Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/4A**

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. The group S_4 is the set of all possible permutations that can be performed on the four numbers 1, 2, 3 and 4, under the operation of composition.

For the group S_4

(a) write down the identity element,

(1)

(b) write down the inverse of the element a, where

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

(1)

(c) demonstrate that the operation of composition is associative using the following elements

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \quad \text{and } c = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

(2)

(d) Explain why it is possible for the group S_4 to have a subgroup of order 4
You do not need to find such a subgroup.

(2)

(a) $e = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

(b) $a^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

(c) (composition: apply the right-hand permutation first)

$(a \circ b) \circ c = \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}\right] \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

$a \circ (b \circ c) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \left[\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}\right]$

$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

So $(a \circ b) \circ c = a \circ (b \circ c)$, i.e. composition is associative for a, b, c

(d) The order of S_4 is $4! = 24$

By Lagrange's theorem the order of a subgroup must divide 24;

4 is a factor of 24, so it is possible for S_4 to have a subgroup of order 4

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Question 1 continued

(Total for Question 1 is 6 marks)

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2. Matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & a \\ -3 & b & 1 \\ 0 & 1 & a \end{pmatrix}$$

where a and b are integers, such that $a < b$

Given that the characteristic equation for **M** is

$$\lambda^3 - 7\lambda^2 + 13\lambda + c = 0$$

where c is a constant,

(a) determine the values of a, b and c.

(5)

(b) Hence, using the Cayley–Hamilton theorem, determine the matrix $\mathbf{M}^{-1}$

(3)

(a) $\det(M - \lambda I) = \begin{vmatrix} 1-\lambda & 0 & a \\ -3 & b-\lambda & 1 \\ 0 & 1 & a-\lambda \end{vmatrix} = 0$

$(1 - \lambda)[(b - \lambda)(a - \lambda) - 1] + a(-3 - 0) = 0$

$(1 - \lambda)[\lambda^2 - (a + b)\lambda + ab - 1] - 3a = 0$

$\lambda^2 - (a + b)\lambda + ab - 1 - \lambda^3 + (a + b)\lambda^2 - (ab - 1)\lambda - 3a = 0$

$\Rightarrow \lambda^3 - (a + b + 1)\lambda^2 + (a + b + ab - 1)\lambda + (3a + 1 - ab) = 0$

Compare with $\lambda^3 - 7\lambda^2 + 13\lambda + c = 0$:

λ^2: $a + b + 1 = 7 \Rightarrow a + b = 6$

λ: $a + b + ab - 1 = 13 \Rightarrow ab = 8$

$a(6 - a) = 8 \Rightarrow a^2 - 6a + 8 = 0 \Rightarrow (a - 2)(a - 4) = 0$

$a < b \Rightarrow a = 2,\ b = 4$

constant: $c = 3a + 1 - ab = 6 + 1 - 8 = -1$

$a = 2,\quad b = 4,\quad c = -1$

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Question 2 continued

(b) Cayley–Hamilton: $M^3 - 7M^2 + 13M - I = 0$

$\Rightarrow I = M^3 - 7M^2 + 13M$

Multiply by M^{-1}: $M^{-1} = M^2 - 7M + 13I$ ($I = 3 \times 3$ identity)

$$M^2 = \begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 6 \\ -15 & 17 & 0 \\ -3 & 6 & 5 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 1 & 2 & 6 \\ -15 & 17 & 0 \\ -3 & 6 & 5 \end{pmatrix} - 7\begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 7 & 2 & -8 \\ 6 & 2 & -7 \\ -3 & -1 & 4 \end{pmatrix}$$

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Question 2 continued

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Question 2 continued

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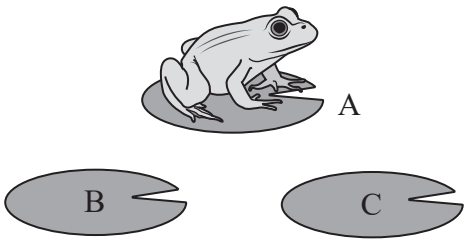


Figure 1

There are three lily pads on a pond. A frog hops repeatedly from one lily pad to another.

The frog starts on lily pad A, as shown in Figure 1.

In a model, the frog hops from its position on one lily pad to either of the other two lily pads with equal probability.

Let p_n be the probability that the frog is on lily pad A after n hops.

(a) Explain, with reference to the model, why $p_1 = 0$

(1)

The probability p_n satisfies the recurrence relation

$$p_{n+1} = \frac{1}{2}(1 - p_n) \qquad n \geqslant 1 \quad \text{where } p_1 = 0$$

(b) Prove by induction that, for $n \geqslant 1$

$$p_n = \frac{2}{3}\left(-\frac{1}{2}\right)^n + \frac{1}{3}$$

(6)

(c) Use the result in part (b) to explain why, in the long term, the probability that the frog is on lily pad A is $\frac{1}{3}$

(1)

(a) The frog starts on A and must hop to one of the other pads (B or C), so after 1 hop it cannot be on A. Hence $p_1 = 0$

(b) $n = 1$: $\frac{2}{3}\left(-\frac{1}{2}\right)^1 + \frac{1}{3} = -\frac{1}{3} + \frac{1}{3} = 0 = p_1$, so the result is true for $n = 1$

Assume true for $n = k$: $p_k = \frac{2}{3}\left(-\frac{1}{2}\right)^k + \frac{1}{3}$

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Question 3 continued

Then $p_{k+1} = \frac{1}{2}(1 - p_k) = \frac{1}{2}\left(1 - \left[\frac{2}{3}\left(-\frac{1}{2}\right)^k + \frac{1}{3}\right]\right)$

$= \frac{1}{2} - \frac{1}{6} - \frac{1}{3}\left(-\frac{1}{2}\right)^k = \frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$

$= \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} + \frac{1}{3}$

which is the result with $n = k + 1$.

If the result is true for $n = k$ then it is true for $n = k + 1$, and as it is true for $n = 1$, by mathematical induction it is true for all $n \geq 1$.

(c) As $n \to \infty$, $\left(-\frac{1}{2}\right)^n \to 0$ since $\left|-\frac{1}{2}\right| < 1$

$\therefore p_n \to \frac{2}{3} \times 0 + \frac{1}{3} = \frac{1}{3}$, so in the long term P(frog on A) $= \frac{1}{3}$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

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4. (a) Use the Euclidean algorithm to show that 124 and 17 are relatively prime (coprime).

(2)

(b) Hence solve the equation

$$124x + 17y = 10$$

(3)

(c) Solve the congruence equation

$$124x \equiv 6 \bmod 17$$

(2)

(a) $124 = 7 \times 17 + 5$

$17 = 3 \times 5 + 2$

$5 = 2 \times 2 + 1$

$2 = 2 \times 1 + 0$

The last non-zero remainder is 1, so $\gcd(124, 17) = 1$

$\therefore$ 124 and 17 are relatively prime (coprime)

(b) $1 = 5 - 2 \times 2$

$= 5 - 2(17 - 3 \times 5) = 7 \times 5 - 2 \times 17$

$= 7(124 - 7 \times 17) - 2 \times 17$

$\Rightarrow 1 = 7 \times 124 - 51 \times 17$

$\times 10$: $10 = 70 \times 124 - 510 \times 17$

$x = 70, \quad y = -510$

(general solution: $x = 70 + 17t, \quad y = -510 - 124t, \quad t \in \mathbb{Z}$)

(c) From (b): $7 \times 124 \equiv 1 \pmod{17}$, so 7 is the inverse of 124 mod 17

$7 \times 124x \equiv 7 \times 6 \pmod{17}$

$x \equiv 42 \equiv 8 \pmod{17}$

$x \equiv 8 \pmod{17}$

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Question 4 continued

(Total for Question 4 is 7 marks)

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5. The locus of points z satisfies

$$|z + ai| = 3|z - a|$$

where a is an integer.

The locus is a circle with its centre in the third quadrant and radius $\frac{3}{2}\sqrt{2}$

Determine

(a) the value of a,

(4)

(b) the coordinates of the centre of the circle.

(2)

(a) $z = x + iy$: $|x + i(y + a)| = 3|(x - a) + iy|$

$x^2 + (y + a)^2 = 9[(x - a)^2 + y^2]$

$x^2 + y^2 + 2ay + a^2 = 9x^2 - 18ax + 9a^2 + 9y^2$

$8x^2 - 18ax + 8y^2 - 2ay + 8a^2 = 0$

$x^2 - \frac{9}{4}ax + y^2 - \frac{1}{4}ay + a^2 = 0$

$(x - \frac{9}{8}a)^2 + (y - \frac{1}{8}a)^2 = (\frac{9}{8}a)^2 + (\frac{1}{8}a)^2 - a^2$

$r^2 = \frac{81a^2}{64} + \frac{a^2}{64} - \frac{64a^2}{64} = \frac{18a^2}{64} = \frac{9a^2}{32}$

$r = \frac{3}{2}\sqrt{2} \Rightarrow r^2 = \frac{9}{2} \Rightarrow \frac{9a^2}{32} = \frac{9}{2}$

$a^2 = 16 \Rightarrow a = \pm 4$

Centre $(9a/8, a/8)$ is in the third quadrant $\Rightarrow a < 0$

$a = -4$

(b) Centre $= (\frac{9(-4)}{8}, \frac{-4}{8}) = (-\frac{9}{2}, -\frac{1}{2})$

Question 5 continued

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(Total for Question 5 is 6 marks)

6. (a) Determine the general solution of the recurrence relation

$$u_n = 2u_{n-1} - u_{n-2} + 2^n \qquad n \geqslant 2$$

(4)

(b) Hence solve this recurrence relation given that $u_0 = 2u_1$ and $u_4 = 3u_2$

(2)

(a) Auxiliary equation: $\lambda^2 - 2\lambda + 1 = 0 \Rightarrow (\lambda - 1)^2 = 0 \Rightarrow \lambda = 1$ (repeated)

Complementary function: $u_n = A + Bn$

Particular solution: try $u_n = \mu(2)^n$

$\mu(2)^n = 2\mu(2)^{n-1} - \mu(2)^{n-2} + (2)^n$

$\div 2^{n-2}$: $4\mu = 4\mu - \mu + 4 \Rightarrow \mu = 4$

General solution: $u_n = A + Bn + 4(2)^n \quad (= A + Bn + 2^{n+2})$

(b) $u_0 = A + 4, \quad u_1 = A + B + 8$

$u_0 = 2u_1 \Rightarrow A + 4 = 2A + 2B + 16 \Rightarrow A + 2B = -12$

$u_2 = A + 2B + 16, \quad u_4 = A + 4B + 64$

$u_4 = 3u_2 \Rightarrow A + 4B + 64 = 3A + 6B + 48 \Rightarrow 2A + 2B = 16$

$\Rightarrow A + B = 8$; subtracting from $A + 2B = -12$: $B = -20, \quad A = 28$

$u_n = 28 - 20n + 4(2)^n$

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Question 6 continued

(Total for Question 6 is 6 marks)

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7. (i) The polynomial $F(x)$ is a quartic such that

$$F(x) = px^4 + qx^3 + 2x^2 + rx + s$$

where p, q, r and s are distinct constants.

Determine the number of possible quartics given that

(a) the constants p, q, r and s belong to the set $\{-4, -2, 1, 3, 5\}$

(1)

(b) the constants p, q, r and s belong to the set $\{-4, -2, 0, 1, 3, 5\}$

(1)

(ii) A 3-digit positive integer $N = abc$ has the following properties

- N is divisible by 11
- the sum of the digits of N is even
- $N \equiv 8 \bmod 9$

(a) Use the first two properties to show that

$$a - b + c = 0$$

(2)

(b) Hence determine all possible integers N, showing all your working and reasoning.

(4)

(i)(a) p, q, r, s distinct from 5 values: ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$

(i)(b) 6 values, but $p \neq 0$ (otherwise F is not a quartic): p has 5 choices, then q, r, s from the remaining 5: $5 \times 5 \times 4 \times 3 = 300$

(ii)(a) $N = 100a + 10b + c = 99a + 11b + (a - b + c)$

$11 \mid N \Rightarrow 11 \mid (a - b + c)$, so $a - b + c = 11k$, $k \in \mathbb{Z}$

$1 \leq a \leq 9$, $0 \leq b, c \leq 9 \Rightarrow -8 \leq a - b + c \leq 18 \Rightarrow a - b + c = 0$ or 11

$a - b + c = (a + b + c) - 2b$, where $a + b + c$ is even and $2b$ is even

$\Rightarrow a - b + c$ is even, so $a - b + c \neq 11$

$\therefore a - b + c = 0$

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Question 7 continued

(ii)(b) $N = 99a + 9b + (a + b + c) \Rightarrow N \equiv a + b + c \pmod 9$

$N \equiv 8 \pmod 9 \Rightarrow a + b + c \equiv 8 \pmod 9$

From (a): $a + c = b$, so $a + b + c = 2b$

$2b \equiv 8 \pmod 9$ and $0 \leq 2b \leq 18 \Rightarrow 2b = 8$ or 17; $2b$ is even $\Rightarrow 2b = 8$

$\Rightarrow b = 4$ and $a + c = 4$

$a \geq 1$: $(a, c) = (1, 3), (2, 2), (3, 1), (4, 0)$

$N = 143, 242, 341, 440$

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Question 7 continued

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Question 7 continued

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8. The locus of points $z = x + iy$ that satisfy

$$\arg\left(\frac{z-8-5i}{z-2-5i}\right)=\frac{\pi}{3}$$

is an arc of a circle C.

(a) On an Argand diagram sketch the locus of z.

(2)

(b) Explain why the centre of C has x coordinate 5

(1)

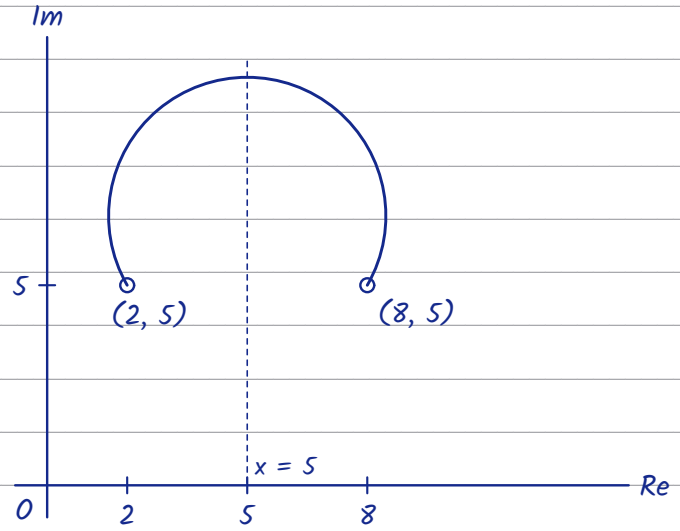
(c) Determine the radius of C.

(2)

(d) Determine the y coordinate of the centre of C.

(2)

(a)



(b) $2 + 5i$ and $8 + 5i$ both lie on C, so the centre is on the perpendicular bisector of the chord joining them: $x = (2 + 8)/2 = 5$

(c) Chord length $= 8 - 2 = 6$; angle at circumference $= \pi/3$, so the angle at the centre is $2\pi/3$ and each half-chord subtends $\pi/3$ at the centre

$$\sin\left(\frac{\pi}{3}\right)=\frac{3}{r} \Rightarrow r=\frac{3}{\sqrt{3}/2}=\frac{6}{\sqrt{3}}=2\sqrt{3}$$

(d) Height of centre above chord: $h=\frac{3}{\tan(\pi/3)}=\frac{3}{\sqrt{3}}=\sqrt{3}$

Centre is above the chord (major arc above): $y = 5 + \sqrt{3}$

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Question 8 continued

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9.

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n 2x \, dx$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{n-1}{n} I_{n-2}$$

(4)

(b) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} 64 \sin^5 x \cos^5 x \, dx$$

(3)

(a) $I_n = \int_0^{\pi/2} \sin^{n-1} 2x \sin 2x \, dx$

$u = \sin^{n-1} 2x \Rightarrow du/dx = 2(n-1)\sin^{n-2} 2x \cos 2x$

$dv/dx = \sin 2x \Rightarrow v = -\frac{1}{2}\cos 2x$

$I_n = \left[-\frac{1}{2} \sin^{n-1} 2x \cos 2x \right]_0^{\pi/2} + \int_0^{\pi/2} (n-1)\sin^{n-2} 2x \cos^2 2x \, dx$

The bracket is $0 - 0 = 0$, since $\sin \pi = \sin 0 = 0$ $(n \geq 2)$

$I_n = (n-1)\int_0^{\pi/2} \sin^{n-2} 2x (1 - \sin^2 2x) \, dx$

$= (n-1)\int_0^{\pi/2} \sin^{n-2} 2x \, dx - (n-1)\int_0^{\pi/2} \sin^n 2x \, dx$

$I_n = (n-1)I_{n-2} - (n-1)I_n$

$nI_n = (n-1)I_{n-2} \Rightarrow I_n = \frac{n-1}{n} I_{n-2}$ as required

(b) $\sin 2x = 2\sin x \cos x \Rightarrow 64\sin^5 x \cos^5 x = 64\left(\frac{1}{2}\sin 2x\right)^5 = 2\sin^5 2x$

$\therefore \int_0^{\pi/2} 64\sin^5 x \cos^5 x \, dx = 2I_5$

$I_5 = \frac{4}{5} I_3, \quad I_3 = \frac{2}{3} I_1$

$I_1 = \int_0^{\pi/2} \sin 2x \, dx = \left[-\frac{1}{2}\cos 2x \right]_0^{\pi/2} = \frac{1}{2} + \frac{1}{2} = 1$

