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Candidate surname StemUp Model Solution | Other names

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Pearson Edexcel Level 3 GCE

Friday 7 June 2024

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle A has mass 3 kg and a particle B has mass 2 kg.

The particles are moving on a smooth horizontal plane when they collide directly.

Immediately **before** the collision, the velocity of A is $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-6\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

Immediately **after** the collision the velocity of A is $\left(-2\mathbf{i} + \frac{2}{3}\mathbf{j}\right)\text{m s}^{-1}$

(a) Find the total kinetic energy of the two particles **before** the collision.

(3)

(b) Find, in terms of **i** and **j**, the impulse exerted on A by B in the collision.

(3)

(c) Find, in terms of **i** and **j**, the velocity of B immediately **after** the collision.

(3)

(a) Speed^2 of A = $3^2 + (-1)^2 = 10$, speed^2 of B = $(-6)^2 + 2^2 = 40$

Total KE = $\frac{1}{2} \times 3 \times (3^2 + (-1)^2) + \frac{1}{2} \times 2 \times ((-6)^2 + 2^2)$

= 15 + 40

= 55 J

(b) Impulse on A = change in momentum of A

$I = 3((-2i + 2/3\,j) - (3i - j))$

$I = 3(-5i + 5/3\,j)$

$I = (-15i + 5j)$ N s

(c) Impulse on B = −(impulse on A) = $(15i - 5j)$ N s

$15i - 5j = 2(v_B - (-6i + 2j))$

$v_B + 6i - 2j = 7.5i - 2.5j$

$v_B = (1.5i - 0.5j)$ m s^{-1}

Check by CLM: $3(3i - j) + 2(-6i + 2j) = -3i + j$

$3(-2i + 2/3\,j) + 2(1.5i - 0.5j) = -6i + 2j + 3i - j = -3i + j$ ✓

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Question 1 continued

(Total for Question 1 is 9 marks)

2. A rough plane is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{3}{4}$

A particle P of mass m is at rest at a point on the plane.

The particle is projected **up** the plane with speed $\sqrt{2ag}$

The particle moves up a line of greatest slope of the plane and comes to instantaneous rest after moving a distance d.

The coefficient of friction between P and the plane is $\frac{1}{7}$

(a) Show that the magnitude of the frictional force acting on P as it moves up the plane is $\frac{4mg}{35}$

(3)

Air resistance is assumed to be negligible.

Using the work-energy principle,

(b) find d in terms of a.

(4)

(a) $\tan\theta = 3/4 \Rightarrow \sin\theta = 3/5, \cos\theta = 4/5$

Resolving perpendicular to the plane: $R = mg\cos\theta = 4mg/5$

Friction is limiting as P slides: $F = \mu R$

$F = 1/7 \times mg\cos\theta = 1/7 \times mg \times 4/5$

$F = 4mg/35$ as required

(b) Work-energy principle from projection to instantaneous rest:

Initial KE = gain in GPE + work done against friction

$\frac{1}{2} \times m \times (\sqrt{(2ag)})^2 = mg \times d\sin\theta + 4mg/35 \times d$

$\frac{1}{2}m \times 2ag = mgd \times 3/5 + 4mgd/35$

$a = 3d/5 + 4d/35 = 21d/35 + 4d/35 = 25d/35 = 5d/7$

$d = 7a/5$

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Question 2 continued

(Total for Question 2 is 7 marks)

3. A car of mass 1000 kg moves in a straight line along a horizontal road at a constant speed of 72 km h^{-1}

- The resistance to the motion of the car is modelled as a constant force of magnitude 900 N

The engine of the car is working at a constant rate of P kW.

Using the model,

(a) find the value of P.

(3)

The car now travels in a straight line up a road which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{2}{49}$

- In a refined model, the resistance to the motion of the car from non-gravitational forces is now modelled as a force of magnitude $20v$ newtons, where v m s^{-1} is the speed of the car

At the instant when the engine of the car is working at a constant rate of 30 kW and the car is moving up the road at 10 m s^{-1}, the acceleration of the car is a m s^{-2}

Using the refined model,

(b) find the value of a.

(4)

Later on, when the engine of the car is again working at a constant rate of 30 kW, the car is moving up the road at a constant speed U m s^{-1}

Using the refined model,

(c) find the value of U.

(5)

(a) $v = 72$ km $h^{-1} = 72000/3600 = 20$ m s^{-1}

Constant speed, so driving force = resistance: $1000P/20 = 900$

$1000P = 18000$

$P = 18$

(b) Driving force $= 30000/10 = 3000$ N, resistance $= 20 \times 10 = 200$ N

Weight component down the slope $= 1000g \sin\alpha = 1000 \times 9.8 \times 2/49 = 400$ N

N2L up the slope: $30000/10 - 1000g \sin\alpha - 20 \times 10 = 1000a$

$3000 - 400 - 200 = 1000a$

$a = 2.4$

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Question 3 continued

(c) Driving force $F = 30000/U$

Constant speed, so resultant force up the slope is zero:

$F - 1000g \sin\alpha - 20U = 0$

$30000/U - 1000 \times 9.8 \times 2/49 - 20U = 0$

$30000/U - 400 - 20U = 0$

$30000 - 400U - 20U^2 = 0$

$U^2 + 20U - 1500 = 0$

$(U - 30)(U + 50) = 0$

$U = 30$ or $U = -50$ (reject, $U > 0$)

$U = 30$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 12 marks)

4. A particle A of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. Particle A collides directly with a particle B of mass m which is at rest on the plane.

