Pearson Edexcel Level 3 GCE

Monday 26 June 2023

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/4D**

Further Mathematics

Advanced

PAPER 4D: Decision Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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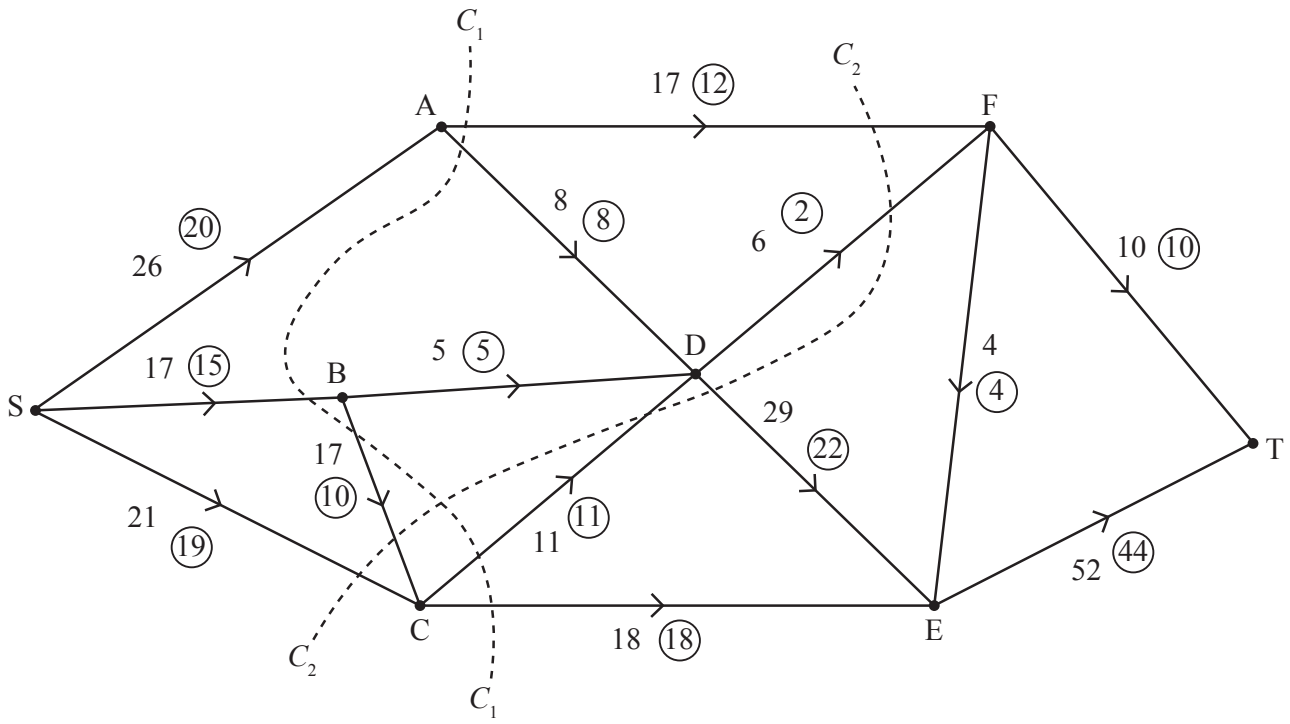


Figure 1

Figure 1 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of that pipe. The numbers in circles represent a feasible flow from S to T.

(a) State the value of the feasible flow.

(1)

(b) Find the capacity of cut C_1 and the capacity of cut C_2

(2)

(c) By inspection, find a flow-augmenting route to increase the flow by two units. You must state your route.

(1)

(d) Prove that, once the flow-augmenting route found in (c) has been applied, the flow is now maximal.

(3)

(Total for Question 1 is 7 marks)

2. An outdoor theatre is holding a summer gala performance. The theatre owner must decide whether to take out insurance against rain for this performance.

The theatre owner estimates that

- on a fine day, the total profit will be £15 000
- on a wet day, the total loss will be £20 000

Insurance against rain costs £2 000. If the performance must be cancelled due to rain, then the theatre owner will receive £16 000 from the insurer. If the performance is not cancelled due to rain, then the theatre owner will receive nothing from the insurer.

The probability of rain on the day of the gala performance is 0.2

Draw a decision tree and hence determine whether the theatre owner should take out the insurance against rain for this performance.

