Pearson Edexcel Level 3 GCE

Monday 24 June 2024

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/4D**

Further Mathematics

Advanced

PAPER 4D: Decision Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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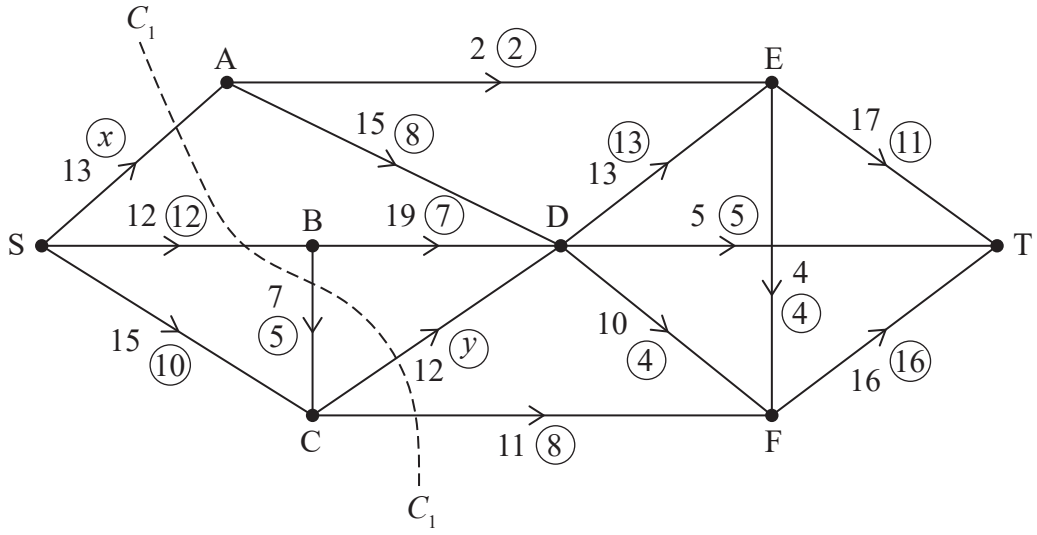


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The numbers in circles represent an initial flow from S to T. The other number on each arc represents the capacity, in litres per second, of the corresponding pipe.

(a) (i) State the value of x

(ii) State the value of y

(2)

(b) State the value of the initial flow.

(1)

(c) State the capacity of cut C_1

(1)

(d) Find, by inspection, a flow-augmenting route to increase the flow by four units. You must state your route.

(1)

The flow-augmenting route from (d) is used to increase the flow from S to T.

(e) Prove that the flow is now maximal.

(3)

A vertex restriction is now applied so that no more than 12 litres per second can flow through E.

(f) (i) Complete Diagram 1 in the answer book to show this restriction.

(ii) State the **value** of the maximum flow through the network with this restriction.

(2)

(Total for Question 1 is 10 marks)

2. The general solution of the first order recurrence relation

$$u_{n+1} + au_n = bn^2 + cn + d \qquad n \geqslant 0$$

is given by

$$u_n = A(3)^n + 5n^2 + 1$$

where A is an arbitrary non-zero constant.

By considering expressions for u_{n+1} and u_n, find the values of the constants a, b, c and d.

(Total for Question 2 is 3 marks)

3. The table below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, E, F, G and H, to three sales points, A, B and C. It also shows the stock held at each supply point and the amount required at each sales point.
A minimum cost solution is required.

	A	B	C	Supply
E	23	28	22	21
F	26	19	29	32
G	29	24	20	29
H	24	26	19	23
Demand	45	19	23	

(a) Explain why it is necessary to add a dummy demand point.

(1)

(b) On Table 1 in the answer book, insert appropriate values in the dummy demand column, D.

(1)

After finding an initial feasible solution and applying one iteration of the stepping-stone method, the table becomes

	A	B	C	D
E	21			
F	19	13		
G		6	23	
H	5			18

(c) Starting with GD as the next entering cell, perform two further iterations of the stepping-stone method to obtain an improved solution. You must make your method clear by showing your routes and stating the

- shadow costs
- improvement indices
- entering and exiting cells

(6)

(d) State the cost of the solution found in (c).

