} = 3 - \frac{2}{n} - \frac{2}{n+1}$

$= \frac{3n(n+1) - 2(n+1) - 2n}{n(n+1)} = \frac{3n^2 + 3n - 2n - 2 - 2n}{n(n+1)}$

$= \frac{3n^2 - n - 2}{n(n+1)} = \frac{(3n+2)(n-1)}{n(n+1)}$

$\therefore \; p = 3, \; q = 2$

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Question 5 continued

(Total for Question 5 is 5 marks)

6. $f(z) = z^3 + az^2 + bz + c$ where a, b and c are real constants

The roots of the equation $f(z) = 0$ are z_1, z_2 and z_3

When plotted on an Argand diagram, the points representing these roots form the vertices of a triangle.

Given that

- $z_1 = 2 + 4i$
- the area of the triangle is 12

Determine the two possible functions $f(z)$

(5)

Coefficients real $\Rightarrow$ $z_2 = z_1^* = 2 - 4i$, so z_3 is real

z_1 and z_2 are 8 apart (on the line Re $z = 2$): base of triangle = 8

Area $= \frac{1}{2} \times 8 \times h = 12 \Rightarrow h = 3$

$z_3 = 2 + 3 = 5$ or $z_3 = 2 - 3 = -1$

$(z - (2 + 4i))(z - (2 - 4i)) = (z - 2)^2 + 16 = z^2 - 4z + 20$

$z_3 = 5$: $f(z) = (z - 5)(z^2 - 4z + 20)$

$= z^3 - 4z^2 + 20z - 5z^2 + 20z - 100$

$= z^3 - 9z^2 + 40z - 100$

$z_3 = -1$: $f(z) = (z + 1)(z^2 - 4z + 20)$

$= z^3 - 4z^2 + 20z + z^2 - 4z + 20$

$= z^3 - 3z^2 + 16z + 20$

$f(z) = z^3 - 9z^2 + 40z - 100$ or $f(z) = z^3 - 3z^2 + 16z + 20$

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 5 marks)

7. (a) Express

$$\frac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)}$$

in partial fractions.

(4)

(b) Hence, show that

$$\int_0^1 \frac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)}\,dx = 2 + \ln\left(\frac{27}{4}\right) - \frac{\pi}{6\sqrt{3}}$$

(4)

(a) $\dfrac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)} \equiv A + \dfrac{B}{x + 2} + \dfrac{Cx + D}{x^2 + 3}$

$2x^3 + 10x^2 + 9x + 22 \equiv A(x + 2)(x^2 + 3) + B(x^2 + 3) + (Cx + D)(x + 2)$

coeff x^3: $2 = A \Rightarrow A = 2$

$x = -2$: $-16 + 40 - 18 + 22 = 7B \Rightarrow 28 = 7B \Rightarrow B = 4$

coeff x^2: $10 = 2A + B + C = 4 + 4 + C \Rightarrow C = 2$

$x = 0$: $22 = 6A + 3B + 2D = 12 + 12 + 2D \Rightarrow D = -1$

$\dfrac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)} \equiv 2 + \dfrac{4}{x + 2} + \dfrac{2x - 1}{x^2 + 3}$

(b) $\int_0^1 \left(2 + \dfrac{4}{x + 2} + \dfrac{2x}{x^2 + 3} - \dfrac{1}{x^2 + 3}\right) dx$

$= \left[2x + 4\ln(x + 2) + \ln(x^2 + 3) - \dfrac{1}{\sqrt{3}} \arctan \dfrac{x}{\sqrt{3}}\right]_0^1$

$= \left[2 + 4\ln 3 + \ln 4 - \dfrac{1}{\sqrt{3}} \arctan \dfrac{1}{\sqrt{3}}\right] - \left[0 + 4\ln 2 + \ln 3 - 0\right]$

$= 2 + 4\ln 3 + \ln 4 - \dfrac{1}{\sqrt{3}} \times \dfrac{\pi}{6} - 4\ln 2 - \ln 3$

$= 2 + 3\ln 3 + \ln 4 - 4\ln 2 - \dfrac{\pi}{6\sqrt{3}}$

$= 2 + \ln 27 + \ln 4 - \ln 16 - \dfrac{\pi}{6\sqrt{3}}$

$= 2 + \ln\left(\dfrac{27 \times 4}{16}\right) - \dfrac{\pi}{6\sqrt{3}} = 2 + \ln\left(\dfrac{27}{4}\right) - \dfrac{\pi}{6\sqrt{3}}$ as required

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8. A rambler starts a walk at the bottom of a hill at 10 am.

The rambler walks all the way to the top of the hill and then turns around and walks back down to the bottom of the hill.

The differential equation

$$\sin t \frac{\mathrm{d}x}{\mathrm{d}t} - x\cos t = A\sin 2t \sin t \qquad t \geqslant 0$$

where A is a constant, is used to model the vertical displacement, x metres, of the rambler from the bottom of the hill, t hours after the start of the walk.