(5)

(Total for Question 2 is 5 marks)

3. The table below shows the stock held at each supply point and the stock required at each demand point in a standard transportation problem. The table also shows the cost, in pounds, of transporting the stock from each supply point to each demand point.

	Q	R	S	Supply
A	23	18	12	45
B	8	10	14	27
C	11	14	21	34
D	19	15	11	50
Demand	75	37	44	

The problem is partially described by the linear programming formulation below.

Let x_{ij} be the number of units transported from i to j

where $i \in \{A, B, C, D\}$

$j \in \{Q, R, S\}$ and $x_{ij} \geqslant 0$

Minimise $P = 23x_{AQ} + 18x_{AR} + 12x_{AS} + 8x_{BQ} + 10x_{BR} + 14x_{BS}$
$+ 11x_{CQ} + 14x_{CR} + 21x_{CS} + 19x_{DQ} + 15x_{DR} + 11x_{DS}$

(a) Write down, as inequalities, the constraints of the linear program.

(2)

(b) Use the north-west corner method to obtain an initial solution to this transportation problem.

(1)

(c) Taking AS as the entering cell, use the stepping-stone method to find an improved solution. Make your route clear.

(2)

(d) Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and the

- shadow costs
- improvement indices
- entering cell and exiting cell

(4)

(Total for Question 3 is 9 marks)

4. Four students, A, B, C and D, are to be allocated to four rounds, 1, 2, 3 and 4, in a competition. Each student is to take part in exactly one round and no two students may play in the same round.

Each student has been given an estimated score for each round. The estimated scores for each student are shown in the table below.

	1	2	3	4
A	34	20	18	15
B	49	31	12	34
C	48	27	23	26
D	52	45	42	42

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total estimated score. You must make your method clear and show the table after each stage.

(7)

(b) Find this total estimated score.

(1)

(Total for Question 4 is 8 marks)

5. A sequence $\{u_n\}$, where $n \geqslant 0$, satisfies the second order recurrence relation

$$u_{n+2} = \frac{1}{2}(u_{n+1} + u_n) + 3 \quad \text{where} \quad u_0 = 15 \quad u_1 = 20$$

(a) By considering the sequence $\{v_n\}$, where $u_n = v_n + 2n$ for $n \geqslant 0$, determine an expression for u_n as a function of n.

(7)

(b) Describe the long-term behaviour of u_n

(1)

(Total for Question 5 is 8 marks)

6. Polly is a motivational speaker who is planning her engagements for the next four weeks.

Polly will

- visit four different countries in these four weeks
- visit just one country each week
- leave from her home, S, and return there only after visiting the four countries
- travel directly from one country to the next

Polly wishes to determine a schedule of four countries to visit.

Table 1 shows the countries Polly could visit each week.

Week	1	2	3	4
Possible countries to visit	A or B	C, D or E	F or G	H, I or J

Table 1

Table 2 shows the speaker fee, in £100s, Polly would expect to earn in each country.

Country	A	B	C	D	E	F	G	H	I	J
Earnings in £100s	47	45	48	47	49	44	45	47	49	48

Table 2

Table 3 shows the cost, in £100s, of travelling between the countries.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
S	5	2						7	8	8
A			3	4	5					
B			5	4	6					
C						7	5			
D						6	7			
E						7	6			
F								6	7	8
G								7	8	6

Table 3

Polly's expected income is the value of the speaker fee minus the cost of travel.

She wants to find a schedule that maximises her total expected income for the four weeks.

Use dynamic programming to determine the optimal schedule. Complete the table provided in the answer book and state the maximum expected income.

(13)

(Total for Question 6 is 13 marks)

7. Martina decides to open a bank account to help her to save for a holiday. Each month she puts £k into the account and allows herself to spend one quarter of what was in the account at the end of the previous month.

Let u_n (where $n \geqslant 1$) represent the amount in the account at the end of month n.

Martina has £k in the account at the end of the first month.

(a) By setting up a first order recurrence relation for u_{n+1} in terms of u_n, determine an expression for u_n in terms of n and k.

(6)

At the end of the 8th month, Martina needs to have at least £1750 in the account to pay for her holiday.

(b) Determine, to the nearest penny, the minimum amount of money that Martina should put into the account each month.

