Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 22 June 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/3A**

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. (a) Use Simpson's rule with 4 intervals to find an estimate for

$$\int_0^2 e^{\sin^2 x}\, dx$$

Give your answer to 3 significant figures.

(4)

Given that $\int_0^2 e^{\sin^2 x}\, dx = 3.855$ to 4 significant figures,

(b) comment on the accuracy of your answer to part (a).

(1)

(a) $h = (2 - 0) \div 4 = 0.5$

$x_0 = 0$: $y_0 = e^{\sin^2 0} = 1$

$x_1 = 0.5$: $y_1 = e^{\sin^2 0.5} = 1.25841...$

$x_2 = 1$: $y_2 = e^{\sin^2 1} = 2.03008...$

$x_3 = 1.5$: $y_3 = e^{\sin^2 1.5} = 2.70471...$

$x_4 = 2$: $y_4 = e^{\sin^2 2} = 2.28604...$

$\int_0^2 e^{\sin^2 x}\, dx \approx \frac{h}{3}[y_0 + 4(y_1 + y_3) + 2y_2 + y_4]$

$= \frac{0.5}{3}[1 + 4(1.25841 + 2.70471) + 2(2.03008) + 2.28604]$

$= \frac{0.5}{3} \times 23.19869... = 3.86644...$

Estimate = 3.87 (3 s.f.)

(b) Percentage error $= \frac{3.87 - 3.855}{3.855} \times 100\% \approx 0.4\%$

Very accurate: the estimate is correct to 2 significant figures (3.9) but it is not correct to 3 significant figures.

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Question 1 continued

(Total for Question 1 is 5 marks)

2. The vertical height, h m, above horizontal ground, of a passenger on a fairground ride, t seconds after the ride starts, where $t \leqslant 5$, is modelled by the differential equation

$$t^2 \frac{\mathrm{d}^2h}{\mathrm{d}t^2} - 2t\frac{\mathrm{d}h}{\mathrm{d}t} + 2h = t^3 \qquad \text{(I)}$$

(a) Given that $t = \mathrm{e}^x$, show that

(i) $t\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}h}{\mathrm{d}x}$

(ii) $t^2\dfrac{\mathrm{d}^2h}{\mathrm{d}t^2} = \dfrac{\mathrm{d}^2h}{\mathrm{d}x^2} - \dfrac{\mathrm{d}h}{\mathrm{d}x}$

(4)

(b) Hence show that the transformation $t = \mathrm{e}^x$ transforms equation (I) into the equation

$$\frac{\mathrm{d}^2h}{\mathrm{d}x^2} - 3\frac{\mathrm{d}h}{\mathrm{d}x} + 2h = \mathrm{e}^{3x}$$

(1)

(c) Hence show that

$$h = At + Bt^2 + \frac{1}{2}t^3$$

where A and B are constants.

(6)

Given that when $t = 1$, $h = 2.5$ and when $t = 2$, $\dfrac{\mathrm{d}h}{\mathrm{d}t} = -1$

(d) determine the height of the passenger above the ground 5 seconds after the start of the ride.

(5)

(a)(i) $t = e^x \Rightarrow \frac{dt}{dx} = e^x = t$

$\frac{dh}{dx} = \frac{dh}{dt} \times \frac{dt}{dx} = e^x \frac{dh}{dt} \Rightarrow t\frac{dh}{dt} = \frac{dh}{dx}$ as required

(a)(ii) Differentiate $\frac{dh}{dx} = t\frac{dh}{dt}$ with respect to x:

$\frac{d^2h}{dx^2} = \frac{dt}{dx}\frac{dh}{dt} + t\frac{d^2h}{dt^2}\frac{dt}{dx} = t\frac{dh}{dt} + t^2\frac{d^2h}{dt^2}$

$\Rightarrow t^2\frac{d^2h}{dt^2} = \frac{d^2h}{dx^2} - t\frac{dh}{dt} = \frac{d^2h}{dx^2} - \frac{dh}{dx}$ as required

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Question 2 continued

(b) $t^2\frac{d^2h}{dt^2} - 2t\frac{dh}{dt} + 2h = t^3$

$\Rightarrow \left(\frac{d^2h}{dx^2} - \frac{dh}{dx}\right) - 2\frac{dh}{dx} + 2h = (e^x)^3$

$\Rightarrow \frac{d^2h}{dx^2} - 3\frac{dh}{dx} + 2h = e^{3x}$ as required

(c) Auxiliary equation: $m^2 - 3m + 2 = 0 \Rightarrow (m-1)(m-2) = 0 \Rightarrow m = 1, 2$

Complementary function: $h = Ae^x + Be^{2x}$

Particular integral: $h = ke^{3x}$

$\frac{dh}{dx} = 3ke^{3x}$, $\frac{d^2h}{dx^2} = 9ke^{3x}$

$9ke^{3x} - 9ke^{3x} + 2ke^{3x} = e^{3x} \Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$

General solution: $h = Ae^x + Be^{2x} + \frac{1}{2}e^{3x}$

$e^x = t$: $h = At + Bt^2 + \frac{1}{2}t^3$ as required

(d) $t = 1, h = 2.5$: $2.5 = A + B + \frac{1}{2} \Rightarrow A + B = 2$

$\frac{dh}{dt} = A + 2Bt + \frac{3}{2}t^2$

$t = 2, \frac{dh}{dt} = -1$: $-1 = A + 4B + 6 \Rightarrow A + 4B = -7$

Subtract: $3B = -9 \Rightarrow B = -3, \quad A = 5$

$h = 5t - 3t^2 + \frac{1}{2}t^3$

$t = 5$: $h = 5(5) - 3(5)^2 + \frac{1}{2}(5)^3 = 25 - 75 + 62.5 = 12.5$

The height of the passenger after 5 seconds is 12.5 m

Question 2 continued

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Question 2 continued

(Total for Question 2 is 16 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

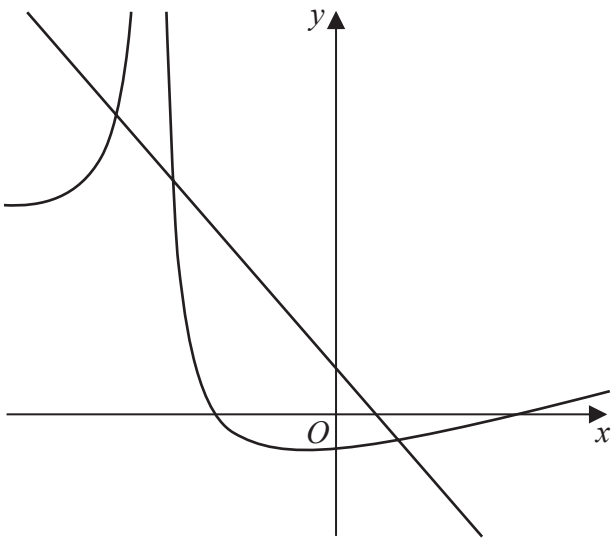


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{x^2 - 2x - 24}{|x + 6|}$ and the line with equation $y = 5 - 4x$

Use algebra to determine the values of x for which

$$\frac{x^2 - 2x - 24}{|x + 6|} < 5 - 4x$$

(7)

Multiply both sides by $|x + 6| > 0$: $x^2 - 2x - 24 < (5 - 4x)|x + 6|$

Case 1: $x > -6$, so $|x + 6| = x + 6$

$x^2 - 2x - 24 < (5 - 4x)(x + 6) = -4x^2 - 19x + 30$

$5x^2 + 17x - 54 < 0 \Rightarrow (5x + 27)(x - 2) < 0$

Critical values $x = -\frac{27}{5}$, $x = 2 \Rightarrow -\frac{27}{5} < x < 2$ (both > -6)

Case 2: $x < -6$, so $|x + 6| = -(x + 6)$

$x^2 - 2x - 24 < -(5 - 4x)(x + 6) = 4x^2 + 19x - 30$

$0 < 3x^2 + 21x - 6 \Rightarrow x^2 + 7x - 2 > 0$

Critical values $x = \dfrac{-7 \pm \sqrt{(49 + 8)}}{2} = \dfrac{-7 \pm \sqrt{57}}{2}$

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Question 3 continued

$x^2 + 7x - 2 > 0 \Rightarrow x < \frac{-7 - \sqrt{57}}{2}$ or $x > \frac{-7 + \sqrt{57}}{2}$

$\frac{-7 - \sqrt{57}}{2} = -7.27\ldots < -6$ (valid), $\frac{-7 + \sqrt{57}}{2} = 0.27\ldots > -6$ (reject)

so $x < \frac{-7 - \sqrt{57}}{2}$

Combining both cases, the solution is

$x < \frac{-7 - \sqrt{57}}{2}$ or $-\frac{27}{5} < x < 2$

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4. The ellipse E has equation

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

(a) Determine the exact value of the eccentricity of E

(2)