$2I_5 = 2 \times \frac{4}{5} \times \frac{2}{3} \times 1 = \frac{16}{15}$

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Question 9 continued

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Question 9 continued

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Question 9 continued

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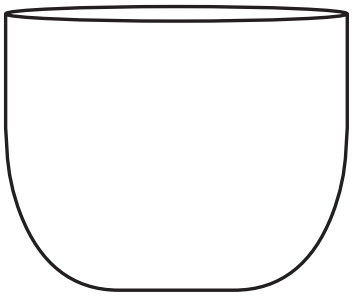


Figure 2

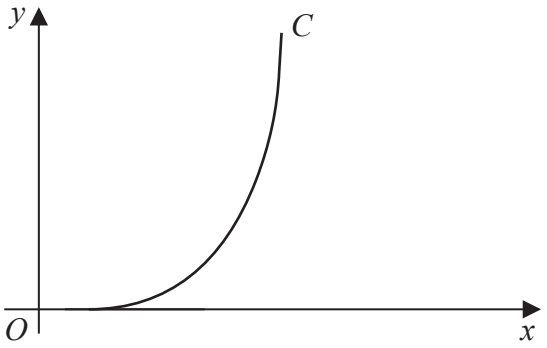


Figure 3

Figure 2 shows a picture of a plant pot.

The plant pot has

- a flat circular base of radius 10 cm
- a height of 15 cm

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 10 + 15t - 5t^3 \qquad y = 15t^2 \qquad 0 \leqslant t \leqslant 1$$

The curved inner surface of the plant pot is modelled by the surface of revolution formed by rotating curve C through 2π radians about the y-axis.

(a) Show that, according to the model, the area of the curved inner surface of the plant pot is given by

$$150\pi \int_0^1 \left(2 + 3t + 2t^2 + 2t^3 - t^5\right) \mathrm{d}t$$

(5)

(b) Determine, according to the model, the total area of the inner surface of the plant pot.

(4)

Each plant pot will be painted with one coat of paint, both inside and outside.
The paint in one tin will cover an area of $12\,\mathrm{m}^2$

(c) Use the answer to part (b) to estimate how many plant pots can be painted using one tin of paint.

(2)

(d) Give a reason why the model might not give an accurate answer to part (c).

(1)

(a) $S = 2\pi \int x \sqrt{((dx/dt)^2 + (dy/dt)^2)}\, dt$

$dx/dt = 15 - 15t^2, \quad dy/dt = 30t$

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Question 10 continued

$(dx/dt)^2 + (dy/dt)^2 = (15 - 15t^2)^2 + (30t)^2$

$= 225 - 450t^2 + 225t^4 + 900t^2$

$= 225 + 450t^2 + 225t^4 = (15 + 15t^2)^2$

$S = 2\pi\int_0^1 (10 + 15t - 5t^3)(15 + 15t^2)\, dt$

$= 2\pi\int_0^1 (150 + 150t^2 + 225t + 225t^3 - 75t^3 - 75t^5)\, dt$

$= 2\pi\int_0^1 (150 + 225t + 150t^2 + 150t^3 - 75t^5)\, dt$

$= 2\pi \times 75\int_0^1 (2 + 3t + 2t^2 + 2t^3 - t^5)\, dt$

$= 150\pi\int_0^1 (2 + 3t + 2t^2 + 2t^3 - t^5)\, dt$ as required

(b) $S = 150\pi\left[2t + \frac{3}{2}t^2 + \frac{2}{3}t^3 + \frac{1}{2}t^4 - \frac{1}{6}t^6\right]_0^1$

$= 150\pi\left(2 + \frac{3}{2} + \frac{2}{3} + \frac{1}{2} - \frac{1}{6}\right) - 0 = 150\pi \times \frac{9}{2}$

Curved surface $= 675\pi = 2120.575...$ cm^2

Base area $= \pi \times 10^2 = 100\pi$ cm^2

Total inner surface area $= 675\pi + 100\pi = 775\pi$

$= 2434.734... \approx 2430$ cm^2

(c) 12 m$^2 = 12 \times 100^2 = 120\,000$ cm^2

Each pot (inside and outside) $\approx 2 \times 775\pi = 4869.468...$ cm^2

$120\,000 \div 4869.468... = 24.643...$

So 24 plant pots can be painted with one tin

(d) The outside surface of the pot is larger than the inside (the pot has thickness) and the rim is ignored, so the area painted per pot is more than 2 × the inner area.

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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