



# Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE  
In A Level Further Mathematics (9FM0)  
Paper 4A Further Pure Mathematics 2

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks | AOs  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>1(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | $\{e=\} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | B1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (1)   |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | B1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (1)   |      |
| <b>(c)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Demonstrates that, for example:<br>$[a \circ b] \circ c = \left[ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \right] \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ $a \circ [b \circ c] = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \left[ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \right]$ $= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ | M1    | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | So $[a \circ b] \circ c = a \circ [b \circ c]$ or associative                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | A1    | 2.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (2)   |      |
| <b>(d)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | The order of the group is 24 or 4!                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | B1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | <b>4 is a factor of 24</b> or <b>4/24</b> therefore <b>it is possible</b> for a subgroup to have order 4.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | B1ft  | 2.4  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (2)   |      |
| <b>(6 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>(a)</b><br>B1: See scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>(b)</b><br>B1: See scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>(c)</b><br><b>M1:</b> Shows two calculations in an attempt to show associative, e.g, $[a \circ b] \circ c$ and $a \circ [b \circ c]$ . There must be an intermediate line of working with evidence of using the permutations. Condone the wrong order for this mark.<br><b>A1:</b> Correct calculations leading to $[a \circ b] \circ c = a \circ [b \circ c]$ or states associative<br><br><b>Note Incorrect order scores M1 A0</b><br>$[a \circ b] \circ c = \left[ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \right] \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

**(d)**

**B1:** Order is 24 or 4!

**B1ft:** Follow through on their order of the group, draws the correct conclusion

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks | AOs  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| <b>2(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | $\begin{vmatrix} 1-\lambda & 0 & a \\ -3 & b-\lambda & 1 \\ 0 & 1 & a-\lambda \end{vmatrix}$ $= (1-\lambda)[(b-\lambda)(a-\lambda) - 1] + a(-3) (= 0)$                                                                                                                                                                                                                                                                                                                                                                                       | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\lambda^3 - (a+b+1)\lambda^2 + (a+b+ab-1)\lambda + (3a+1-ab) (= 0)$ o.e.<br>$-\lambda^3 + (a+b+1)\lambda^2 - (a+b+ab-1)\lambda + (ab-3a-1) (= 0)$ o.e.                                                                                                                                                                                                                                                                                                                                                                                      | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $\lambda^2 \Rightarrow a+b+1 = 7 \text{ and } \lambda \Rightarrow a+b+ab-1 = 13$ Solves simultaneously e.g. $a+b = 6, ab = 8$<br>For example: leading to $a^2 - 6a + 8 = 0 \Rightarrow a = \dots$                                                                                                                                                                                                                                                                                                                                            | M1    | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $a = 2, b = 4$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $c = -1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (5)   |      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | $\mathbf{M}^3 - 7\mathbf{M}^2 + 13\mathbf{M} + \text{'their c'} \mathbf{I} = 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                              | B1ft  | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $I = M^3 - 7M^2 + 13M \Rightarrow M^{-1} = M^2 - 7M + 13I$ $\Rightarrow M^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix}^2 - 7 \begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{pmatrix} = \dots$ $= \begin{pmatrix} 1 & 2 & 6 \\ -15 & 17 & 0 \\ -3 & 6 & 5 \end{pmatrix} - 7 \begin{pmatrix} 1 & 0 & 2 \\ -3 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{pmatrix} = \dots$ | M1    | 3.1a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $M^{-1} = \begin{pmatrix} 7 & 2 & -8 \\ 6 & 2 & -7 \\ -3 & -1 & 4 \end{pmatrix}$                                                                                                                                                                                                                                                                                                                                                                                                                                                             | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | (3)   |      |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>(a)</b><br><b>M1:</b> Correct method to find the characteristic equation for <b>M</b> , condone missing = 0, and one slip as long as the intention is clear<br><b>A1:</b> Multiplies out to achieve a correct characteristic equation, condone missing = 0<br><b>M1:</b> A complete method to find the values of the constants $a$ or $b$ . Equates their coefficients for $\lambda^2$ and $\lambda$ and solves simultaneously to find values for $a$ or $b$ .<br><b>A1:</b> Deduces the correct values for $a$ and $b$ . ( $a < b$ ) following correct simultaneous equations<br><b>A1:</b> Deduces the correct value for $c$ . |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |
| <b>(b)</b><br><b>B1ft:</b> Uses Cayley-Hamilton theorem to produce equation replacing $\lambda$ with <b>M</b> and constant term with constant multiple of the identity matrix <b>I</b> . Follow through on their value for $c$ . This mark may be implied by the M mark.                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |      |