(1)

(e) Determine whether the solution obtained in (c) is optimal, giving a reason for your answer.

(3)

(Total for Question 3 is 12 marks)

4. Four workers, A, B, C and D, are to be assigned to four tasks, P, Q, R and S.

Each task must be assigned to just one worker and each worker can do only one task.

Worker B cannot be assigned to task Q and worker D cannot be assigned to task R.

The amount, in pounds, that each worker would earn when assigned to each task is shown in the table below.

	P	Q	R	S
A	65	72	69	75
B	71	–	68	65
C	70	69	73	77
D	73	70	–	71

The Hungarian algorithm can be used to find the maximum total amount that would be earned by the four workers.

(a) (i) Explain how to modify the table so that the Hungarian algorithm could be applied.

(ii) Modify the table as described in (a)(i).

(3)

(b) Formulate the above situation as a linear programming problem. You must define the decision variables and make the objective function and constraints clear.

(6)

(Total for Question 4 is 9 marks)

5. Sebastien needs to make a journey. He can choose between travelling by plane, by train or by coach.

Sebastien knows the exact costs of all three travel options, but he also wants to account for his travel time, including any possible delays.

The cost of Sebastien's time is £50 per hour.

The table below shows the costs, the journey times (without delays), and the corresponding probabilities of delays, for each travel option.

	Cost of travel option	Journey time (in hours) without delays	Probability of a 1-hour delay	Probability of a 2-hour delay	Probability of a 3-hour delay	Probability of a 24-hour delay
Plane	£200	3	0.09	0.05	0	0.03
Train	£130	5	0.07	0.03	0	0
Coach	£70	6	0.15	0.1	0.05	0

(a) By drawing a decision tree, evaluate the EMV of the total cost of Sebastien's journey for each node of your tree.

(6)

(b) Hence state the travel option that minimises the EMV of the total cost of Sebastien's journey.

(1)

(c) A cube root utility function is applied to the total costs of each option. Determine the travel option with the best expected utility and state the corresponding value.

(3)

(Total for Question 5 is 10 marks)

6.

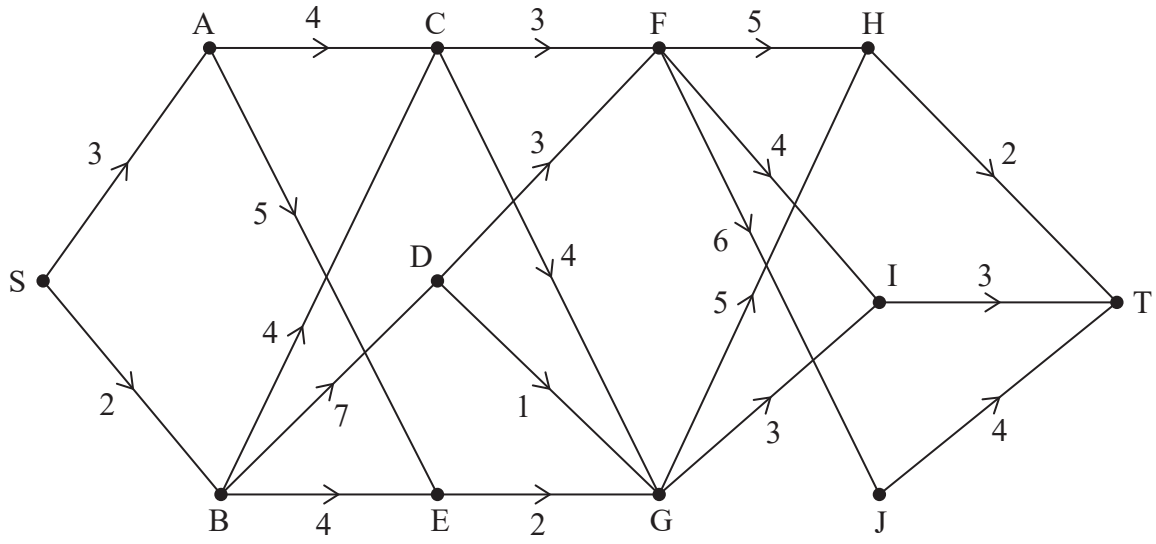


Figure 2

The staged, directed network in Figure 2 represents the roads that connect 12 towns, S, A, B, C, D, E, F, G, H, I, J and T. The number on each arc shows the time, in hours, it takes to drive between these towns.