(2)

(Total for Question 7 is 8 marks)

8. A two-person zero-sum game is represented by the pay-off matrix for player A shown below.

		Player B		
		Option X	Option Y	Option Z
Player A	Option Q	−3	2	5
	Option R	2	−1	0
	Option S	4	−2	−1
	Option T	−4	0	2

(a) Verify that there is no stable solution to this game.

(2)

(b) Explain why player A should never play option T. You must make your reasoning clear.

(2)

Player A intends to make a random choice between options Q, R and S, choosing option Q with probability p_1, option R with probability p_2 and option S with probability p_3

Player A wants to calculate the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.

(c) (i) Formulate the game as a linear programming problem for player A. You should write the constraints as equations.

(ii) Write down an initial Simplex tableau for this linear programming problem, making your variables clear.

(7)

The linear programming problem is solved using the Simplex algorithm. The optimal value of p_1 is $\frac{6}{11}$ and the optimal value of p_2 is 0

(d) Find the best strategy for player B, defining any variables you use.

(6)

(Total for Question 8 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

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Answer Book

Do not return the question paper with the answer book.

Total Marks

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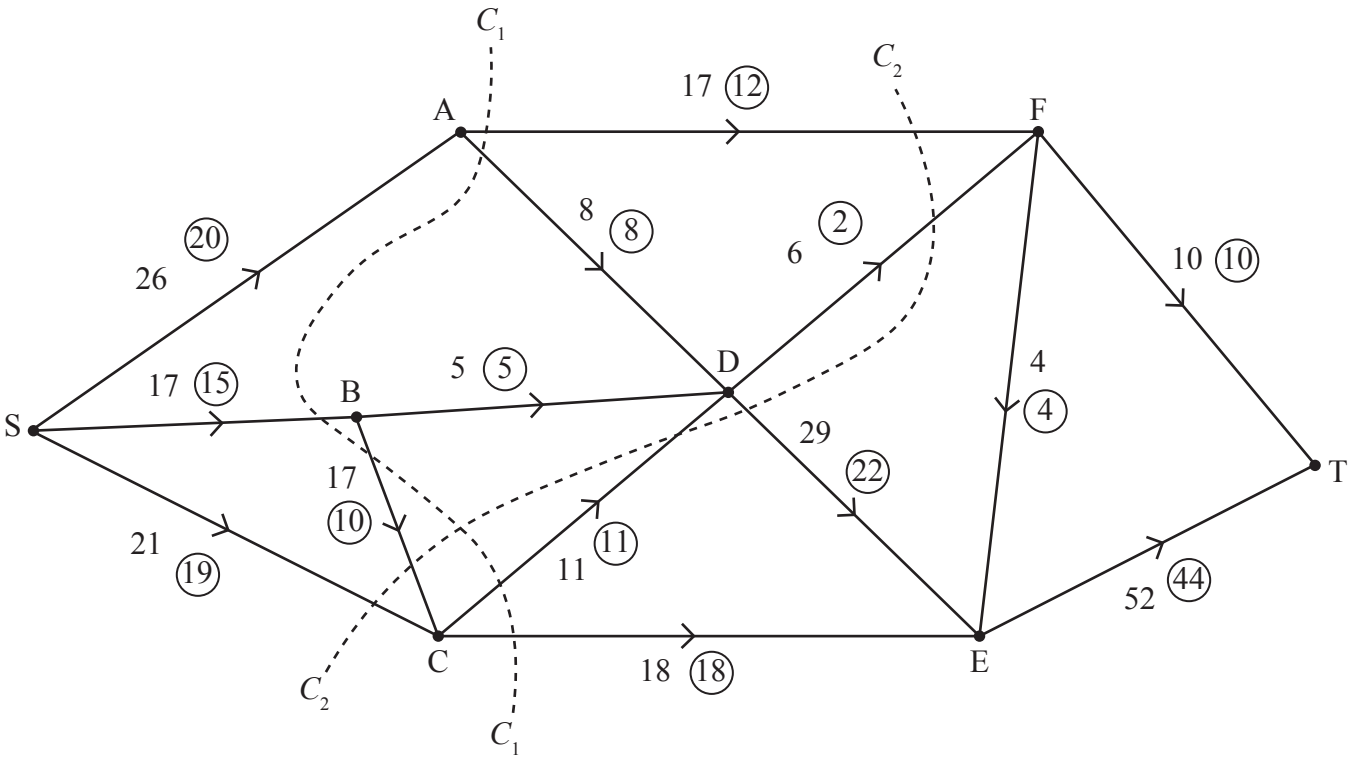