**M1:** A complete method to find  $M^{-1}$  using the Cayley-Hamilton theorem. The minimum is for writing an expression for  $M^{-1}$  from their characteristic equation, for example  $M^{-1} = M^2 - 7M + 13I$  and then stating an answer for  $M^{-1}$ , they may have used their calculator, there is no need to check.

**A1:** Correct  $M^{-1}$

| Question | Scheme                                                                                                                                                                                                                                                                                                                                                             | Marks | AOs  |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 3 (a)    | Implies moves from A <ul style="list-style-type: none"> <li>Has to hop to different lily pad to A</li> <li>Will not hop on the same pad</li> <li>They can't stay on lily pad A</li> <li>Goes to B or C but not A</li> <li>Can't be where it started</li> <li>Can't stay on A</li> </ul>                                                                            | B1    | 2.4  |
|          |                                                                                                                                                                                                                                                                                                                                                                    | (1)   |      |
| (b)      | $p_1 = \frac{2}{3}\left(-\frac{1}{2}\right)^1 + \left(\frac{1}{3}\right) = 0$<br>Minimum needed $p_1 = \frac{2}{3}\left(-\frac{1}{2}\right) + \left(\frac{1}{3}\right) = 0$                                                                                                                                                                                        | B1    | 2.1  |
|          | Assume result is true for $n = k$<br>Or $p_k = \frac{2}{3}\left(-\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)$                                                                                                                                                                                                                                                  | M1    | 2.4  |
|          | Finds an expression for $p_{k+1} = \frac{1}{2}\left(1 - \left[\frac{2}{3}\left(-\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)\right]\right)$                                                                                                                                                                                                                     | M1    | 1.1b |
|          | $p_{k+1} = \frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$ or $\frac{1}{2} - \frac{1}{6} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$<br>$p_{k+1} = \frac{1}{3} - \frac{1}{3} \times -2\left(-\frac{1}{2}\right)^{k+1}$ or $\frac{1}{2} - \frac{1}{6} - \frac{1}{3} \times -2\left(-\frac{1}{2}\right)^{k+1}$ | M1    | 2.1  |
|          | $p_{k+1} = \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} + \left(\frac{1}{3}\right)$                                                                                                                                                                                                                                                                                  | A1    | 1.1b |
|          | If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$ , the statement is true for all $n$ . (Allow 'for all values')                                                                                                                                                                                                                   | A1    | 2.2a |
|          |                                                                                                                                                                                                                                                                                                                                                                    | (6)   |      |
| (c)      | As $n \rightarrow \infty, \left(-\frac{1}{2}\right)^n \rightarrow 0 \therefore p_n \rightarrow \frac{1}{3}$                                                                                                                                                                                                                                                        | B1    | 3.4  |
|          |                                                                                                                                                                                                                                                                                                                                                                    | (1)   |      |

(8 marks)

Notes:

(a)

**B1:** Explains has to move from A

(b)

**B1:** Shows the statement is true for  $n = 1$ . Needs to show that  $p_1 = 0$  and conclusion true for  $n = 1$ , this statement can be recovered in their conclusion if says e.g. true for  $n = 1$