The coefficient of restitution between A and B is e, where $e > 0$

(a) Show that the speed of B immediately after the collision is $2u(1 + e)$.

(6)

After the collision, B hits a smooth fixed vertical wall which is perpendicular to the direction of motion of B.

(b) Show that there will be a second collision between A and B.

(3)

The coefficient of restitution between B and the wall is $\frac{1}{2}$

Find, **in simplified form**, in terms of m, u and e,

(c) the magnitude of the impulse received by B in its collision with the wall,

(3)

(d) the loss in kinetic energy of B due to its collision with the wall.

(3)

(a) Take the direction of motion of A as positive; after the collision A has velocity v_A and B has velocity v_B.

CLM: $2m \times 3u = 2mv_A + mv_B \Rightarrow 6u = 2v_A + v_B$ ... (1)

NLR: $v_B - v_A = e \times 3u \Rightarrow v_B - v_A = 3ue$ ... (2)

(1) + 2 × (2): $3v_B = 6u + 6ue$

$v_B = 2u(1 + e)$ so the speed of B is $2u(1 + e)$ as required

(b) From (2): $v_A = v_B - 3ue = 2u + 2ue - 3ue$

$v_A = u(2 - e)$

Since $0 < e \leq 1$, $2 - e > 0$, so $v_A > 0$: A continues to move towards the wall. B rebounds from the wall and moves back towards A, so A and B are moving towards each other and there will be a second collision.

(c) Rebound speed of B = ½ × $2u(1 + e) = u(1 + e)$

Impulse = change in momentum of B (taking away from the wall as positive)

$I = m(u(1 + e) - (-2u(1 + e)))$

$I = m \times 3u(1 + e)$

Magnitude of impulse = $3mu(1 + e)$

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Question 4 continued

(d) Loss in KE of B = $\frac{1}{2}m(2u(1 + e))^2 - \frac{1}{2}m(u(1 + e))^2$

$= \frac{1}{2}m[4u^2(1 + e)^2 - u^2(1 + e)^2]$

$= \frac{1}{2}m \times 3u^2(1 + e)^2$

$= 3mu^2(1 + e)^2/2$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 15 marks)

5. A light elastic string has natural length $2a$ and modulus of elasticity $2mg$.
One end of the string is attached to a fixed point A on a horizontal ceiling.
The other end is attached to a particle P of mass m.

The particle P hangs in equilibrium at the point E, where $AE = 3a$.

The particle P is then projected vertically downwards from E with speed $\frac{3}{2}\sqrt{ag}$

Air resistance is assumed to be negligible.

Find the elastic energy stored in the string, when P first comes to instantaneous rest.
Give your answer in the form $kmga$, where k is a constant to be found.

(7)

Equilibrium at E: $T = mg$, with extension $e_0 = 3a - 2a = a$

check: $T = \lambda x/l = 2mg \times a/(2a) = mg$ ✓

Let P first come to instantaneous rest a distance x below E.

Then the extension is $a + x$. Conservation of energy from E to rest:

KE at E + GPE lost + EPE at E = EPE at rest

$\frac{1}{2}m(3/2\sqrt{(ag)})^2 + mgx + 2mg \times a^2/(2 \times 2a) = 2mg(a + x)^2/(2 \times 2a)$

$9mag/8 + mgx + mga/2 = mg(a + x)^2/(2a)$

Multiply by $2a/(mg)$:

$9a^2/4 + 2ax + a^2 = a^2 + 2ax + x^2$

$x^2 = 9a^2/4$

$x = 3a/2 \quad (x > 0)$

Extension at instantaneous rest $= a + 3a/2 = 5a/2$

$EPE = \lambda x^2/(2l) = 2mg \times (5a/2)^2/(2 \times 2a)$

$= 2mg \times 25a^2/4 \div 4a$

$= 25mga/8$

Elastic energy stored $= 25mga/8$, $k = 25/8$

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Question 5 continued

(Total for Question 5 is 7 marks)

6. [*In this question,* **i** *and* **j** *are horizontal perpendicular unit vectors.*]

A particle P is moving with velocity $(4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ on a smooth horizontal plane.
The particle collides with a smooth vertical wall and rebounds with velocity $(\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$

The coefficient of restitution between P and the wall is e.

(a) Find the value of e.

(6)

After the collision, P goes on to hit a second smooth vertical wall, which is parallel to **i**.

The coefficient of restitution between P and this second wall is $\frac{1}{3}$

The angle through which the direction of motion of P has been deflected by its collision with this second wall is $\alpha°$.

(b) Find the value of α, giving your answer to the nearest whole number.

