Please check the examination details below before entering your candidate information

Candidate surname: StemUp Model Solution | Other names:

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Pearson Edexcel Level 3 GCE

Thursday 25 May 2023

Afternoon
(Time: 1 hour 30 minutes)

Paper reference **9FM0/01**

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The cubic equation

$$x^3 - 7x^2 - 12x + 6 = 0$$

has roots α, β and γ.

Without solving the equation, determine a cubic equation whose roots are $(\alpha + 2)$, $(\beta + 2)$ and $(\gamma + 2)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where p, q and r are integers to be found.

(5)

$w = x + 2 \quad \Rightarrow \quad x = w - 2$

$(w - 2)^3 - 7(w - 2)^2 - 12(w - 2) + 6 = 0$

$(w^3 - 6w^2 + 12w - 8) - 7(w^2 - 4w + 4) - 12w + 24 + 6 = 0$

$w^3 - 6w^2 + 12w - 8 - 7w^2 + 28w - 28 - 12w + 30 = 0$

$w^3 - 13w^2 + 28w - 6 = 0$

$p = -13, \quad q = 28, \quad r = -6$

Equation: $w^3 - 13w^2 + 28w - 6 = 0$

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Question 1 continued

(Total for Question 1 is 5 marks)

2. (a) Write $x^2 + 4x - 5$ in the form $(x + p)^2 + q$ where p and q are integers.

(1)

(b) Hence use a standard integral from the formula book to find

$$\int \frac{1}{\sqrt{x^2 + 4x - 5}} \, dx$$

(2)

(c) Determine the mean value of the function

$$f(x) = \frac{1}{\sqrt{x^2 + 4x - 5}} \qquad 3 \leqslant x \leqslant 13$$

giving your answer in the form $A \ln B$ where A and B are constants in simplest form.

(3)

(a) $x^2 + 4x - 5 = (x + 2)^2 - 9$

$p = 2, \quad q = -9$

(b) $\int \frac{1}{\sqrt{(x^2 + 4x - 5)}} \, dx = \int \frac{1}{\sqrt{((x + 2)^2 - 3^2)}} \, dx$

$= \operatorname{arcosh}\left(\frac{x + 2}{3}\right) + c$

(c) Mean value $= \frac{1}{13 - 3} \int_3^{13} \frac{1}{\sqrt{(x^2 + 4x - 5)}} \, dx$

$= \frac{1}{10} \left[\operatorname{arcosh}\left(\frac{x + 2}{3}\right) \right]_3^{13}$

$= \frac{1}{10} \left(\operatorname{arcosh} 5 - \operatorname{arcosh}\left(\frac{5}{3}\right) \right)$

$= \frac{1}{10} \left(\ln(5 + \sqrt{24}) - \ln\left(\frac{5}{3} + \frac{4}{3}\right) \right)$

$= \frac{1}{10} \left(\ln(5 + 2\sqrt{6}) - \ln 3 \right)$

Mean value $= \frac{1}{10} \ln\left(\frac{5 + 2\sqrt{6}}{3}\right)$

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Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$z_1 = -4 + 4\mathrm{i}$$

(a) Express z_1 in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r \in \mathbb{R}$, $r > 0$ and $0 \leqslant \theta < 2\pi$

(2)

$$z_2 = 3\left(\cos\frac{17\pi}{12} + \mathrm{i}\sin\frac{17\pi}{12}\right)$$

(b) Determine in the form $a + \mathrm{i}b$, where a and b are exact real numbers,

(i) $\dfrac{z_1}{z_2}$

(2)

(ii) $(z_2)^4$

(2)

(c) Show on a single Argand diagram

(i) the complex numbers z_1, z_2 and $\dfrac{z_1}{z_2}$

(ii) the region defined by $\{z \in \mathbb{C} : |z - z_1| < |z - z_2|\}$

(4)

(a) $|z_1| = \sqrt{((-4)^2 + 4^2)} = \sqrt{32} = 4\sqrt{2}$

$\arg z_1 = \pi - \arctan\left(\frac{4}{4}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$z_1 = 4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(b)(i) $\frac{z_1}{z_2} = \frac{4\sqrt{2}}{3}\left(\cos\left(\frac{3\pi}{4} - \frac{17\pi}{12}\right) + i\sin\left(\frac{3\pi}{4} - \frac{17\pi}{12}\right)\right)$

$\frac{3\pi}{4} - \frac{17\pi}{12} = \frac{9\pi - 17\pi}{12} = -\frac{8\pi}{12} = -\frac{2\pi}{3}$

$= \frac{4\sqrt{2}}{3}\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$

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Question 3 continued

$= \frac{4\sqrt{2}}{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$\frac{z_1}{z_2} = -\frac{2\sqrt{2}}{3} - \frac{2\sqrt{6}}{3}i$

(ii) $z_2^4 = 3^4\left(\cos\left(4 \times \frac{17\pi}{12}\right) + i\sin\left(4 \times \frac{17\pi}{12}\right)\right)$ (de Moivre)

$= 81\left(\cos\frac{17\pi}{3} + i\sin\frac{17\pi}{3}\right)$

$\frac{17\pi}{3} = 6\pi - \frac{\pi}{3}$ so

$= 81\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) = 81\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$z_2^4 = \frac{81}{2} - \frac{81\sqrt{3}}{2}i$

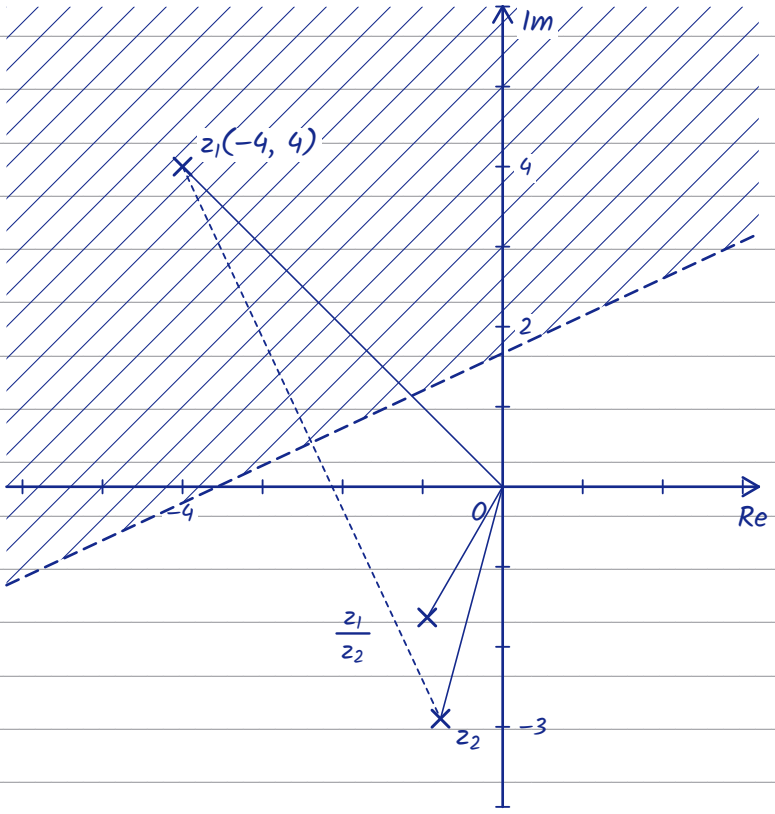
(c) $z_1 = -4 + 4i$

$z_2 = 3\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right) \approx -0.78 - 2.90i$ (third quadrant, below $y = x$, closer to 0 than z_1)

$\frac{z_1}{z_2} \approx -0.94 - 1.63i$ (third quadrant)

Region $|z - z_1| < |z - z_2|$: points closer to z_1 than to z_2, i.e. on the z_1 side of the perpendicular bisector of z_1z_2 (shaded)

Question 3 continued



Shaded region:

$|z - z_1| < |z - z_2|$

dashed line = perpendicular bisector of z_1z_2 (not included)

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Question 3 continued

(Total for Question 3 is 10 marks)

4. Prove by induction that for $n \in \mathbb{N}$

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & -2n \\ 0 & 1 \end{pmatrix}$$

(5)

$n = 1$: LHS $= \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ RHS $= \begin{pmatrix} 1 & -2 \times 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

LHS = RHS, so the result is true for $n = 1$

Assume true for $n = k$: $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix}$

Then $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}^k \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & -2 - 2k \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2(k+1) \\ 0 & 1 \end{pmatrix}$$

which is the result with $n = k + 1$, so it is true for $n = k + 1$.

If the result is true for $n = k$ then it is true for $n = k + 1$.
As it is true for $n = 1$, by mathematical induction it is true for all $n \in \mathbb{N}$.

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Question 4 continued

(Total for Question 4 is 5 marks)

5. The line l_1 has equation $\dfrac{x+5}{1} = \dfrac{y+4}{-3} = \dfrac{z-3}{5}$

The plane Π_1 has equation $2x + 3y - 2z = 6$

(a) Find the point of intersection of l_1 and Π_1

(2)

The line l_2 is the reflection of the line l_1 in the plane Π_1

(b) Show that a vector equation for the line l_2 is

$$\mathbf{r} = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

where μ is a scalar parameter.

(5)

The plane Π_2 contains the line l_1 and the line l_2

(c) Determine a vector equation for the line of intersection of Π_1 and Π_2

(2)

The plane Π_3 has equation $\mathbf{r}.\begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = b$ where a and b are constants.

Given that the planes Π_1, Π_2 and Π_3 form a sheaf,

(d) determine the value of a and the value of b.

(3)

(a) l_1: $x = \lambda - 5$, $y = -3\lambda - 4$, $z = 5\lambda + 3$

$2(\lambda - 5) + 3(-3\lambda - 4) - 2(5\lambda + 3) = 6$

$2\lambda - 10 - 9\lambda - 12 - 10\lambda - 6 = 6 \Rightarrow -17\lambda = 34 \Rightarrow \lambda = -2$

Point of intersection $(-7, 2, -7)$

(b) $(-5, -4, 3)$ lies on l_1. Line through it perpendicular to Π_1:

$$r = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 + 2t \\ -4 + 3t \\ 3 - 2t \end{pmatrix}$$

meets Π_1: $2(-5 + 2t) + 3(-4 + 3t) - 2(3 - 2t) = 6$

$17t - 28 = 6 \Rightarrow t = 2$

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Question 5 continued

Mirror point: $t = 2 \times 2 = 4$: $\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}$

l_2 passes through $(-7, 2, -7)$ and $(3, 8, -5)$

direction: $\begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} - \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} = \begin{pmatrix} 3-(-7) \\ 8-2 \\ -5-(-7) \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$

$\therefore$ l_2: $r = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + \mu\begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$ as required

(c) Foot of perpendicular from $(-5, -4, 3)$ to Π_1 $(t = 2)$:

$\begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ lies in Π_1 and in Π_2

$(-7, 2, -7)$ also lies in both planes, so the common line has direction

$\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$r = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(d) Sheaf $\Rightarrow$ Π_3 contains the line $r = (-7, 2, -7) + s(1, 0, 1)$

so its direction is perpendicular to the normal of Π_3:

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ a \end{pmatrix} = 0 \Rightarrow 1 + 0 + a = 0 \Rightarrow a = -1$

Question 5 continued

$(-7, 2, -7)$ lies on Π_3: $b = \begin{pmatrix} -7 \\ 2 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = -7 + 2 + 7 = 2$

$a = -1, \quad b = 2$

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Question 5 continued

(Total for Question 5 is 12 marks)

6. Water is flowing into and out of a large tank.

Initially the tank contains 10 litres of water.

The rate of flow of the water is modelled so that

- there are V litres of water in the tank at time t minutes after the water begins to flow
- water enters the tank at a rate of $\left(3 - \frac{4}{1 + e^{0.8t}}\right)$ litres per minute
- water leaves the tank at a rate proportional to the volume of water remaining in the tank

Given that when $t = 0$ the volume of water in the tank is decreasing at a rate of 3 litres per minute, use the model to

(a) show that the volume of water in the tank at time t satisfies

$$\frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - 0.4V$$

(3)

(b) Determine $\frac{d}{dt}(\arctan e^{0.4t})$

(2)

Hence, by solving the differential equation from part (a),

(c) determine an equation for the volume of water in the tank at time t.

Give your answer in simplest form as $V = f(t)$

(6)

After 10 minutes, the volume of water in the tank was 8 litres.

(d) Evaluate the model in light of this information.

(1)

(a) $\frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - kV$ (k a constant)

$t = 0$: $V = 10$, $\frac{dV}{dt} = -3 \Rightarrow -3 = 3 - \frac{4}{1+1} - 10k$

$-3 = 1 - 10k \Rightarrow 10k = 4 \Rightarrow k = \frac{2}{5} = 0.4$

$\therefore \frac{dV}{dt} = 3 - \frac{4}{1 + e^{0.8t}} - 0.4V$

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(b) $\frac{d}{dt}(\arctan e^{0.4t}) = \frac{1}{1+(e^{0.4t})^2} \times 0.4e^{0.4t}$

$= \frac{0.4e^{0.4t}}{1+e^{0.8t}}$

(c) $\frac{dV}{dt} + 0.4V = 3 - \frac{4}{1+e^{0.8t}}$

Integrating factor $= e^{\int 0.4\,dt} = e^{0.4t}$

$\frac{d}{dt}(Ve^{0.4t}) = 3e^{0.4t} - \frac{4e^{0.4t}}{1+e^{0.8t}}$

$Ve^{0.4t} = \int \left(3e^{0.4t} - 10 \times \frac{0.4e^{0.4t}}{1+e^{0.8t}}\right) dt$

$Ve^{0.4t} = 7.5e^{0.4t} - 10\arctan(e^{0.4t}) + c$

$t = 0, V = 10$: $10 = 7.5 - 10\arctan 1 + c = 7.5 - \frac{10\pi}{4} + c$

$c = 2.5 + 2.5\pi$

$V = 7.5 - 10e^{-0.4t}\arctan(e^{0.4t}) + 2.5(\pi+1)e^{-0.4t}$

(d) $t = 10$: $V = 7.5 - 10e^{-4}\arctan(e^4) + 2.5(\pi+1)e^{-4}$

$= 7.4052\ldots \approx 7.41$ litres

The model predicts about 7.4 litres but the actual volume was 8 litres, about 7% below the actual level, so the model is not very accurate.

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Explain why, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} (-1)^r \, \mathrm{f}(r) = \sum_{r=1}^{n} \left(\mathrm{f}(2r) - \mathrm{f}(2r-1) \right)$$

for any function $\mathrm{f}(r)$.

(2)

(b) Use the standard summation formulae to show that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{2n} r\left((-1)^r + 2r\right)^2 = n(2n+1)(8n^2 + 4n + 5)$$

(6)

(c) Hence evaluate

$$\sum_{r=14}^{50} r\left((-1)^r + 2r\right)^2$$

(4)

(a) $\sum_{r=1}^{2n} (-1)^r f(r) = -f(1) + f(2) - f(3) + f(4) - \ldots - f(2n-1) + f(2n)$

the even terms are positive and the odd terms are negative, so

$= [\, f(2) + f(4) + \ldots + f(2n) \,] - [\, f(1) + f(3) + \ldots + f(2n-1) \,]$

$= \sum_{r=1}^{n} (f(2r) - f(2r-1))$

(b) $r((-1)^r + 2r)^2 = r((-1)^{2r} + 4r(-1)^r + 4r^2) = r + 4(-1)^r r^2 + 4r^3$

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Question 7 continued

$$\sum_{r=1}^{2n} r((-1)^r + 2r)^2 = \sum_{r=1}^{2n} r + 4\sum_{r=1}^{2n} r^3 + 4\sum_{r=1}^{2n} (-1)^r r^2$$

$$= \frac{1}{2}(2n)(2n+1) + 4 \times \frac{1}{4}(2n)^2(2n+1)^2 + 4\sum_{r=1}^{2n} (-1)^r r^2$$

Using (a) with $f(r) = r^2$:

$$\sum_{r=1}^{2n} (-1)^r r^2 = \sum_{r=1}^{n} \left((2r)^2 - (2r-1)^2 \right) = \sum_{r=1}^{n} (4r-1)$$

$$= 4 \times \frac{1}{2} n(n+1) - n = 2n^2 + n = n(2n+1)$$

$$\therefore \text{ Sum} = n(2n+1) + 4n^2(2n+1)^2 + 4n(2n+1)$$

$$= n(2n+1)[\,1 + 4n(2n+1) + 4\,]$$

$$= n(2n+1)(8n^2 + 4n + 5) \quad \text{as required}$$

(c) $\displaystyle\sum_{r=14}^{50} = \sum_{r=1}^{50} - \sum_{r=1}^{12} - (\text{term with } r = 13)$

$r = 13$ term: $13((-1)^{13} + 26)^2 = 13 \times 25^2 = 8125$

$2n = 50$, $n = 25$: $\displaystyle\sum_{r=1}^{50} = 25 \times 51 \times (8 \times 625 + 100 + 5)$

$$= 25 \times 51 \times 5105 = 6508875$$

$2n = 12$, $n = 6$: $\displaystyle\sum_{r=1}^{12} = 6 \times 13 \times (288 + 24 + 5) = 6 \times 13 \times 317 = 24726$

$$\sum_{r=14}^{50} r((-1)^r + 2r)^2 = 6508875 - 24726 - 8125 = 6476024$$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

8. A colony of small mammals is being studied.
In the study, the mammals are divided into 3 categories

N (newborns)	0 to less than 1 month old
J (juveniles)	1 to 3 months old
B (breeders)	over 3 months old

(a) State one limitation of the model regarding the division into these categories.

(1)

A model for the population of the colony is given by the matrix equation

$$\begin{pmatrix} N_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} N_n \\ J_n \\ B_n \end{pmatrix}$$

where a and b are constants, and N_n, J_n and B_n are the respective numbers of the mammals in each category n months after the start of the study.

At the start of the study the colony has breeders only, with no newborns or juveniles.

According to the model, after 2 months the number of newborns is 48 and the number of juveniles is 40

(b) (i) Determine the number of mammals in the colony at the start of the study.

(ii) Show that $a = 0.8$

(4)

(c) Determine, in terms of b,

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1}$$

(3)

Given that the model predicts approximately 1015 mammals **in total** at the start of a particular month, and approximately 596 **newborns**, 464 **juveniles** and 437 **breeders** at the start of the next month,

(d) determine the value of b, giving your answer to 2 decimal places.

(3)

It is decided to monitor the number of **newborn** males and females as a part of the study. Assuming that 42% of newborns are male,

(e) refine the matrix equation for the model to reflect this information, giving a reason for your answer.
(*There is no need to estimate any unknown values for the refined model, but any known values should be made clear.*)

(2)

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Question 8 continued

(a) Not all mammals over 3 months old will be able to breed (some may be infertile or too old to breed), so "breeders" is not accurate.

(b)(i) At the start $N_0 = 0$, $J_0 = 0$, $B_0 = k$

After 1 month: $\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix} = \begin{pmatrix} 2k \\ 0 \\ 0.96k \end{pmatrix}$

After 2 months: $\begin{pmatrix} 0 & 0 & 2 \\ a & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} \begin{pmatrix} 2k \\ 0 \\ 0.96k \end{pmatrix} = \begin{pmatrix} 1.92k \\ 2ak \\ 0.9216k \end{pmatrix}$

$N_2 = 2 \times 0.96k = 1.92k = 48 \Rightarrow k = 25$

25 mammals at the start of the study

(ii) $J_2 = 2ak = 2a \times 25 = 50a = 40 \Rightarrow a = 0.8$

(c) $\det \begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix} = 0 - 0 + 2(0.8 \times 0.48 - b \times 0) = 0.768$

cofactors: $\begin{pmatrix} 0.96b & -0.768 & 0.384 \\ 0.96 & 0 & 0 \\ -2b & 1.6 & 0 \end{pmatrix}$ adj $= \begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix}$

Question 8 continued

$$\begin{pmatrix} 0 & 0 & 2 \\ 0.8 & b & 0 \\ 0 & 0.48 & 0.96 \end{pmatrix}^{-1} = \frac{1}{0.768}\begin{pmatrix} 0.96b & 0.96 & -2b \\ -0.768 & 0 & 1.6 \\ 0.384 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1.25b & 1.25 & -125b/48 \\ -1 & 0 & 25/12 \\ 0.5 & 0 & 0 \end{pmatrix}$$

(d) Let x, y, z be the numbers of N, J, B at the start of the month:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.25b & 1.25 & -125b/48 \\ -1 & 0 & 25/12 \\ 0.5 & 0 & 0 \end{pmatrix}\begin{pmatrix} 596 \\ 464 \\ 437 \end{pmatrix}$$

$x = 745b + 580 - 1138.02...b = 580 - 393.02...b$

$y = -596 + 910.41... = 314.41...,\quad z = 0.5 \times 596 = 298$

$x + y + z = 1015$: $580 - 393.0208b + 314.4167 + 298 = 1015$

$393.0208b = 177.4167 \Rightarrow b = 0.45141...$

$b = 0.45$ (2 d.p.)

(e) Let NM_n and NF_n be the numbers of newborn males and females.

Newborns = $2B_n$: males $0.42 \times 2 = 0.84$, females $0.58 \times 2 = 1.16$

$$\begin{pmatrix} NM_{n+1} \\ NF_{n+1} \\ J_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0.84 \\ 0 & 0 & 0 & 1.16 \\ p & q & 0.45 & 0 \\ 0 & 0 & 0.48 & 0.96 \end{pmatrix}\begin{pmatrix} NM_n \\ NF_n \\ J_n \\ B_n \end{pmatrix}$$

where p and q are the unknown proportions of male and female newborns surviving to become juveniles ($b = 0.45$ from (d)).

Reason: 42% of the $2B_n$ newborns are male and 58% are female, so the newborn row splits into $0.84B_n$ males and $1.16B_n$ females.

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Question 8 continued

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(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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