**M1:** Makes a statement that assumes the result is true for  $n = k$ . Assume (true for)  $n = k$  is sufficient. This mark may be recovered in their conclusion if they say e.g. if true for  $n = k$  then ...etc

**M1:** Finds an expression for  $p_{k+1}$  using the recurrence relation and substitutes in for  $p_k$

**M1:** Shows a correct intermediate stage  $p_{k+1} = \frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$

Condone  $\frac{1}{3} - \frac{2}{3}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)^k$

**A1:**  $p_{k+1} = \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} + \left(\frac{1}{3}\right)$  cso

**A1:** Correct complete conclusion. This mark is dependent on previous four marks, the B mark is not required. It is gained by conveying the ideas of **all** underlined points either at the end of their solution **or** as a narrative in their solution.

**Note** Using  $p_{k+1} = \frac{1}{2} - \frac{1}{2}p_k = \frac{1}{2} - \frac{1}{2}\left[\frac{2}{3}\left(-\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)\right] = \frac{1}{2} + \frac{2}{3}\left(-\frac{1}{2}\right)^{k+1} - \left(\frac{1}{6}\right)$  scores 2<sup>nd</sup> M1 3<sup>rd</sup> M1

**(c)**

**B1:** see scheme

| Question                                                                                                                                                                                                                                                                                                                         | Scheme                                                                                                                                                                             | Marks                                                                                                                                                                               | AOs         |             |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-------------|
| 4(a)                                                                                                                                                                                                                                                                                                                             | $124 = 17 \times 7 + 5$ $17 = 3 \times 5 + 2$ $5 = 2 \times 2 + 1$ $\{2 = 2 \times 1\}$                                                                                            | M1                                                                                                                                                                                  | 2.1         |             |
|                                                                                                                                                                                                                                                                                                                                  | Since the <b>gcd is 1</b> , 124 and 17 are <b>relatively prime (coprime)</b>                                                                                                       | A1                                                                                                                                                                                  | 2.4         |             |
|                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                    | <b>(2)</b>                                                                                                                                                                          |             |             |
| (b)                                                                                                                                                                                                                                                                                                                              | $1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 1 = 7 \times 5 - 2 \times 17$ $1 = 7(124 - 17 \times 7) - 2 \times 17$ $\Rightarrow 1 = 7 \times 124 - 51 \times 17$  | M1<br>A1                                                                                                                                                                            | 2.1<br>1.1b |             |
|                                                                                                                                                                                                                                                                                                                                  | $\Rightarrow 10 = 70 \times 124 - 510 \times 17$ $x = 70, y = -510$                                                                                                                | A1                                                                                                                                                                                  | 2.2a        |             |
|                                                                                                                                                                                                                                                                                                                                  | <b>Alternative</b><br>For example<br>$124 = 17 \times 7 + 5$ $5 = 124 - 7 \times 17$                                                                                               | $1 = 5 - 2 \times 2$ $1 = 5 - 2(17 - 3 \times 5) \Rightarrow 4$ $= 2 \times 17 - 6 \times 5$ $4 = 2 \times 17 - 6(124 - 17 \times 7)$ $\Rightarrow 4 = 44 \times 17 - 6 \times 124$ | M1<br>A1    | 2.1<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                  | $\Rightarrow 10 = 2 \times 124$ $- 14$ $\times 17$ $x = 2, y = -14$                                                                                                                | $\Rightarrow 10 = 110 \times 17 - 15 \times 124$ $x = -15, y = 110$                                                                                                                 | A1          | 2.2a        |
|                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                    | <b>(3)</b>                                                                                                                                                                          |             |             |
| (c)                                                                                                                                                                                                                                                                                                                              | A complete method using modulo arithmetic to achieve $x \equiv \dots \pmod{17}$<br>For example<br>$7 \times 124x \equiv 7 \times 6 \pmod{17} \Rightarrow x \equiv \dots \pmod{17}$ | M1                                                                                                                                                                                  | 2.1         |             |
|                                                                                                                                                                                                                                                                                                                                  | $x \equiv 8 \pmod{17} \text{ o.e.}$                                                                                                                                                | A1                                                                                                                                                                                  | 1.1b        |             |
|                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                    | <b>(2)</b>                                                                                                                                                                          |             |             |
| <b>(7 marks)</b>                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                    |                                                                                                                                                                                     |             |             |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                    |                                                                                                                                                                                     |             |             |
| <b>(a)</b><br><b>M1:</b> Complete attempt at the Euclidean algorithm to find the gcd (hcf) of 124 and 17, condone a numerical slip.<br><b>A1:</b> A fully correct application of the Euclidean algorithm and draws the conclusion that since the <b>gcd/hcf = 1</b> therefore 124 and 17 are <b>relatively prime (coprime)</b> . |                                                                                                                                                                                    |                                                                                                                                                                                     |             |             |
| <b>(b)</b><br><b>M1:</b> Attempts to find the Bezout's identity, condone sign slips<br><b>A1:</b> Correct Bezout's identity.<br><b>A1:</b> Deduces a set of correct values of x and y.                                                                                                                                           |                                                                                                                                                                                    |                                                                                                                                                                                     |             |             |

**Alternatively**

**M1:** Uses the solution to (a) to find an identity of the form  $a \times 124 + b \times 17 = c$

**A1:** Correct identity

**A1:** Deduces a set of correct values of  $x$  and  $y$ .

**(c)**

**M1:** A complete method to reach using modulo arithmetic to achieve  $x \equiv \dots \pmod{17}$

For example

Multiplies through by their multiplicative inverse of 124 and reaches  $x \equiv \dots \pmod{17}$ . Bezout's identity used in part (b) follow through on their multiple of 124

**A1:**  $x \equiv 8 \pmod{17}$  o.e.

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Scheme                                                                                                                                                                                                                                                                                                                                                     | Marks | AOs  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 5(a)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | $(x)^2 + (y + a)^2 = 9[(x - a)^2 + y^2]$ or $\sqrt{(x)^2 + (y + a)^2} = 3\sqrt{(x - a)^2 + y^2}$                                                                                                                                                                                                                                                           | M1    | 2.1  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $8x^2 - 18ax + 8y^2 - 2ay + 8a^2 \{= 0\} \text{ o.e.}$                                                                                                                                                                                                                                                                                                     | A1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $x^2 - \frac{9}{4}ax + y^2 - \frac{1}{4}ay + a^2 = 0$ $\Rightarrow \left(x - \frac{9}{8}a\right)^2 - \left(\frac{9}{8}a\right)^2 + \left(y - \frac{1}{8}a\right)^2 - \left(\frac{1}{8}a\right)^2 + a^2 = 0$ $\Rightarrow r^2 = \left(\frac{9}{8}a\right)^2 + \left(\frac{1}{8}a\right)^2 - a^2 = \left(\frac{3}{2}\sqrt{2}\right)^2 \Rightarrow a = \dots$ | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $a = -4 \text{ cso}$                                                                                                                                                                                                                                                                                                                                       | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                            | (4)   |      |
| (b)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | $\left(x - \frac{9}{8}(-4)\right)^2 + \left(y - \frac{1}{8}(-4)\right)^2 = \dots$ $(x - \alpha)^2 + (y - \beta)^2 = \dots \text{ implies centre}(\alpha, \beta)$                                                                                                                                                                                           | M1    | 1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | centre $\left(-\frac{9}{2}, -\frac{1}{2}\right)$                                                                                                                                                                                                                                                                                                           | A1    | 2.2a |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                            | (2)   |      |
| (6 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                            |       |      |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                            |       |      |
| <p><b>(a)</b></p> <p><b>M1:</b> Obtains an equation in terms of <math>x</math> and <math>y</math> using the given information. Condone <math>(x)^2 + (y + a)^2 = 3[(x - a)^2 + y^2]</math> for this mark</p> <p><b>A1:</b> Expands and simplifies the algebra, collects terms and obtains a correct simplified equation. Condone missing <math>= 0</math></p> <p><b>M1:</b> Completes the square for their equation using <math>x^2 + Ax = \left(x + \frac{A}{2}\right)^2 + \dots</math> Sets their radius squared <math>= \left(\frac{3}{2}\sqrt{2}\right)^2 = \frac{18}{4} = \frac{9}{2}</math> or radius <math>= \left(\frac{3}{2}\sqrt{2}\right)</math> and finds a value for <math>a</math>.</p> <p><b>Note</b> the correct values are <math>r^2 = \frac{9}{32}a^2</math>, <math>r = \frac{3a\sqrt{2}}{8}</math></p> <p><b>A1:</b> Deduces that <math>a = -4\text{cso}</math></p> |                                                                                                                                                                                                                                                                                                                                                            |       |      |
| <p><b>(b)</b></p> <p><b>M1:</b> Substitutes their value for <math>a</math> into their equation and finds their centre.</p> <p><b>A1:</b> Deduces the correct centre.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                            |       |      |

| Question                                                                                                                                                                                                                                                | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Marks | AOs  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 6 (a)                                                                                                                                                                                                                                                   | $l^2 - 2l + 1 = 0 \Rightarrow l = \dots \{\lambda = 1\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | M1    | 1.1b |
|                                                                                                                                                                                                                                                         | $u_n = A + Bn$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | A1    | 2.2a |
|                                                                                                                                                                                                                                                         | $u_n = \mu(2)^n \Rightarrow \mu(2)^n = 2\mu(2)^{n-1} - \mu(2)^{n-2} + (2)^n$<br>$\Rightarrow 4\mu(2)^{n-2} = 4\mu(2)^{n-2} - \mu(2)^{n-2} + 4(2)^{n-2}$<br>$\Rightarrow \mu = \dots \{4\}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | M1    | 1.1b |
|                                                                                                                                                                                                                                                         | $u_n = A + Bn + 4(2)^n$ or $u_n = A + Bn + (2)^{n+2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | A1    | 1.1b |
|                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | (4)   |      |
| (b)                                                                                                                                                                                                                                                     | Uses the information to find the values of the constants<br>For example<br>$u_0 = 2u_1 \Rightarrow A + 4 = 2(A + B + 4(2)) \Rightarrow \dots \{A + 2B = -12\}$<br>$u_4 = 3u_2 \Rightarrow A + 4B + 4(2)^4 = 3(A + 2B + 4(2)^2) \Rightarrow \dots \{2A + 2B = 16\}$<br>Solves simultaneous equations to find values for A and B.<br><br><b>Alternatively</b><br>$u_2 = 2u_1 - u_0 + 2^2 = u_0 - u_0 + 4 = 4$ leading to $u_4 = 3u_2 = 3 \times 4 = 12$<br>$u_3 = 2u_2 - u_1 + 2^3 = 2 \times 4 - \left(\frac{A + 4}{2}\right) + 8 = 16 - \left(\frac{A + 4}{2}\right)$<br>$u_4 = 2u_3 - u_2 + 2^4 = 2\left(16 - \left(\frac{A + 4}{2}\right)\right) - 4 + 16 = 12$<br>Leading $A = \dots u_2 = '28' + 2B + 4(2^2) = 4 \Rightarrow B = \dots$ | M1    | 3.1a |
|                                                                                                                                                                                                                                                         | $u_n = 28 - 20n + 4(2)^n$ or $u_n = 28 - 20n + (2)^{n+2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | A1    | 1.1b |
|                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | (2)   |      |
|                                                                                                                                                                                                                                                         | (6 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |      |
| Notes:                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |       |      |
| (a)<br>M1: Forms and solves the auxiliary equation.<br>A1: Correct complementary function.<br>M1: Correct form for the particular solution, substitutes into the recurrence relation to find the PS.<br>A1: Correct general solution                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |       |      |
| (b)<br>Note: They must have two constants to score any marks in this part<br>M1: A complete method to find the constants using the information given. Form two equations and solves simultaneously to find values for A and B.<br>A1: Correct solution. |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |       |      |