Elena plans to drive from S to T. She must arrive at T by 9 pm.

(a) By completing the table in the answer book, use dynamic programming to find the latest time that Elena can start her journey from S to arrive at T by 9 pm.

(9)

(b) Hence write down the route that Elena should take.

(1)

(Total for Question 6 is 10 marks)

7.

		Player B		
		Option X	Option Y	Option Z
Player A	Option R	3	2	–3
	Option S	4	–2	1
	Option T	–1	3	6

A two person zero-sum game is represented by the pay-off matrix for player A, shown above.

(a) Verify that there is no stable solution to this game.

(2)

Player A intends to make a random choice between options R, S and T, choosing option R with probability p_1, option S with probability p_2 and option T with probability p_3

Player A wants to find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.

Player A formulates the following objective function for the corresponding linear programme.

Maximise $P = V$ where V = the value of the game + 3

(b) Determine an initial Simplex tableau, making your variables and working clear.

(5)

After several iterations of the Simplex algorithm, a possible final tableau is

b.v.	V	p_1	p_2	p_3	r	s	t	u	Value
p_3	0	0	0	1	$\frac{1}{10}$	$-\frac{3}{80}$	$-\frac{1}{16}$	$\frac{33}{80}$	$\frac{33}{80}$
p_2	0	0	1	0	$-\frac{1}{10}$	$\frac{13}{80}$	$-\frac{1}{16}$	$\frac{17}{80}$	$\frac{17}{80}$
V	1	0	0	0	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{73}{16}$	$\frac{73}{16}$
p_1	0	1	0	0	0	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
P	0	0	0	0	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{73}{16}$	$\frac{73}{16}$

(c) (i) State the best strategy for player A.

(ii) Calculate the value of the game for player B.

(3)

Player B intends to make a random choice between options X, Y and Z.

(d) Determine the best strategy for player B, making your method and working clear.

(3)

(Total for Question 7 is 13 marks)

8. A sequence $\{u_n\}$, where $n \geqslant 0$, satisfies the recurrence relation

$$2u_{n+2} + 5u_{n+1} = 3u_n + 8n + 2$$

(a) Find the general solution of this recurrence relation.

(5)

A particular solution of this recurrence relation has $u_0 = 1$ and $u_1 = k$, where k is a positive constant. All terms of the sequence are positive.

(b) Determine the value of k.

(3)

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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Please check the examination details below before entering your candidate information

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Further Mathematics

Advanced

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Answer Book

Do not return the question paper with the answer book.

Total Marks

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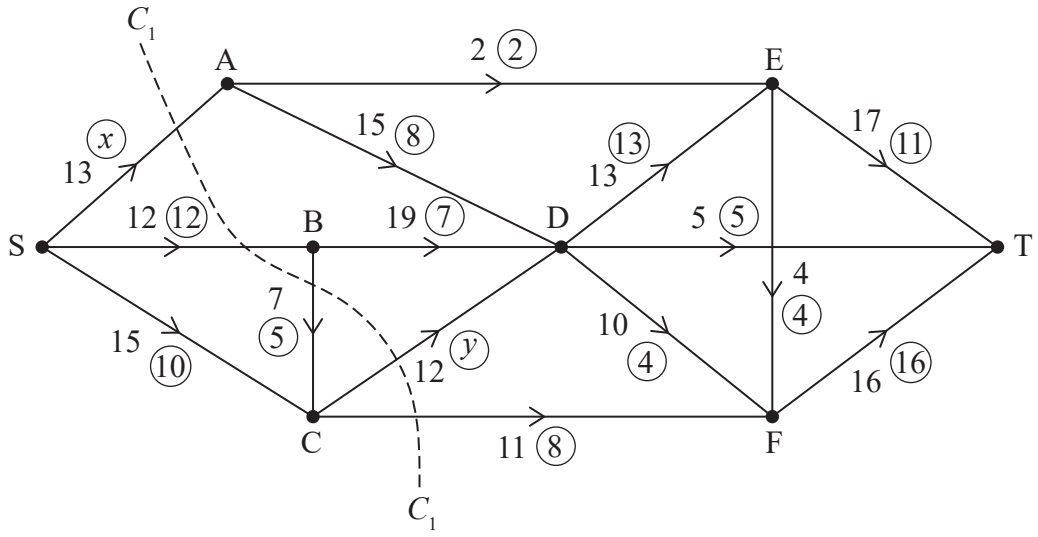


Figure 1

(a)(i) Flow out of A = 2 + 8 = 10, so x = 10

(ii) Flow out of C = 8 + y = flow into C = 10 + 5 = 15, so y = 7

(b) Initial flow = 10 + 12 + 10 = 32

(c) Capacity of C_1 = 13 + 12 + 12 + 11 = 48

(d) Flow-augmenting route: S – C – D – F – E – T (SCDFET), flow 4

(4 is sent back along EF, so the flow in EF drops from 4 to 0)

(e) New flow = 32 + 4 = 36

Consider the cut through AE, DE, DT, EF and FT

({S, A, B, C, D, F} on the source side, {E, T} on the sink side):

capacity = 2 + 13 + 5 + 0 + 16 = 36

(EF goes from the sink side to the source side, so it counts 0)

Value of flow = 36 = capacity of this cut, so it is a minimum cut,

and by the maximum flow – minimum cut theorem the flow of 36

is maximal.

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Question 1 continued

(f) (i)

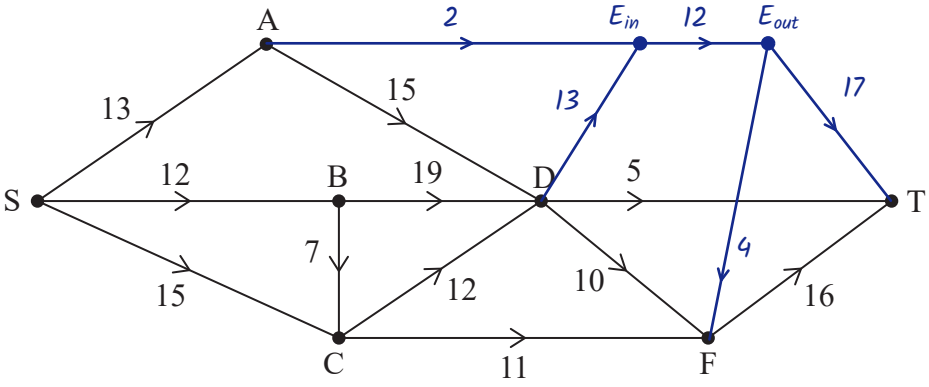


Diagram 1

(f)(ii) Cut through $E_{in}E_{out}$, DT and FT = 12 + 5 + 16 = 33

(the flow through E drops from 15 to 12, a loss of 3)

Maximum flow = 33

(Total for Question 1 is 10 marks)

2.

$u_{n+1} = A(3)^{n+1} + 5(n+1)^2 + 1$

$= 3A(3)^n + 5n^2 + 10n + 6$

From u_n: $A(3)^n = u_n - 5n^2 - 1$

So $u_{n+1} = 3(u_n - 5n^2 - 1) + 5n^2 + 10n + 6$

$u_{n+1} = 3u_n - 15n^2 - 3 + 5n^2 + 10n + 6$

$u_{n+1} - 3u_n = -10n^2 + 10n + 3$

Compare with $u_{n+1} + au_n = bn^2 + cn + d$:

$a = -3, \quad b = -10, \quad c = 10, \quad d = 3$

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Question 2 continued

(Total for Question 2 is 3 marks)

3.

(a)

Total supply = 21 + 32 + 29 + 23 = 105, total demand = 45 + 19 + 23 = 87

Supply ≠ demand (supply exceeds demand), so a dummy demand point with demand 105 − 87 = 18 is needed to balance the problem.

(b)

	A	B	C	D	Supply
E	23	28	22	0	21
F	26	19	29	0	32
G	29	24	20	0	29
H	24	26	19	0	23
Demand	45	19	23	18	

Table 1

You may not need to use all of these tables.