Figure 1

(a) Value of feasible flow = 20 + 15 + 19 = 54

(b) C_1 = 17 + 8 + 17 + 11 + 18 = 71

C_2 = 17 + 6 + 29 + 17 + 21 = 90

(c) Flow-augmenting route: S – A – F – D – E – T (2 units)

(the 2 units along F – D cancel the flow of 2 in arc DF)

(d) New flow value = 54 + 2 = 56

Consider the cut through arcs FT, FE, DF, AD, BD, CD and CE

(S, A, B, C, F on the source side; D, E, T on the sink side)

DF goes from the sink side to the source side, so it

contributes 0 to the capacity of the cut:

Capacity = 10 + 4 + 0 + 8 + 5 + 11 + 18 = 56

Value of flow = 56 = capacity of this cut

By the maximum flow minimum cut theorem, since the flow

equals the capacity of a cut, the flow of 56 is maximal

(56 is the maximum flow and the cut is a minimum cut).

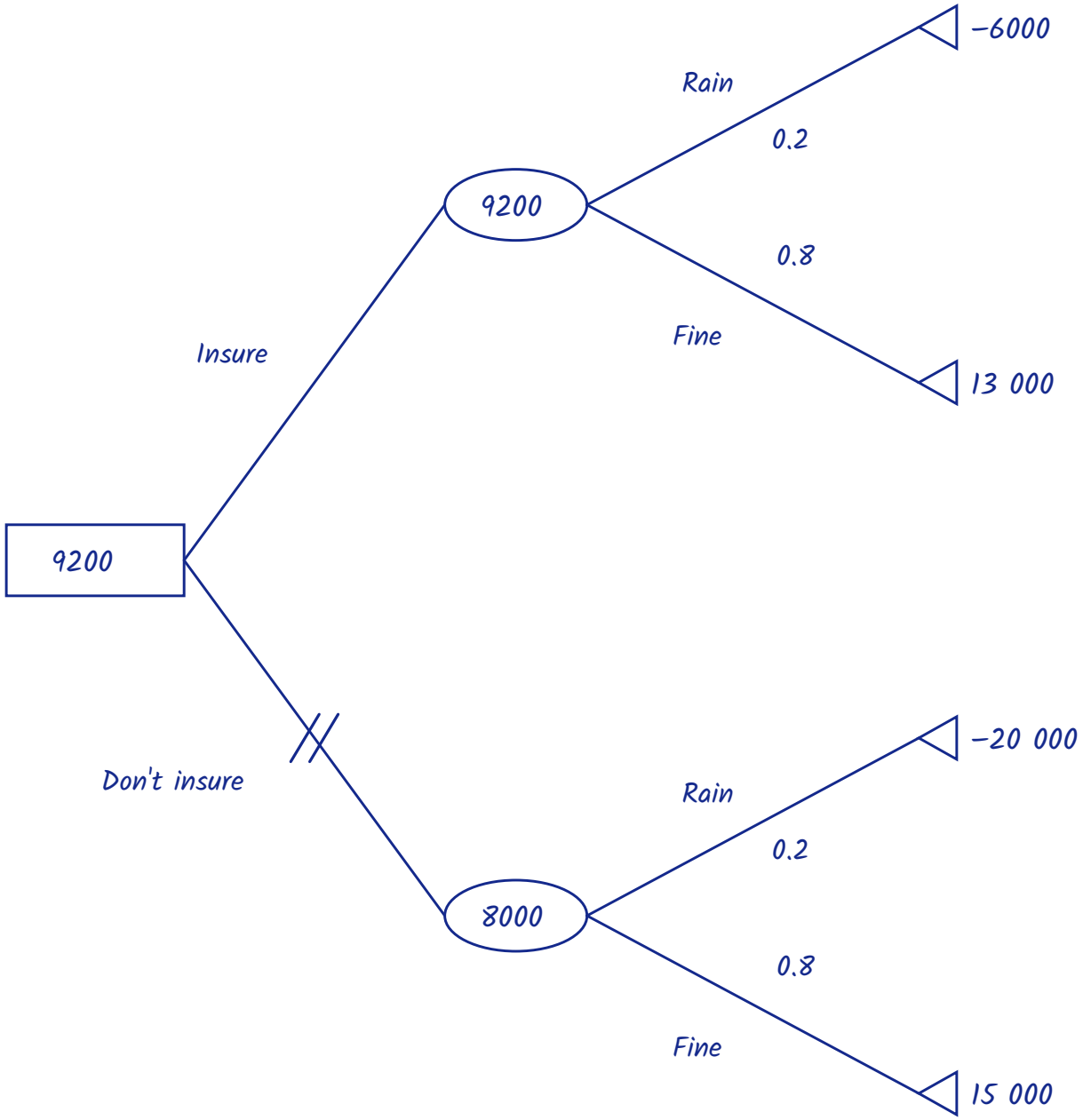
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Question 1 continued

(Total for Question 1 is 7 marks)

2.



Insure: rain −20 000 − 2000 + 16 000 = −6000, fine 15 000 − 2000 = 13 000

EMV(insure) = 0.2(−6000) + 0.8(13 000) = −1200 + 10 400 = £9200

EMV(don't insure) = 0.2(−20 000) + 0.8(15 000) = −4000 + 12 000 = £8000

9200 > 8000, so the theatre owner should take out the insurance against rain (expected profit £9200).

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Question 2 continued

Use the space below if you need to redraw your decision tree.

(Total for Question 2 is 5 marks)

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3.

(a)

$x_{AQ} + x_{AR} + x_{AS} \leq 45$

$x_{BQ} + x_{BR} + x_{BS} \leq 27$

$x_{CQ} + x_{CR} + x_{CS} \leq 34$

$x_{DQ} + x_{DR} + x_{DS} \leq 50$

$x_{AQ} + x_{BQ} + x_{CQ} + x_{DQ} \geq 75$

$x_{AR} + x_{BR} + x_{CR} + x_{DR} \geq 37$

$x_{AS} + x_{BS} + x_{CS} + x_{DS} \geq 44$

(b)

	Q	R	S
A	45		
B	27		
C	3	31	
D		6	44

You may not need to use all of these tables.

	Q	R	S
A	45 − θ		θ
B	27		
C	3 + θ	31 − θ	
D		6 + θ	44 − θ

	Q	R	S
A	14		31
B	27		
C	34		
D		37	13

(c) Entering cell AS

Route: AS(+) AQ(−) CQ(+)

CR(−) DR(+) DS(−)

θ = min(45, 31, 44) = 31

Exiting cell CR

Improved solution:

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Question 3 continued

	Q	R	S
A	X	2	X
B	X	9	17
C	X	10	21
D	−3	X	X

(d) Shadow costs ($c_{ij} = R_i + K_j$):

$R_A = 0$, $R_B = -15$, $R_C = -12$, $R_D = -1$

$K_Q = 23$, $K_R = 16$, $K_S = 12$

Improvement indices $I_{ij} = c_{ij} - R_i - K_j$

shown in table (X = occupied cell)

e.g. DQ: $19 - (-1) - 23 = -3$

	Q	R	S
A	14 − θ		31 + θ
B	27		
C	34		
D	θ	37	13 − θ

Most negative II is DQ (−3)

Entering cell DQ

Route: DQ(+) AQ(−) AS(+) DS(−)

$\theta = \min(14, 13) = 13$

Exiting cell DS

	Q	R	S
A	1		44
B	27		
C	34		
D	13	37	

Improved solution:

AQ 1, AS 44, BQ 27, CQ 34,

DQ 13, DR 37

	Q	R	S
A			
B			
C			
D			

	Q	R	S
A			
B			
C			
D			

(Total for Question 3 is 9 marks)

4.

	1	2	3	4
A	34	20	18	15
B	49	31	12	34
C	48	27	23	26
D	52	45	42	42

You may not need to use all of these tables.

	1	2	3	4
A	18	32	34	37
B	3	21	40	18
C	4	25	29	26
D	0	7	10	10

Maximising, so subtract every element from 52 (the largest)

	1	2	3	4
A	0	14	16	19
B	0	18	37	15
C	0	21	25	22
D	0	7	10	10

Reduce rows: subtract 18, 3, 4, 0 from rows A, B, C, D

	1	2	3	4
A	0	7	6	9
B	0	11	27	5
C	0	14	15	12
D	0	0	0	0

Reduce columns: subtract 0, 7, 10, 10
Zeros covered by 2 lines (column 1, row D): 2 < 4 so not optimal
Smallest uncovered element = 5

	1	2	3	4
A	0	2	1	4
B	0	6	22	0
C	0	9	10	7
D	5	0	0	0

Subtract 5 from uncovered elements, add 5 to D1 (covered twice)
Now 3 lines needed (column 1, column 4, row D): 3 < 4, not optimal
Smallest uncovered element = 1

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Question 4 continued

	1	2	3	4
A	0	1	0	4
B	0	5	21	0
C	0	8	9	7
D	6	0	0	1

Subtract 1 from uncovered elements, add 1 to D1 and D4 (covered twice) Now 4 lines are needed to cover the zeros, so this is optimal

	1	2	3	4
A				
B				
C				
D				

Allocation:

A = round 3, B = round 4, C = round 1, D = round 2

	1	2	3	4
A				
B				
C				
D				

(b) Total score = 18 + 34 + 48 + 45 = 145

	1	2	3	4
A				
B				
C				
D				

	1	2	3	4
A				
B				
C				
D				

(Total for Question 4 is 8 marks)

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5.

(a) $u_{n+2} = \frac{1}{2}(u_{n+1} + u_n) + 3$, with $u_n = v_n + 2n$:

$v_{n+2} + 2(n + 2) = \frac{1}{2}(v_{n+1} + 2(n + 1) + v_n + 2n) + 3$

$v_{n+2} + 2n + 4 = \frac{1}{2}v_{n+1} + \frac{1}{2}v_n + 2n + 1 + 3$

$v_{n+2} = \frac{1}{2}v_{n+1} + \frac{1}{2}v_n$

$\Rightarrow 2v_{n+2} - v_{n+1} - v_n = 0$

Auxiliary equation: $2m^2 - m - 1 = 0$

$(2m + 1)(m - 1) = 0 \Rightarrow m = 1$ or $m = -\frac{1}{2}$

$v_n = A(1)^n + B(-\frac{1}{2})^n = A + B(-\frac{1}{2})^n$

$u_0 = 15 \Rightarrow v_0 = 15 - 2(0) = 15$

$u_1 = 20 \Rightarrow v_1 = 20 - 2(1) = 18$

$n = 0$: $A + B = 15$

$n = 1$: $A - \frac{1}{2}B = 18$

Subtracting: $1.5B = -3 \Rightarrow B = -2$, $A = 17$

$v_n = 17 - 2(-\frac{1}{2})^n$

$\therefore u_n = 17 - 2(-\frac{1}{2})^n + 2n$

(b) As $n \to \infty$, $(-\frac{1}{2})^n \to 0$, so $u_n \approx 17 + 2n$

i.e. for large n the terms are (approximately) in arithmetic progression, increasing by 2 each term ($u_n \to \infty$ linearly).

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Question 5 continued

(Total for Question 5 is 8 marks)

6.

Stage	State	Action	Destination	Value
1	H	HS	S	47 − 7 = 40*
	I	IS	S	49 − 8 = 41*
	J	JS	S	48 − 8 = 40*
2	F	FH	H	44 − 6 + 40 = 78*
		FI	I	44 − 7 + 41 = 78*
		FJ	J	44 − 8 + 40 = 76
	G	GH	H	45 − 7 + 40 = 78
		GI	I	45 − 8 + 41 = 78
		GJ	J	45 − 6 + 40 = 79*
3	C	CF	F	48 − 7 + 78 = 119
		CG	G	48 − 5 + 79 = 122*
	D	DF	F	47 − 6 + 78 = 119*
		DG	G	47 − 7 + 79 = 119*
	E	EF	F	49 − 7 + 78 = 120
		EG	G	49 − 6 + 79 = 122*
4	A	AC	C	47 − 3 + 122 = 166*
		AD	D	47 − 4 + 119 = 162
		AE	E	47 − 5 + 122 = 164
	B	BC	C	45 − 5 + 122 = 162*
		BD	D	45 − 4 + 119 = 160
		BE	E	45 − 6 + 122 = 161
5	S	SA	A	−5 + 166 = 161*
		SB	B	−2 + 162 = 160

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Question 6 continued

Stage	State	Action	Destination	Value

Values in £100s (fee − travel cost + best value onwards)

Optimal schedule: S − A − C − G − J − S

(week 1 A, week 2 C, week 3 G, week 4 J)

Maximum expected income = 161 × £100 = £16 100

(Total for Question 6 is 13 marks)

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7.

(a) Each month she spends ¼ of the previous amount and adds £k:

$u_{n+1} = u_n - \frac{1}{4}u_n + k$

$u_{n+1} = 0.75u_n + k, \quad u_1 = k$

Auxiliary equation $m - 0.75 = 0$, so the complementary function is $A(0.75)^n$

Particular solution: try $u_n = \lambda$

$\lambda = 0.75\lambda + k \Rightarrow 0.25\lambda = k \Rightarrow \lambda = 4k$

General solution: $u_n = A(0.75)^n + 4k$

$u_1 = k$: $0.75A + 4k = k \Rightarrow 0.75A = -3k \Rightarrow A = -4k$

$\therefore u_n = 4k - 4k(0.75)^n = 4k(1 - 0.75^n)$

(b) Need $u_8 \geq 1750$:

$4k(1 - 0.75^8) \geq 1750$

$4k(0.8998870\ldots) \geq 1750$

$3.599548\ldots k \geq 1750$

$k \geq 1750 \div 3.599548\ldots = 486.1721068\ldots$

Minimum amount each month = £486.18

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Question 7 continued

(Total for Question 7 is 8 marks)

8.

		Player B		
		Option X	Option Y	Option Z
Player A	Option Q	−3	2	5
	Option R	2	−1	0
	Option S	4	−2	−1
	Option T	−4	0	2

(a) Row minima: Q −3, R −1, S −2, T −4; row maximin = −1

Column maxima: X 4, Y 2, Z 5; column minimax = 2

Row maximin −1 ≠ column minimax 2, so the game has no stable solution.

(b) Option Q dominates option T, because −3 > −4, 2 > 0 and 5 > 2: whatever B plays, Q gives A a better pay-off than T.

(c)(i) Add 3 to every element so that all pay-offs are non-negative:

Q: 0, 5, 8 R: 5, 2, 3 S: 7, 1, 2 (against X, Y, Z)

Let V = value of the augmented game to player A

(V = value of original game + 3)

Maximise $P = V$, i.e. $P - V = 0$

subject to

$V - 5p_2 - 7p_3 + r = 0$

$V - 5p_1 - 2p_2 - p_3 + s = 0$

$V - 8p_1 - 3p_2 - 2p_3 + t = 0$

$p_1 + p_2 + p_3 + u = 1$

$p_1, p_2, p_3, V, r, s, t, u \geq 0$

where r, s, t, u are slack variables

(ii) Initial Simplex tableau (on the next page)

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Question 8 continued

(c)(ii) Initial Simplex tableau:

b.v.	V	p_1	p_2	p_3	r	s	t	u	Value
r	1	0	−5	−7	1	0	0	0	0
s	1	−5	−2	−1	0	1	0	0	0
t	1	−8	−3	−2	0	0	1	0	0
u	0	1	1	1	0	0	0	1	1
P	−1	0	0	0	0	0	0	0	0

Question 8 continued

(d) $p_3 = 1 - 6/11 - 0 = 5/11$

In the augmented game:

B plays X: $V \leq 0(6/11) + 5(0) + 7(5/11) = 35/11$

B plays Y: $V \leq 5(6/11) + 2(0) + 1(5/11) = 35/11$

B plays Z: $V \leq 8(6/11) + 3(0) + 2(5/11) = 58/11$

$\Rightarrow V = 35/11$ (value of original game = $35/11 - 3 = 2/11$)

Let player B play option X with probability q_1, option Y with probability q_2 and option Z with probability q_3

Against Z, A expects $58/11 > 35/11$, so B should never play option Z: $q_3 = 0$

A plays Q: $0q_1 + 5q_2 + 8q_3 = 35/11$

A plays S: $7q_1 + q_2 + 2q_3 = 35/11$

$q_1 + q_2 + q_3 = 1$

With $q_3 = 0$: $5q_2 = 35/11 \Rightarrow q_2 = 7/11$

$q_1 = 1 - 7/11 = 4/11$

Check (A plays S): $7(4/11) + 7/11 = 35/11$ ✓

Player B should play option X with probability 4/11, option Y with probability 7/11 and never play option Z.

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Question 8 continued

(Total for Question 8 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

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