Given that after 1 hour

- the rambler has a vertical displacement of 213 m
- $\frac{\mathrm{d}x}{\mathrm{d}t} = 273.2$

(a) determine the value of A to 3 significant figures.

(1)

(b) Hence, determine the particular solution of the differential equation, giving your answer in the form $x = \mathrm{f}(t)$

(5)

(c) Use the model to determine the time at which the rambler will return to the bottom of the hill.

(3)

Given that the vertical displacement of the top of the hill is 300.68 m

(d) use the model to find the value of t when the rambler reaches the top of the hill.

(2)

(e) Using your answers to parts (c) and (d), give a limitation of the model.

(1)

(a) $t = 1$: $(\sin 1)(273.2) - 213\cos 1 = A(\sin 2)(\sin 1)$

$A = \dfrac{273.2\sin 1 - 213\cos 1}{\sin 2 \sin 1} = 150.0436\ldots \Rightarrow A = 150$

(b) $\div \sin t$: $\dfrac{dx}{dt} - x\dfrac{\cos t}{\sin t} = 150\dfrac{\sin 2t \sin t}{\sin t} \Rightarrow \dfrac{dx}{dt} - x\cot t = 150\sin 2t$

$IF = e^{-\int \cot t\, dt} = e^{-\ln \sin t} = \dfrac{1}{\sin t} = \operatorname{cosec} t$

$\dfrac{d}{dt}(x \operatorname{cosec} t) = 150\sin 2t \operatorname{cosec} t = \dfrac{150 \times 2 \sin t \cos t}{\sin t} = 300\cos t$

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Question 8 continued

$x \operatorname{cosec} t = \int 300 \cos t \, dt = 300 \sin t + c$

$t = 1, x = 213: \quad \frac{213}{\sin 1} = 300 \sin 1 + c$

$c = 253.132... - 252.441... = 0.6868...$

$x \operatorname{cosec} t = 300 \sin t + 0.687$

$x = 300 \sin^2 t + 0.687 \sin t$

(c) $x = 0: \quad 300 \sin^2 t + 0.687 \sin t = 0$

$\sin t (300 \sin t + 0.687) = 0 \Rightarrow \sin t = 0 \text{ or } \sin t = -\frac{0.687}{300}$

$\sin t = 0, t > 0 \Rightarrow t = \pi = 3.1415...$ hours

$\left(\sin t = -\frac{0.687}{300} \Rightarrow t = \pi + \sin^{-1}\left(\frac{0.687}{300}\right) = 3.1439...\right)$

3.14 hours = 3 hours 8.5 minutes after 10 am

The rambler returns to the bottom at 13:08 (1:08 pm)

(d) $300 \sin^2 t + 0.687 \sin t = 300.68$

$300 \sin^2 t + 0.687 \sin t - 300.68 = 0$

$\sin t = \frac{-0.687 + \sqrt{(0.687^2 + 4 \times 300 \times 300.68)}}{600} = 0.99998...$

$t = \sin^{-1}(0.99998...) = 1.5659... \Rightarrow t = 1.57$

(e) Time going up ≈ 1.57 h and time coming down $\approx 3.14 - 1.57 = 1.57$ h, so the model says it takes the same time to walk up the hill as to walk down. In reality it would take longer to go up the hill than down.

Question 8 continued

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Question 8 continued

(Total for Question 8 is 12 marks)

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9. (i) The curves with equations

$$y = \frac{3}{4}\sinh x \text{ and } y = \tanh x + \frac{1}{5}$$

intersect at just one point P

(a) Use algebra to show that the x coordinate of P satisfies the equation

$$15e^{4x} - 48e^{3x} + 32e^{x} - 15 = 0$$

(3)

(b) Show that $e^{x} = 3$ is a solution of this equation.

(1)

(c) Hence state the exact coordinates of P.

(1)

(ii) Show that

$$\int_{-4}^{0} \frac{e^{\frac{1}{x}}}{x^2}\,dx = e^{-\frac{1}{4}}$$

(4)

(i)(a) $\sinh x = \frac{e^x - e^{-x}}{2}$, $\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$

$\frac{3}{4} \times \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{e^{2x} + 1} + \frac{1}{5}$

$\times 40e^x(e^{2x} + 1)$:

$15(e^{2x} - 1)(e^{2x} + 1) = 40e^x(e^{2x} - 1) + 8e^x(e^{2x} + 1)$

$15e^{4x} - 15 = 40e^{3x} - 40e^x + 8e^{3x} + 8e^x$

$15e^{4x} - 15 = 48e^{3x} - 32e^x$

$\Rightarrow 15e^{4x} - 48e^{3x} + 32e^x - 15 = 0$ as required

(b) $15(3)^4 - 48(3)^3 + 32(3) - 15 = 1215 - 1296 + 96 - 15 = 0$

$\therefore e^x = 3$ is a solution

(c) $x = \ln 3$, $y = \frac{3}{4}\sinh(\ln 3) = \frac{3}{4} \times \frac{3 - \frac{1}{3}}{2} = \frac{3}{4} \times \frac{4}{3} = 1$