	A	B	C	D
E	21			
F	19−θ	13+θ		
G		6−θ	23	θ
H	5+θ			18−θ

(c) Entering cell GD

Route: GD(+) → HD(−) → HA(+)

→ FA(−) → FB(+) → GB(−)

θ = min(18, 19, 6) = 6

Exiting cell GB

New solution: second table

	A	B	C	D
E	21			
F	13	19		
G			23	6
H	11			12

Shadow costs ($R_i + K_j = C_{ij}$):

$R_E = 0$, $K_A = 23$, $R_F = 3$, $K_B = 16$,

$R_H = 1$, $K_D = -1$, $R_G = 1$, $K_C = 19$

Improvement indices $I_{ij} = C_{ij} - R_i - K_j$:

EB = 12, EC = 3, ED = 1, FC = 7,

FD = −2, GA = 5, GB = 7, HB = 9,

HC = −1 (third table)

Most negative index is FD = −2,

so the entering cell is FD

		23	16	19	−1
		A	B	C	D
0	E	X	12	3	1
3	F	X	X	7	−2
1	G	5	7	X	X
1	H	X	9	−1	X

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Question 3 continued

	A	B	C	D
E	21			
F	13−θ	19		θ
G			23	6
H	11+θ			12−θ

	A	B	C	D
E	21			
F	1	19		12
G			23	6
H	23			

		23	16	17	−3
	A	B	C	D	
0	E	X	12	5	3
3	F	X	X	9	X
3	G	3	5	X	X
1	H	X	9	1	2

	A	B	C	D
E				
F				
G				
H				

	A	B	C	D
E				
F				
G				
H				

(c) continued

Route: FD(+) → HD(−) → HA(+) → FA(−)

θ = min(12, 13) = 12

Exiting cell HD

(entering cell FD, exiting cell HD)

Improved solution (second table):

EA 21, FA 1, FB 19, FD 12,

GC 23, GD 6, HA 23

(d) Cost = 21×23 + 1×26 + 19×19

+ 23×20 + 23×24 (dummy costs 0)

= 483 + 26 + 361 + 460 + 552

= £1882

(e) Shadow costs for this solution:

$R_E = 0$, $K_A = 23$, $R_F = 3$, $K_B = 16$,

$K_D = -3$, $R_G = 3$, $K_C = 17$, $R_H = 1$

Improvement indices (third table):

EB = 12, EC = 5, ED = 3, FC = 9,

GA = 3, GB = 5, HB = 9, HC = 1, HD = 2

All improvement indices are non-negative,

so the solution is optimal.

(Total for Question 3 is 12 marks)

4.

(a)(i)

Subtract every entry from a constant at least as large as the largest entry, e.g. 77, to turn the maximisation problem into a minimisation. Then place a sufficiently large number (e.g. 100) in the cells BQ and DR so that these assignments can never be chosen.

(a)(ii)

	P	Q	R	S
A	12	5	8	2
B	6	100	9	12
C	7	8	4	0
D	4	7	100	6

(b) Let $x_{ij} = 1$ if worker i is assigned to task j, and $x_{ij} = 0$ otherwise, where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R, S\}$

Minimise $C = 12x_{AP} + 5x_{AQ} + 8x_{AR} + 2x_{AS} + 6x_{BP} + 100x_{BQ} + 9x_{BR} + 12x_{BS} + 7x_{CP} + 8x_{CQ} + 4x_{CR} + 0x_{CS} + 4x_{DP} + 7x_{DQ} + 100x_{DR} + 6x_{DS}$

Subject to:

$x_{AP} + x_{AQ} + x_{AR} + x_{AS} = 1$ (each worker does one task)

$x_{BP} + x_{BQ} + x_{BR} + x_{BS} = 1$

$x_{CP} + x_{CQ} + x_{CR} + x_{CS} = 1$

$x_{DP} + x_{DQ} + x_{DR} + x_{DS} = 1$

$x_{AP} + x_{BP} + x_{CP} + x_{DP} = 1$ (each task is done once)