The points $P(4\cos\theta, 3\sin\theta)$ and $Q(4\cos\theta, -3\sin\theta)$ lie on E where $0 < \theta < \frac{\pi}{2}$
The line l_1 is the normal to E at the point P

(b) Use calculus to show that l_1 has equation

$$4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta$$

(4)

The line l_2 passes through the origin and the point Q
The lines l_1 and l_2 intersect at the point R

(c) Determine, in simplest form, the coordinates of R

(4)

(d) Hence show that, as θ varies, R lies on an ellipse which has the same eccentricity as ellipse E

(2)

(a) $b^2 = a^2(1 - e^2) \Rightarrow 9 = 16(1 - e^2)$

$\Rightarrow e^2 = 1 - \frac{9}{16} = \frac{7}{16} \Rightarrow e = \frac{\sqrt{7}}{4} \quad (e > 0)$

(b) $\frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \frac{2x}{16} + \frac{2y}{9}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{9x}{16y}$

At P: $\frac{dy}{dx} = -\frac{9(4\cos\theta)}{16(3\sin\theta)} = -\frac{3\cos\theta}{4\sin\theta}$

Gradient of normal $= \frac{4\sin\theta}{3\cos\theta}$

l_1: $y - 3\sin\theta = \frac{4\sin\theta}{3\cos\theta}(x - 4\cos\theta)$

$3y\cos\theta - 9\sin\theta\cos\theta = 4x\sin\theta - 16\sin\theta\cos\theta$

$\Rightarrow 4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta$ as required

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Question 4 continued

(c) l_2 passes through O and Q$(4\cos\theta, -3\sin\theta)$: $y = -\dfrac{3\sin\theta}{4\cos\theta}x$

Substitute into l_1:

$4x\sin\theta - 3\cos\theta\left(-\dfrac{3\sin\theta}{4\cos\theta}x\right) = 7\sin\theta\cos\theta$

$4x\sin\theta + \dfrac{9}{4}x\sin\theta = 7\sin\theta\cos\theta \Rightarrow \dfrac{25}{4}x\sin\theta = 7\sin\theta\cos\theta$

$\Rightarrow x = \dfrac{28\cos\theta}{25} \quad (\sin\theta \neq 0)$

$y = -\dfrac{3\sin\theta}{4\cos\theta} \times \dfrac{28\cos\theta}{25} = -\dfrac{21\sin\theta}{25}$

$R\left(\dfrac{28\cos\theta}{25}, -\dfrac{21\sin\theta}{25}\right)$

(d) $R = (a\cos\varphi, b\sin\varphi)$ with $a = \dfrac{28}{25}$, $b = \dfrac{21}{25}$, $\varphi = -\theta$

so R lies on the ellipse $\dfrac{x^2}{(28/25)^2} + \dfrac{y^2}{(21/25)^2} = 1$

$b^2 = a^2(1 - e^2)$: $\dfrac{441}{625} = \dfrac{784}{625}(1 - e^2)$

$\Rightarrow 1 - e^2 = \dfrac{441}{784} = \dfrac{9}{16} \Rightarrow e^2 = \dfrac{7}{16} \Rightarrow e = \dfrac{\sqrt{7}}{4}$

This is the same eccentricity as ellipse E, as required

Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

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5. (a) Show that the substitution $t = \tan\left(\frac{x}{2}\right)$ transforms the integral

$$\int \frac{1}{2\sin x - \cos x + 5}\,\mathrm{d}x$$

into the integral

$$\int \frac{1}{3t^2 + 2t + 2}\,\mathrm{d}t$$

(4)

(b) Hence determine

$$\int \frac{1}{2\sin x - \cos x + 5}\,\mathrm{d}x$$

(4)

(a) $t = \tan\left(\frac{x}{2}\right) \Rightarrow \frac{dt}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) = \frac{1+t^2}{2} \Rightarrow dx = \frac{2}{1+t^2}\,dt$

$\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

$2\sin x - \cos x + 5 = \frac{4t - (1-t^2) + 5(1+t^2)}{1+t^2} = \frac{6t^2+4t+4}{1+t^2}$

$\int \frac{1}{2\sin x - \cos x + 5}\,dx = \int \frac{1+t^2}{6t^2+4t+4} \times \frac{2}{1+t^2}\,dt$

$= \int \frac{2}{6t^2+4t+4}\,dt = \int \frac{1}{3t^2+2t+2}\,dt$ as required

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Question 5 continued

(b) $3t^2 + 2t + 2 = 3\left[\left(t + \frac{1}{3}\right)^2 + \frac{5}{9}\right]$

$$\int \frac{1}{3t^2 + 2t + 2}\,dt = \frac{1}{3}\int \frac{1}{(t + 1/3)^2 + 5/9}\,dt$$

$$= \frac{1}{3} \times \frac{3}{\sqrt{5}} \tan^{-1}\left(\frac{t + 1/3}{\sqrt{5}/3}\right) + c$$

$$= \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{3t + 1}{\sqrt{5}}\right) + c$$

$$\int \frac{1}{2\sin x - \cos x + 5}\,dx = \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{3\tan(x/2) + 1}{\sqrt{5}}\right) + c$$

Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

6. $y = \ln(e^{2x}\cos 3x) \qquad -\frac{1}{2} < x < \frac{1}{2}$

(a) Show that

$$\frac{dy}{dx} = 2 - 3\tan 3x$$

(2)

(b) Determine $\frac{d^4y}{dx^4}$

(3)

(c) Hence determine the first 3 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(e^{2x}\cos 3x)$, giving each coefficient in simplest form.

(3)

(d) Use the Maclaurin series expansion for $\ln(1 + x)$ to write down the first 4 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(1 + kx)$, where k is a constant.

(1)

(e) Hence determine the value of k for which

$$\lim_{x\to 0}\left(\frac{1}{x^2}\ln\frac{e^{2x}\cos 3x}{1+kx}\right)$$

exists.

(3)

(a) $y = \ln(e^{2x}) + \ln(\cos 3x) = 2x + \ln(\cos 3x)$

$\frac{dy}{dx} = 2 + \frac{-3\sin 3x}{\cos 3x} = 2 - 3\tan 3x$ as required

(b) $\frac{d^2y}{dx^2} = -9\sec^2 3x$

$\frac{d^3y}{dx^3} = -9 \times 2\sec 3x \times 3\sec 3x \tan 3x = -54\sec^2 3x \tan 3x$

$\frac{d^4y}{dx^4} = -54(6\sec^2 3x \tan 3x)\tan 3x - 54\sec^2 3x(3\sec^2 3x)$

$\frac{d^4y}{dx^4} = -324\sec^2 3x \tan^2 3x - 162\sec^4 3x$

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Question 6 continued

(c) At $x = 0$: $y = \ln 1 = 0$, $\frac{dy}{dx} = 2$, $\frac{d^2y}{dx^2} = -9$, $\frac{d^3y}{dx^3} = 0$, $\frac{d^4y}{dx^4} = -162$

$$y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{(4)}(0) + \ldots$$

$$\ln(e^{2x}\cos 3x) = 0 + 2x + \frac{-9}{2}x^2 + 0 + \frac{-162}{24}x^4 + \ldots$$

$$= 2x - \frac{9}{2}x^2 - \frac{27}{4}x^4 + \ldots$$

(d) $$\ln(1 + kx) = kx - \frac{k^2x^2}{2} + \frac{k^3x^3}{3} - \frac{k^4x^4}{4} + \ldots$$

(e) $$\ln\left(\frac{e^{2x}\cos 3x}{1 + kx}\right) = \ln(e^{2x}\cos 3x) - \ln(1 + kx)$$

$$= 2x - \frac{9}{2}x^2 - \frac{27}{4}x^4 + \ldots - \left(kx - \frac{k^2x^2}{2} + \frac{k^3x^3}{3} - \ldots\right)$$

$$= (2 - k)x - \frac{9 - k^2}{2}x^2 - \frac{k^3}{3}x^3 + \ldots$$

$$\frac{1}{x^2}\ln\left(\frac{e^{2x}\cos 3x}{1 + kx}\right) = \frac{2 - k}{x} - \frac{9 - k^2}{2} - \frac{k^3}{3}x + \ldots$$

As $x \to 0$ the $(2 - k)/x$ term is unbounded unless $2 - k = 0$; other terms are finite

For the limit to exist: $2 - k = 0 \Rightarrow k = 2$

Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

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7. With respect to a fixed origin O the point A has coordinates (3, 6, 5) and the line l has equation

$$\left(\mathbf{r} - (12\mathbf{i} + 30\mathbf{j} + 39\mathbf{k})\right) \times (7\mathbf{i} + 13\mathbf{j} + 24\mathbf{k}) = \mathbf{0}$$

The points B and C lie on l such that $AB = AC = 15$

Given that A does not lie on l and that the x coordinate of B is negative,

(a) determine the coordinates of B and the coordinates of C

(4)

(b) Hence determine a Cartesian equation of the plane containing the points A, B and C

(3)

The point D has coordinates $(-2, 1, \alpha)$, where α is a constant.

Given that the volume of the tetrahedron $ABCD$ is 147

(c) determine the possible values of α

(4)

Given that $\alpha > 0$

(d) determine the shortest distance between the line l and the line passing through the points A and D, giving your answer to 2 significant figures.

(4)

(a) X on l: $\begin{pmatrix} 12+7\lambda \\ 30+13\lambda \\ 39+24\lambda \end{pmatrix}$ $\quad AX = \begin{pmatrix} 12+7\lambda \\ 30+13\lambda \\ 39+24\lambda \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} 9+7\lambda \\ 24+13\lambda \\ 34+24\lambda \end{pmatrix}$

$|AX| = 15$: $(9+7\lambda)^2 + (24+13\lambda)^2 + (34+24\lambda)^2 = 15^2$

$81 + 126\lambda + 49\lambda^2 + 576 + 624\lambda + 169\lambda^2 + 1156 + 1632\lambda + 576\lambda^2 = 225$

$794\lambda^2 + 2382\lambda + 1588 = 0 \Rightarrow \lambda^2 + 3\lambda + 2 = 0$

$(\lambda + 1)(\lambda + 2) = 0 \Rightarrow \lambda = -1$ or $\lambda = -2$

$\lambda = -2$: $(12 - 14, 30 - 26, 39 - 48) = (-2, 4, -9)$

$\lambda = -1$: $(12 - 7, 30 - 13, 39 - 24) = (5, 17, 15)$

x coordinate of B is negative: $B(-2, 4, -9)$, $C(5, 17, 15)$

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Question 7 continued

(b) $\overrightarrow{AB} = \begin{pmatrix}-2\\4\\-9\end{pmatrix} - \begin{pmatrix}3\\6\\5\end{pmatrix} = \begin{pmatrix}-5\\-2\\-14\end{pmatrix}$ $\quad \overrightarrow{BC} = \begin{pmatrix}5\\17\\15\end{pmatrix} - \begin{pmatrix}-2\\4\\-9\end{pmatrix} = \begin{pmatrix}7\\13\\24\end{pmatrix}$

$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{BC} = \begin{pmatrix}(-2)(24) - (-14)(13)\\(-14)(7) - (-5)(24)\\(-5)(13) - (-2)(7)\end{pmatrix} = \begin{pmatrix}134\\22\\-51\end{pmatrix}$

$\mathbf{n} \cdot \overrightarrow{OA} = 134(3) + 22(6) - 51(5) = 402 + 132 - 255 = 279$

Plane: $134x + 22y - 51z = 279$

(c) $\overrightarrow{DA} = \begin{pmatrix}3\\6\\5\end{pmatrix} - \begin{pmatrix}-2\\1\\\alpha\end{pmatrix} = \begin{pmatrix}5\\5\\5-\alpha\end{pmatrix}$ $\quad \overrightarrow{AB} \times \overrightarrow{AC} = \overrightarrow{AB} \times \overrightarrow{BC} = \begin{pmatrix}134\\22\\-51\end{pmatrix}$

$(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{DA} = 134(5) + 22(5) - 51(5 - \alpha) = 525 + 51\alpha$

Volume $= \frac{1}{6}|(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{DA}| \Rightarrow \frac{1}{6}|525 + 51\alpha| = 147 \Rightarrow 525 + 51\alpha = \pm 882$

$\alpha = \frac{357}{51} = 7$ or $\alpha = \frac{-1407}{51} = -\frac{469}{17}$

(d) $\alpha = 7$: $D(-2, 1, 7)$, $\overrightarrow{AD} = (-2, 1, 7) - (3, 6, 5) = (-5, -5, 2)$

Vector from A to a point on l: $(12, 30, 39) - (3, 6, 5) = (9, 24, 34)$

$\overrightarrow{AD} \times \mathbf{d} = \begin{pmatrix}-5\\-5\\2\end{pmatrix} \times \begin{pmatrix}7\\13\\24\end{pmatrix} = \begin{pmatrix}(-5)(24) - 2(13)\\2(7) - (-5)(24)\\(-5)(13) - (-5)(7)\end{pmatrix} = \begin{pmatrix}-146\\134\\-30\end{pmatrix}$

Distance $= \frac{|(-146)(9) + 134(24) + (-30)(34)|}{\sqrt{((-146)^2 + 134^2 + (-30)^2)}} = \frac{|-1314 + 3216 - 1020|}{\sqrt{40172}} = \frac{882}{\sqrt{40172}}$

$= 4.40054\ldots = 4.4$ (2 s.f.)

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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