$P(\ln 3, 1)$

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Question 9 continued

(ii) $u = \frac{1}{x} \Rightarrow \frac{du}{dx} = -\frac{1}{x^2}$

$\int \frac{e^{1/x}}{x^2}\,dx = \int -e^u\,du = -e^u = -e^{1/x}\ (+c)$

The integrand is undefined at $x = 0$, so the integral is improper:

$\int_{-4}^{0} \frac{e^{1/x}}{x^2}\,dx = \lim_{t\to 0-} \int_{-4}^{t} \frac{e^{1/x}}{x^2}\,dx$

$= \lim_{t\to 0}\left[-e^{1/x}\right]_{-4}^{t} = \lim_{t\to 0}\left[(-e^{1/t}) - (-e^{-1/4})\right]$

As $t \to 0^-, \frac{1}{t} \to -\infty$ so $e^{1/t} \to 0$

$\therefore \int_{-4}^{0} \frac{e^{1/x}}{x^2}\,dx = 0 + e^{-1/4} = e^{-1/4}$ as required

Question 9 continued

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Question 9 continued

(Total for Question 9 is 9 marks)

10. (a) Given that, for $n \in \mathbb{N}$

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

show that

$$8\sin^4\theta \equiv \cos 4\theta - 4\cos 2\theta + 3$$

(5)



Figure 1

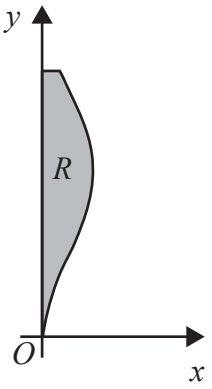


Figure 2

Figure 1 shows the central vertical cross-section of a solid wooden ornament.

Figure 2 shows the curve with equation

$$x = \sin^2\left(\frac{1}{2}y\right) \qquad 0 \leqslant y \leqslant \frac{8\pi}{5}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $y = \frac{8\pi}{5}$ and the y-axis.

The ornament is modelled by the solid of revolution formed when R is rotated 360° about the y-axis. The units are centimetres.

(b) Using algebraic integration and the result in part (a), determine, in cm^3, the volume of wood needed to make the ornament, according to the model. Give your answer to 2 significant figures.

[*Solutions based entirely on calculator technology are not acceptable.*]

(5)

Given that

- the density of the wood is 0.85 $\mathrm{g/cm}^3$
- the mass of the ornament is 6 grams

(c) comment on the suitability of the model.

(2)

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Question 10 continued

(a) Let $z = \cos\theta + i\sin\theta$

$$\left(z - \frac{1}{z}\right)^4 = (2i\sin\theta)^4 = 16\sin^4\theta$$

$$\left(z - \frac{1}{z}\right)^4 = z^4 + 4z^3\left(-\frac{1}{z}\right) + 6z^2\left(-\frac{1}{z}\right)^2 + 4z\left(-\frac{1}{z}\right)^3 + \left(-\frac{1}{z}\right)^4$$

$$= z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}$$

$$= \left(z^4 + \frac{1}{z^4}\right) - 4\left(z^2 + \frac{1}{z^2}\right) + 6$$

$$= 2\cos 4\theta - 4(2\cos 2\theta) + 6$$

$16\sin^4\theta \equiv 2\cos 4\theta - 8\cos 2\theta + 6$

$\Rightarrow 8\sin^4\theta \equiv \cos 4\theta - 4\cos 2\theta + 3$ as required

(b) $V = \pi\int_0^{8\pi/5} x^2\,dy = \pi\int_0^{8\pi/5} \sin^4\left(\frac{1}{2}y\right)dy$

Using (a) with $\theta = \frac{1}{2}y$: $\sin^4\left(\frac{1}{2}y\right) = \frac{1}{8}(\cos 2y - 4\cos y + 3)$

$$V = \frac{\pi}{8}\int_0^{8\pi/5} (\cos 2y - 4\cos y + 3)\,dy$$

$$= \frac{\pi}{8}\left[\frac{1}{2}\sin 2y - 4\sin y + 3y\right]_0^{8\pi/5}$$

$$= \frac{\pi}{8}\left[\frac{1}{2}\sin\frac{16\pi}{5} - 4\sin\frac{8\pi}{5} + \frac{24\pi}{5}\right] - 0$$

$$= \frac{\pi}{8}(-0.29389\ldots + 3.80422\ldots + 15.07964\ldots)$$

$= 7.30026\ldots$

Volume $= 7.3\ \text{cm}^3$ (2 s.f.)

(c) Mass $= 7.3 \times 0.85 = 6.205 \approx 6.2$ grams

6.2 g is close to the actual mass of 6 g, so the model is a good model (suitable) for the ornament.

Question 10 continued

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DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA