Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/4C**

Further Mathematics

Advanced

PAPER 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Three particles of masses $2m$, $3m$ and km are placed at the points with coordinates $(3a, 2a)$, $(a, -4a)$ and $(-3a, 4a)$ respectively.

The centre of mass of the three particles lies at the point with coordinates $(\bar{x}, \bar{y})$.

(a) (i) Find $\bar{x}$ in terms of a and k

(ii) Find $\bar{y}$ in terms of a and k

(4)

Given that the distance of the centre of mass of the three particles from the point (0, 0) is $\frac{1}{3}a$

(b) find the possible values of k

(2)

(a)(i) Moments about the y-axis:

$(2m + 3m + km)\bar{x} = 2m(3a) + 3m(a) + km(-3a)$

$(5 + k)m\,\bar{x} = 6ma + 3ma - 3kma = (9 - 3k)ma$

$\bar{x} = \dfrac{(9 - 3k)a}{5 + k}$

(a)(ii) Moments about the x-axis:

$(5 + k)m\,\bar{y} = 2m(2a) + 3m(-4a) + km(4a)$

$(5 + k)m\,\bar{y} = 4ma - 12ma + 4kma = (4k - 8)ma$

$\bar{y} = \dfrac{(4k - 8)a}{5 + k}$

(b) $\bar{x}^2 + \bar{y}^2 = (a/3)^2$

$\dfrac{(9 - 3k)^2a^2 + (4k - 8)^2a^2}{(5 + k)^2} = \dfrac{a^2}{9}$

$9[(9 - 3k)^2 + (4k - 8)^2] = (5 + k)^2$

$9[81 - 54k + 9k^2 + 16k^2 - 64k + 64] = 25 + 10k + k^2$

$225k^2 - 1062k + 1305 = k^2 + 10k + 25$

$224k^2 - 1072k + 1280 = 0$

$14k^2 - 67k + 80 = 0$

$(2k - 5)(7k - 16) = 0$

$k = 5/2$ or $k = 16/7$

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Question 1 continued

(Total for Question 1 is 6 marks)

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2. A cyclist and her cycle have a combined mass of 60 kg. The cyclist is moving along a straight horizontal road and is working at a constant rate of 200 W.

When she has travelled a distance x metres, her speed is $v\,\text{m s}^{-1}$ and the magnitude of the resistance to motion is $3v^2$ N.

(a) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{200 - 3v^3}{60v^2}$

(4)

The distance travelled by the cyclist as her speed increases from $2\,\text{m s}^{-1}$ to $4\,\text{m s}^{-1}$ is D metres.

(b) Find the exact value of D

(3)

(a) Driving force F: $P = Fv \Rightarrow F = 200/v$

N2L along the road: $F - 3v^2 = 60a$

$\dfrac{200}{v} - 3v^2 = 60v\dfrac{dv}{dx}$ $(a = v\,dv/dx)$

Multiply by v: $200 - 3v^3 = 60v^2\,dv/dx$

$\therefore \dfrac{dv}{dx} = \dfrac{200 - 3v^3}{60v^2}$ as required

(b) Separate the variables:

$\int \dfrac{60v^2}{200 - 3v^3}\,dv = \int 1\,dx$

$x = -\dfrac{60}{9}\ln(200 - 3v^3) + C = -\dfrac{20}{3}\ln(200 - 3v^3) + C$

$D = \dfrac{20}{3}\left[\ln(200 - 3v^3)\right]$ from $v = 4$ to $v = 2$

$D = \dfrac{20}{3}\left[\ln(200 - 3 \times 2^3) - \ln(200 - 3 \times 4^3)\right]$

$D = \dfrac{20}{3}\left[\ln 176 - \ln 8\right] = \dfrac{20}{3}\ln\dfrac{176}{8}$

$D = \dfrac{20}{3}\ln 22$

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

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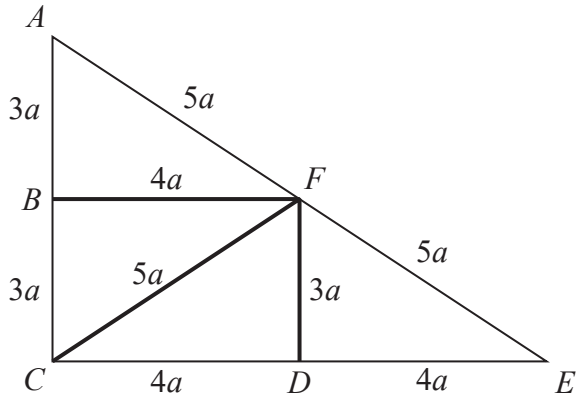


Figure 1

Nine uniform rods are joined together to form the rigid framework $ABCDEFA$, with $AB = BC = DF = 3a$, $BF = CD = DE = 4a$ and $AF = FE = CF = 5a$, as shown in Figure 1. All nine rods lie in the same plane.

The mass per unit length of each of the rods BF, CF and DF is twice the mass per unit length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from AC

(4)

The mass of the framework is M. A particle of mass kM is attached to the framework at E to form a loaded framework.

When the loaded framework is freely suspended from F, it hangs in equilibrium with CE horizontal.

(b) Find the exact value of k

(3)

(a) Take mass per unit length 1 for the six single rods, 2 for BF, CF, DF

Rod: CD DE EF FA AB BC BF DF CF

Mass: 4 4 5 5 3 3 8 6 10

From AC: 2a 6a 6a 2a 0 0 2a 4a 2a

Total mass = 4 + 4 + 5 + 5 + 3 + 3 + 8 + 6 + 10 = 48

Moments about AC:

$4(2a) + 4(6a) + 5(6a) + 5(2a) + 8(2a) + 6(4a) + 10(2a) = 48\bar{x}$

$8a + 24a + 30a + 10a + 16a + 24a + 20a = 48\bar{x}$

$132a = 48\bar{x}$

$\bar{x} = \frac{132a}{48} = \frac{11a}{4}$ from AC

(b) F is 4a from AC and E is 8a from AC.

Hanging from F with CE horizontal, the centre of mass of the loaded framework is vertically below F.

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Question 3 continued

Moments about F:

$Mg(4a - 11a/4) = kMg(8a - 4a)$

$M(5a/4) = 4kMa$

$k = \frac{5}{16}$

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Question 3 continued

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Question 3 continued

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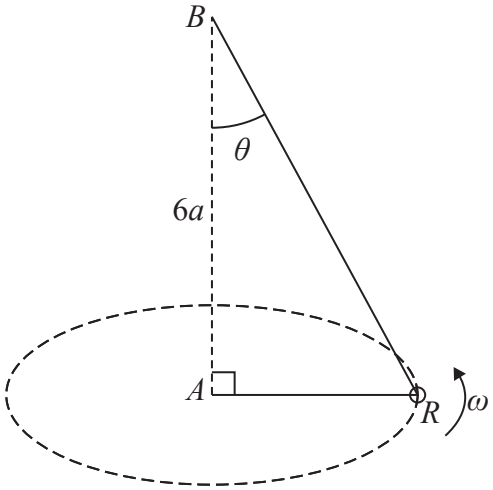


Figure 2

A small smooth ring R of mass m is threaded onto a light inextensible string. One end of the string is attached to a fixed point A and the other end of the string is attached to the fixed point B such that B is vertically above A and $AB = 6a$

The ring moves with constant angular speed ω in a horizontal circle with centre A. The string is taut and BR makes a constant angle θ with the downward vertical, as shown in Figure 2.

The ring is modelled as a particle.

Given that $\tan\theta = \frac{8}{15}$

(a) find, in terms of m and g, the magnitude of the tension in the string,

(3)

(b) find ω in terms of a and g

(5)

$\tan\theta = 8/15 \Rightarrow \sin\theta = 8/17, \cos\theta = 15/17$

Smooth ring, so the tension T is the same in AR and BR.

(a) Resolve vertically for R (AR is horizontal):

$T\cos\theta = mg$

$T = \frac{mg}{15/17} = \frac{17mg}{15}$ $(= 1.13mg)$

(b) Radius $r = AR = 6a\tan\theta = 6a \times 8/15 = 3.2a$

Horizontally (towards A): $T + T\sin\theta = mr\omega^2$

$\frac{17mg}{15}\left(1 + \frac{8}{17}\right) = m(3.2a)\omega^2$

$\frac{17mg}{15} \times \frac{25}{17} = \frac{5mg}{3} = 3.2ma\omega^2$

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Question 4 continued

$\omega^2 = \frac{5g}{3 \times 3.2a} = \frac{25g}{48a}$

$\omega = \frac{5}{4}\sqrt{\frac{g}{3a}}$ $(= 0.722\sqrt{(g/a)})$

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Question 4 continued

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Question 4 continued

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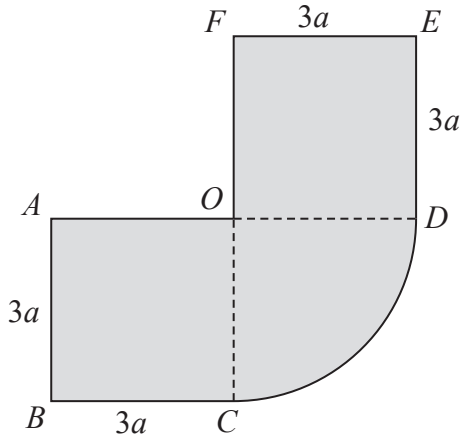


Figure 3

The uniform plane lamina shown in Figure 3 is formed from two squares, *ABCO* and *ODEF*, and a sector *ODC* of a circle with centre *O*. Both squares have sides of length 3*a* and *AO* is perpendicular to *OF*. The radius of the sector is 3*a*

[*In part* (a) *you may use, without proof, any of the centre of mass formulae given in the formulae booklet.*]

(a) Show that the distance of the centre of mass of the sector *ODC* from *OC* is $\frac{4a}{\pi}$

(3)

(b) Find the distance of the centre of mass of the lamina from *FC*

(4)

The lamina is freely suspended from *F* and hangs in equilibrium with *FC* at an angle $\theta°$ to the downward vertical.

(c) Find the value of θ

(4)

(a) Sector of radius 3a, angle 2α = π/2, so α = π/4

OG = $\frac{2r \sin \alpha}{3\alpha} = \frac{2 \times 3a \times \sin(\pi/4)}{3 \times \pi/4} = \frac{4\sqrt{2}\, a}{\pi}$ (G on the line of symmetry)

The line of symmetry OG makes 45° with OC, so

distance from OC = OG sin 45° = $\frac{4\sqrt{2}\, a}{\pi} \times \frac{1}{\sqrt{2}}$

$= \frac{4a}{\pi}$ as required

(b)

Shape:	ABCO	ODEF	ODC
Mass ratio:	$9a^2$	$9a^2$	$9\pi a^2/4$
From FC:	$-3a/2$	$3a/2$	$4a/\pi$

(ABCO is on the opposite side of FC to ODEF and ODC)

Moments about FC:

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$-9 \times \frac{3a}{2} + 9 \times \frac{3a}{2} + \frac{9\pi}{4} \times \frac{4a}{\pi} = (18 + \frac{9\pi}{4})\bar{x}$

$9a = \frac{9(8+\pi)}{4}\bar{x} \Rightarrow \bar{x} = \frac{4a}{8+\pi} \quad (= 0.359a)$

(c) By symmetry, the centre of mass G is $4a/(8 + \pi)$ below OD, so the distance of G below FE (vertically, parallel to FC) is

$\bar{y} = 3a + \frac{4a}{8+\pi} = \frac{(28+3\pi)a}{8+\pi}$

When suspended from F, FG is vertical, so

$\tan\theta° = \frac{\bar{x}}{\bar{y}} = \frac{4a}{(28+3\pi)a} = \frac{4}{28+3\pi} = 0.10688...$

$\theta = 6.1007...$

$\theta = 6.1$

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Question 5 continued

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Question 5 continued

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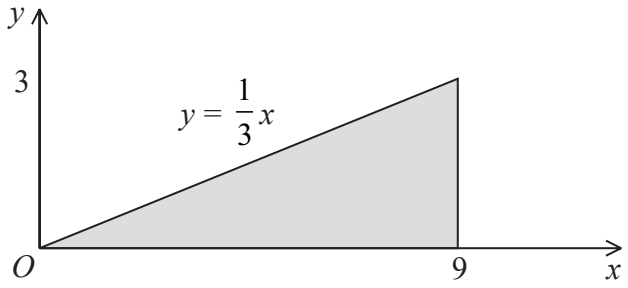


Figure 4

The shaded region shown in Figure 4 is bounded by the x-axis, the line with equation $x = 9$ and the line with equation $y = \frac{1}{3}x$. This shaded region is rotated through 360° about the x-axis to form a solid of revolution. This solid of revolution is used to model a solid right circular cone of height 9 cm and base radius 3 cm.

The cone is non-uniform and the mass per unit volume of the cone at the point (x, y, z) is $\lambda x \,\text{kg cm}^{-3}$, where $0 \leqslant x \leqslant 9$ and λ is constant.

(a) Find the distance of the centre of mass of the cone from its vertex.

(6)

A toy is made by joining the circular plane face of the cone to the circular plane face of a uniform solid hemisphere of radius 3 cm, so that the centres of the two plane surfaces coincide.

The weight of the cone is W newtons and the weight of the hemisphere is kW newtons.

When the toy is placed on a smooth horizontal plane with any point of the curved surface of the hemisphere in contact with the plane, the toy will remain at rest.

(b) Find the value of k

(4)

(a) Mass of cone:

$M = \int \pi y^2 \lambda x \, dx = \pi\lambda \int \frac{x^2}{9} x \, dx$ (from 0 to 9)

$M = \frac{\pi\lambda}{9}\left[\frac{x^4}{4}\right]$ from 0 to 9 $= \frac{\pi\lambda}{9} \times \frac{6561}{4} = \frac{729\pi\lambda}{4}$

Moment about the y-axis (through the vertex O):

$\int \pi y^2 \lambda x \cdot x \, dx = \frac{\pi\lambda}{9} \int x^4 \, dx$ (from 0 to 9)

$= \frac{\pi\lambda}{9}\left[\frac{x^5}{5}\right]$ from 0 to 9 $= \frac{\pi\lambda}{9} \times \frac{59049}{5} = \frac{6561\pi\lambda}{5}$

$d = \frac{6561\pi\lambda/5}{729\pi\lambda/4} = \frac{9 \times 4}{5} = \frac{36}{5}$

Distance of centre of mass from vertex = 7.2 cm

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Question 6 continued

(b) The toy remains at rest in any position, so its centre of mass is at the centre of the common plane face, O'.

Cone: c.o.m. $9 - 7.2 = 1.8$ cm from O' (on the cone side)

Hemisphere: c.o.m. $3r/8 = 3 \times 3/8 = 9/8$ cm from O'

Moments about a diameter of the plane face:

$$W\left(9 - \frac{36}{5}\right) = kW \times \frac{3}{8} \times 3$$

$$1.8 = \frac{9k}{8} \quad \Rightarrow \quad k = \frac{1.8 \times 8}{9}$$

$$k = \frac{8}{5} \quad (= 1.6)$$

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Question 6 continued

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Question 6 continued

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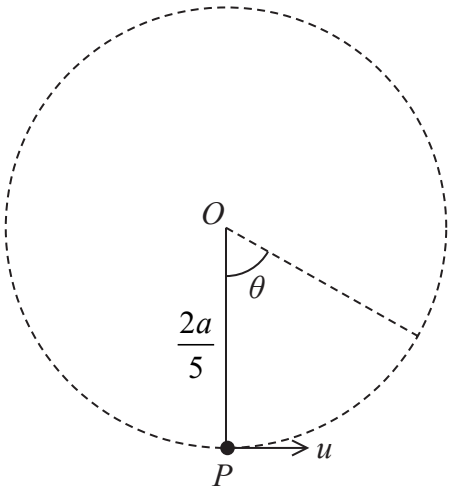


Figure 5

A package P of mass m is attached to one end of a string of length $\frac{2a}{5}$. The other end of the string is attached to a fixed point O. The package hangs at rest vertically below O with the string taut and is then projected horizontally with speed u, as shown in Figure 5.

When OP has turned through an angle θ and the string is still taut, the tension in the string is T

The package is modelled as a particle and the string as being light and inextensible.

(a) Show that $T = 3mg\cos\theta - 2mg + \frac{5mu^2}{2a}$

(6)

Given that P moves in a complete vertical circle with centre O

(b) find, in terms of a and g, the minimum possible value of u

(2)

Given that $u = 2\sqrt{ag}$

(c) find, in terms of g, the magnitude of the acceleration of P at the instant when OP is horizontal.

(3)

(d) Apart from including air resistance, suggest one way in which the model could be refined to make it more realistic.

(1)

(a) Let the speed be v when OP has turned through θ.

P rises (2a/5)(1 − cos θ). Conservation of energy:

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg \times \frac{2a}{5}(1 - \cos\theta)$

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Question 7 continued

$\Rightarrow v^2 = u^2 - \frac{4ag}{5}(1 - \cos\theta)$

N2L towards O (radius 2a/5):

$T - mg\cos\theta = \frac{mv^2}{2a/5} = \frac{5mv^2}{2a}$

$T = mg\cos\theta + \frac{5m}{2a}\left(u^2 - \frac{4ag}{5}(1 - \cos\theta)\right)$

$T = mg\cos\theta + \frac{5mu^2}{2a} - 2mg(1 - \cos\theta)$

$T = mg\cos\theta + \frac{5mu^2}{2a} - 2mg + 2mg\cos\theta$

$\therefore T = 3mg\cos\theta - 2mg + \frac{5mu^2}{2a}$ as required

(b) Complete circle requires $T \geq 0$ at the top, $\theta = \pi$:

$-3mg - 2mg + \frac{5mu^2}{2a} \geq 0 \Rightarrow \frac{5mu^2}{2a} \geq 5mg$

$u^2 \geq 2ag$

Minimum $u = \sqrt{(2ag)}$

(c) $u = 2\sqrt{(ag)}$, so $u^2 = 4ag$. OP horizontal: $\theta = \pi/2$

$T = 0 - 2mg + \frac{5m \times 4ag}{2a} = -2mg + 10mg = 8mg$

Radial acceleration (towards O) = $T/m = 8g$ (weight is tangential)

Tangential acceleration = $mg/m = g$ (vertically downwards)

Magnitude = $\sqrt{((8g)^2 + g^2)} = g\sqrt{(64 + 1)}$

Magnitude of acceleration = $\sqrt{65}\,g$ $(= 8.06g)$

(d) Model the package as a rigid body with dimensions rather than as a particle (or allow the string to be extensible).

Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

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8. **Throughout this question, use $g = 10\,\text{m s}^{-2}$**

A light elastic string has natural length 1.25 m and modulus of elasticity 25 N.

A particle P of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a fixed point A. Particle P hangs freely in equilibrium with P vertically below A

The particle is then pulled vertically down to a point B and released from rest.

(a) Show that, while the string is taut, P moves with simple harmonic motion with period $\frac{\pi}{\sqrt{10}}$ seconds.

(6)

The maximum kinetic energy of P during the subsequent motion is 2.5 J.

(b) Show that $AB = 2\,\text{m}$

(3)

The particle returns to B for the first time T seconds after it was released from rest at B

(c) Find the value of T

(5)

(a) Equilibrium extension e: $T = \lambda e/l$

$0.5g = \frac{25e}{1.25} \Rightarrow e = \frac{0.5 \times 10 \times 1.25}{25} = 0.25$ m

Let x m be the displacement of P below the equilibrium position (string taut). N2L, downwards positive:

$0.5g - \frac{25(0.25 + x)}{1.25} = 0.5\ddot{x}$

$5 - 5 - 20x = 0.5\ddot{x}$

$\ddot{x} = -40x$, which is of the form $\ddot{x} = -\omega^2 x$ with $\omega = \sqrt{40}$,

so P moves with SHM while the string is taut.

Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{40}} = \frac{2\pi}{2\sqrt{10}} = \frac{\pi}{\sqrt{10}}$ s as required

(b) Max KE occurs at the centre of the SHM:

$\frac{1}{2} \times 0.5 \times v_{max}^2 = 2.5 \Rightarrow v_{max}^2 = 10$

$v_{max} = a\omega$: $\sqrt{10} = a\sqrt{40} \Rightarrow a = \sqrt{(10/40)} = 0.5$ m

$AB = l + e + a = 1.25 + 0.25 + 0.5$

$AB = 2$ m as required

(c) Measuring x downwards from the equilibrium position:

$x = 0.5\cos(\sqrt{40}\,t)$

String goes slack at natural length: $x = -0.25$

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Question 8 continued

$-0.25 = 0.5\cos(\sqrt{40}\,t) \Rightarrow \cos(\sqrt{40}\,t) = -1/2$

$\sqrt{40}\,t = \frac{2\pi}{3} \Rightarrow t_1 = \frac{2\pi}{3\sqrt{40}} = 0.33115\ldots\ s$

Speed when slack: $v^2 = \omega^2(a^2 - x^2) = 40(0.5^2 - 0.25^2)$

$v^2 = 7.5 = \frac{15}{2} \Rightarrow v = \sqrt{7.5} = 2.7386\ldots\ m\ s^{-1}$ upwards

String slack: P moves freely under gravity, up and back down to the same point with speed $\sqrt{7.5}$ downwards:

time slack $t_2 = \frac{2v}{g} = \frac{2\sqrt{7.5}}{10} = 0.54772\ldots\ s$

By symmetry, the time from the slack point back to B = t_1

$T = 2t_1 + t_2 = 2(0.33115\ldots) + 0.54772\ldots$

$T = 1.2100\ldots$

$T = 1.21$

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Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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