$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} = 1$

$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$

$x_{AS} + x_{BS} + x_{CS} + x_{DS} = 1$

$x_{ij} \in \{0, 1\}$

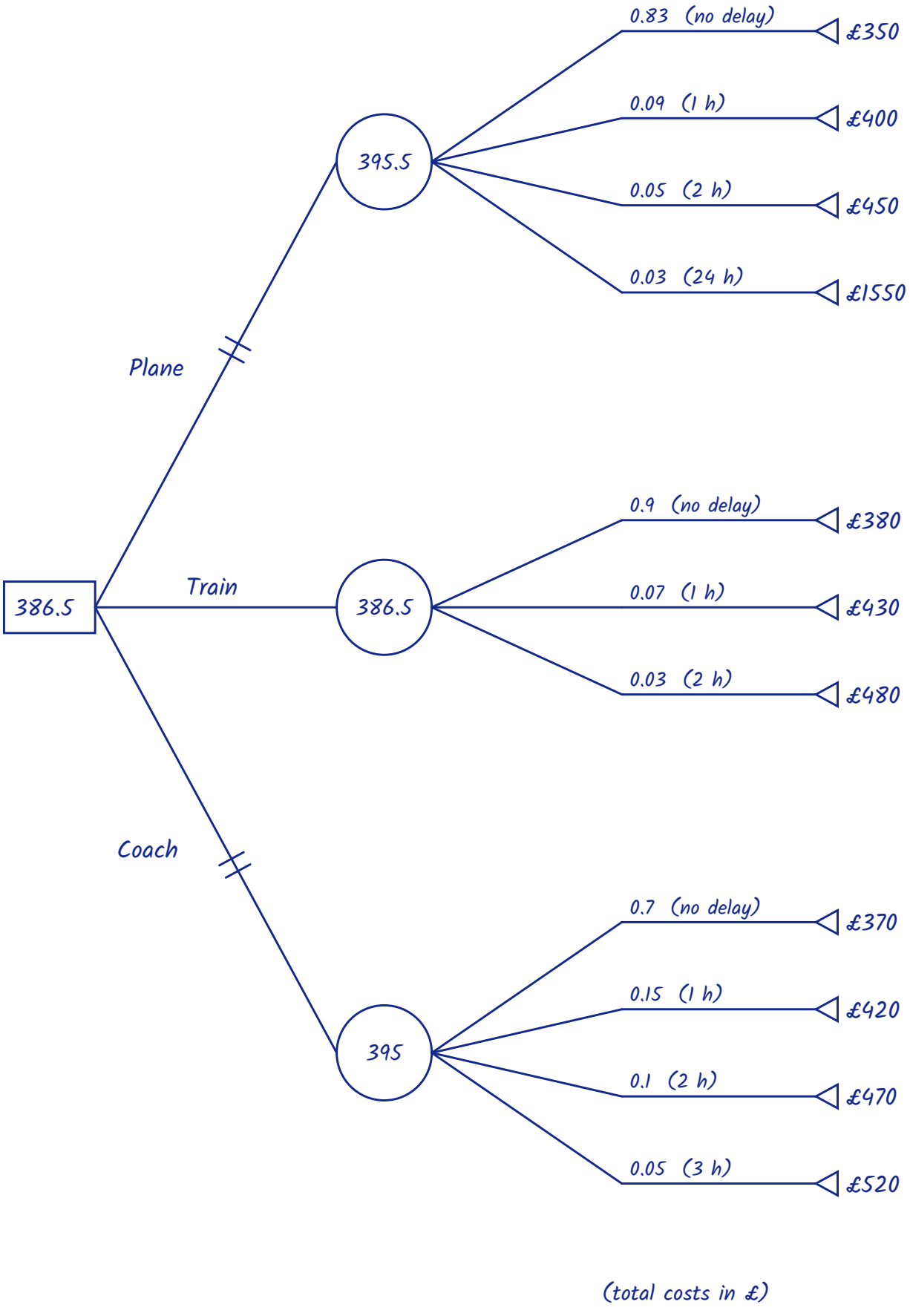
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Question 4 continued

(Total for Question 4 is 9 marks)

5.



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Question 5 continued

(a) End pay-offs: total cost = fare + £50 × (journey time + delay)

Plane: 200 + 50 × 3 = 350, then 400, 450, 350 + 50 × 24 = 1550

Train: 130 + 50 × 5 = 380, then 430, 480

Coach: 70 + 50 × 6 = 370, then 420, 470, 520

P(no delay): plane 1 − 0.17 = 0.83, train 0.9, coach 0.7

EMV(Plane) = 0.83 × 350 + 0.09 × 400 + 0.05 × 450 + 0.03 × 1550

= 290.5 + 36 + 22.5 + 46.5 = £395.50

EMV(Train) = 0.9 × 380 + 0.07 × 430 + 0.03 × 480

= 342 + 30.1 + 14.4 = £386.50

EMV(Coach) = 0.7 × 370 + 0.15 × 420 + 0.1 × 470 + 0.05 × 520

= 259 + 63 + 47 + 26 = £395

Decision node: min(395.5, 386.5, 395) = £386.50

(b) The train minimises the EMV of the total cost (£386.50)

(c) Expected utility = Σ p × $\sqrt[3]{}$(total cost) (lower cost is better)

Plane: $0.83\sqrt[3]{350} + 0.09\sqrt[3]{400} + 0.05\sqrt[3]{450} + 0.03\sqrt[3]{1550} = 7.2427...$

Train: $0.9\sqrt[3]{380} + 0.07\sqrt[3]{430} + 0.03\sqrt[3]{480} = 7.2820...$

Coach: $0.7\sqrt[3]{370} + 0.15\sqrt[3]{420} + 0.1\sqrt[3]{470} + 0.05\sqrt[3]{520} = 7.3282...$

The plane gives the smallest expected utility of cost, so it is the best option: Plane, with expected utility 7.24 (3 s.f.)

(Total for Question 5 is 10 marks)

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6.

(a)

Stage	State	Action	Destination	Value
1	H	HT	T	2*
	I	IT	T	3*
	J	JT	T	4*
2	F	FH	H	5 + 2 = 7*
		FI	I	4 + 3 = 7*
		FJ	J	6 + 4 = 10
	G	GH	H	5 + 2 = 7
		GI	I	3 + 3 = 6*
3	C	CF	F	3 + 7 = 10*
		CG	G	4 + 6 = 10*
	D	DF	F	3 + 7 = 10
		DG	G	1 + 6 = 7*
	E	EG	G	2 + 6 = 8*
4	A	AC	C	4 + 10 = 14
		AE	E	5 + 8 = 13*
	B	BC	C	4 + 10 = 14
		BD	D	7 + 7 = 14
		BE	E	4 + 8 = 12*
5	S	SA	A	3 + 13 = 16
		SB	B	2 + 12 = 14*

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Question 6 continued

Stage	State	Action	Destination	Value

Latest time Elena's journey can start: 14 hours: 9 pm − 14 h = 7 am

(b) Route: S − B − E − G − I − T

(Total for Question 6 is 10 marks)

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7.

		Player B		
		Option X	Option Y	Option Z
Player A	Option R	3	2	–3
	Option S	4	–2	1
	Option T	–1	3	6

(a) Row minima: R: −3, S: −2, T: −1; maximin = −1

Column maxima: X: 4, Y: 3, Z: 6; minimax = 3

−1 ≠ 3, so row maximin ≠ column minimax: no stable solution.

(b) Add 3 to every pay-off so that all entries are positive:

	X	Y	Z
R	6	5	0
S	7	1	4
T	2	6	9

Let V = value of the game + 3, and $p_1, p_2, p_3 \geq 0$ be the probabilities of A playing R, S, T.

Maximise $P = V$ subject to

B plays X: $6p_1 + 7p_2 + 2p_3 \geq V$

B plays Y: $5p_1 + p_2 + 6p_3 \geq V$

B plays Z: $4p_2 + 9p_3 \geq V$

and $p_1 + p_2 + p_3 \leq 1$

With slack variables $r, s, t, u \geq 0$:

$V - 6p_1 - 7p_2 - 2p_3 + r = 0$

$V - 5p_1 - p_2 - 6p_3 + s = 0$

$V - 4p_2 - 9p_3 + t = 0$

$p_1 + p_2 + p_3 + u = 1$

$P - V = 0$

Initial tableau: see the table on page 15

The Simplex tableau can be found at the bottom of page 15.

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Question 7 continued

(c)(i) From the final tableau: $p_1 = 3/8$, $p_2 = 17/80$, $p_3 = 33/80$

Player A should play option R with probability 3/8,

option S with probability 17/80 and option T with probability 33/80.

(ii) $V = 73/16$, so the value of the game for A $= 73/16 - 3 = 25/16$

Zero-sum game, so the value of the game for player B $= -25/16$

(d) Let B play X, Y, Z with probabilities q_1, q_2, q_3.

p_1, p_2, p_3 are all non-zero, so each of A's options gives A the value $V = 73/16$ against B's optimal strategy (augmented pay-offs):

R: $6q_1 + 5q_2 + 0q_3 = 73/16$

S: $7q_1 + q_2 + 4q_3 = 73/16$

T: $2q_1 + 6q_2 + 9q_3 = 73/16$

$q_1 + q_2 + q_3 = 1$

Sub $q_3 = 1 - q_1 - q_2$ into S: $3q_1 - 3q_2 + 4 = 73/16$

$\Rightarrow q_1 - q_2 = 3/16$

Into R: $6(q_2 + 3/16) + 5q_2 = 73/16 \Rightarrow 11q_2 = 55/16$

$\Rightarrow q_2 = 5/16$, $q_1 = 8/16 = 1/2$, $q_3 = 3/16$

Check T: $2(1/2) + 6(5/16) + 9(3/16) = (16 + 30 + 27)/16 = 73/16$ ✓

(these agree with the r, s, t entries 1/2, 5/16, 3/16 in the P row)

Player B should play option X with probability 1/2, option Y with probability 5/16 and option Z with probability 3/16.

b.v.	V	p_1	p_2	p_3	r	s	t	u	Value
r	1	−6	−7	−2	1	0	0	0	0
s	1	−5	−1	−6	0	1	0	0	0
t	1	0	−4	−9	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	−1	0	0	0	0	0	0	0	0

Question 7 continued

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Question 7 continued

(Total for Question 7 is 13 marks)

8.

(a) Auxiliary equation: $2m^2 + 5m - 3 = 0$

$(2m - 1)(m + 3) = 0 \Rightarrow m = 1/2$ or $m = -3$

Complementary function: $u_n = A(1/2)^n + B(-3)^n$

Particular solution: try $u_n = an + b$

$2(a(n + 2) + b) + 5(a(n + 1) + b) = 3(an + b) + 8n + 2$

$7an + 9a + 7b = 3an + 3b + 8n + 2$

n terms: $7a = 3a + 8 \Rightarrow 4a = 8 \Rightarrow a = 2$

constants: $9a + 7b = 3b + 2 \Rightarrow 9a + 4b = 2$

$18 + 4b = 2 \Rightarrow b = -4$

General solution: $u_n = A(1/2)^n + B(-3)^n + 2n - 4$

(b) $u_0 = 1$: $A + B - 4 = 1 \Rightarrow A + B = 5$

$u_1 = k$: $(1/2)A - 3B + 2 - 4 = k \Rightarrow (1/2)A - 3B = k + 2$

$A = 5 - B$: $5/2 - (1/2)B - 3B = k + 2$

$(7/2)B = 1/2 - k \Rightarrow 7B = 1 - 2k$

$B = (1 - 2k)/7$, $A = 5 - B = (34 + 2k)/7$

$u_n = ((34 + 2k)/7)(1/2)^n + ((1 - 2k)/7)(-3)^n + 2n - 4$

If $B \neq 0$, the term $B(-3)^n$ dominates for large n and alternates in sign, so some terms would be negative. All terms are positive, so $B = 0$:

$(1 - 2k)/7 = 0 \Rightarrow k = 1/2$

(Check: $u_n = 5(1/2)^n + 2n - 4$ gives $u_0 = 1$, $u_1 = 0.5$, $u_2 = 1.25$, and for $n \geq 2$, $2n - 4 \geq 0$ so every term is positive.)

$k = 0.5$

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Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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