(4)

(a) The impulse from the wall acts along the normal to the wall:

$m((i + 3j) - (4i - j)) = m(-3i + 4j)$, so the normal is parallel to $-3i + 4j$

unit normal $n = 1/5\,(-3i + 4j)$

Components of velocity along the normal:

approach: $(4i - j)\cdot 1/5(-3i + 4j) = (-12 - 4)/5 = -16/5$

separation: $(i + 3j)\cdot 1/5(-3i + 4j) = (-3 + 12)/5 = 9/5$

e = speed of separation ÷ speed of approach

$e = (9/5) \div (16/5)$

$e = 9/16 \quad (= 0.5625)$

(b) Second wall parallel to i: the i component is unchanged and the j component is reversed and multiplied by 1/3:

velocity after $= i - 1/3 \times 3j = (i - j)\ m\ s^{-1}$

Angle between the velocities before and after this collision:

$\cos \alpha° = (i + 3j)\cdot(i - j) / (\sqrt{(1^2 + 3^2)}\ \sqrt{(1^2 + 1^2)}) = (1 - 3)/(\sqrt{10}\ \sqrt{2})$

$\cos \alpha° = -2/\sqrt{20}$

$\alpha = 116.565\ldots \quad (\text{or } \alpha = \tan^{-1}(3) + \tan^{-1}(1) = 71.565\ldots + 45)$

$\alpha = 117 \quad (\text{nearest whole number})$

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7.

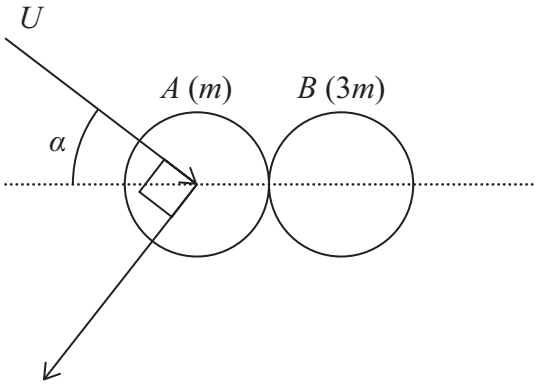


Figure 1

A smooth uniform sphere A of mass m is moving with speed U on a smooth horizontal plane. The sphere A collides obliquely with a smooth uniform sphere B of mass $3m$ which is at rest on the plane. The two spheres have the same radius.

Immediately before the collision, the direction of motion of A makes an angle α, where $0° < \alpha < 90°$, with the line joining the centres of the spheres.

Immediately after the collision, the direction of motion of A is **perpendicular** to its original direction, as shown in Figure 1.

The coefficient of restitution between the spheres is e.

(a) Show that the speed of B immediately after the collision is

$$\frac{1}{4}(1+e)U\cos\alpha$$

(6)

(b) Show that $e > \frac{1}{3}$

(4)

(c) Show that $0 < \tan\alpha \leqslant \frac{1}{\sqrt{2}}$

(5)

(a) Along the line of centres (A → B positive). Before: A has components U cos α along and U sin α perpendicular. Perpendicular component of A is unchanged (smooth spheres). After: A has v_A along BA, B has v_B along AB.

CLM: $mU\cos\alpha = -mv_A + 3mv_B \Rightarrow U\cos\alpha = -v_A + 3v_B$ ... (1)

NLR: $v_B + v_A = eU\cos\alpha$ ... (2)

(1) + (2): $4v_B = U\cos\alpha + eU\cos\alpha$

$v_B = 1/4\,(1+e)U\cos\alpha$ as required

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Question 7 continued

(b) From (2): $v_A = eU\cos\alpha - v_B = eU\cos\alpha - 1/4\,(1+e)U\cos\alpha$

$v_A = 1/4\,(4e - 1 - e)U\cos\alpha = 1/4\,(3e-1)U\cos\alpha$

A's direction after is perpendicular to its original direction, so its component along the line of centres is reversed (directed from B to A):

$v_A > 0 \quad \Rightarrow \quad 1/4\,(3e-1)U\cos\alpha > 0$

Since $0° < \alpha < 90°$, $\cos\alpha > 0$ (and $U > 0$), so

$3e - 1 > 0 \quad \Rightarrow \quad e > 1/3$ as required

(c) After the collision A has components: v_A along BA,

$U\sin\alpha$ perpendicular to the line of centres (unchanged).

Original direction makes angle α with AB; A is deflected through 90°, so its new direction makes angle $90° - \alpha$ with BA, i.e. angle α with the perpendicular to the line of centres:

$\tan\alpha = v_A / (U\sin\alpha) = 1/4\,(3e-1)U\cos\alpha / (U\sin\alpha)$

$\tan\alpha = (3e-1)/(4\tan\alpha)$

$\tan^2\alpha = 1/4\,(3e-1)$

Now $1/3 < e \leq 1$, so $0 < 3e - 1 \leq 2$:

$0 < \tan^2\alpha \leq 1/4 \times 2 = 1/2$

Since $0° < \alpha < 90°$, $\tan\alpha > 0$, so

$0 < \tan\alpha \leq 1/\sqrt{2}$ as required

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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