| Question | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Marks | AOs  |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 7(i) (a) | 120                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | B1    | 1.1b |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | (1)   |      |
| (i) (b)  | 300                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | B1    | 1.1b |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | (1)   |      |
| (ii)(a)  | As divisible by 11 implies $a - b + c = 11p$ where $p$ is an integer<br>Due to the restrictions on $a, b$ and $c$<br>$\Rightarrow a - b + c = 11$ or $0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | M1    | 2.4  |
|          | For example<br>$a + b + c$ is even $\Rightarrow (a + b + c) - (2b)$ is even<br>$\Rightarrow a - b + c$ is even<br>or<br>Sum of digits is even implies that $a + b + c = 2q$<br>$(a - b + c = (a + b + c) - 2b = 2q - 2b = 2n)$<br><br><b>Alternative approach 1</b><br>$a + b + c = 2q$ where $q \in \mathbb{Z}$<br>either<br>$a + b + c = 2q$ and $a - b + c = 0 \Rightarrow 2b = 2q \Rightarrow b = q$ valid<br>$a + b + c = 2q$ and $a - b + c = 11 \Rightarrow 2b = 2q - 11 \Rightarrow b = q - 5.5$ not valid<br>Or<br>$a + b + c = 2q$ and $a - b + c = 0 \Rightarrow 2a + 2c = 2q$ valid<br>$a + b + c = 2q$ and $a - b + c = 11 \Rightarrow 2a + 2c = 2q + 11$ not valid as $2(a + c)$ even<br>or<br>This approach may be in words<br><br><b>Alternative approach 2</b><br>If sum $a - b + c$ is odd/11 then either one or all three numbers odd.<br>This would mean sum $a + b + c$ would be odd/not even<br><br><b>Alternative approach 3</b><br>If $a - b + c = 11$ this implies $a + b + c \equiv 1 \pmod{2}$ (contradiction)<br><br><b>Conclusion</b><br>therefore $a - b + c = 0$ cso | A1*   | 2.1  |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | (2)   |      |
| (ii)(b)  | $N = 100a + 10b + c \Rightarrow a + b + c \equiv 8 \pmod{9}$ o.e<br>or<br>$N + 1 \equiv 0 \pmod{9}$ ( $N + 1$ is a multiple of 9) o.e.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | B1    | 1.1b |

|                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|--------------|
|                                                                                                                                                                                                                                                                                                                                                | $\Rightarrow a + b + c + 1 \equiv 0 \pmod{9}$ ( $a + b + c + 1$ is a multiple of 9) o.e                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |          |              |
|                                                                                                                                                                                                                                                                                                                                                | <p>Solves <math>a - b + c = 0</math> and <math>a + b + c \equiv 8 \pmod{9}</math><br/> either<br/> (<math>a + b + c \equiv 8</math> or <math>26</math>) to find a value for <math>b</math> (<math>= 4</math>) and uses this value<br/> to form an equation <math>a + c = \dots (4)</math><br/> Or<br/> <math>2a + 2c \equiv 8 \pmod{9} \Rightarrow a + c \equiv 4 \pmod{9}</math> leading to <math>a + c = \dots (4)</math><br/> or<br/> <math>2a + 2c \equiv 8 \pmod{9} \Rightarrow a + c \equiv 4 \pmod{9} \Rightarrow b \equiv 4 \pmod{9}</math> leading to<br/> <math>b = \dots (4)</math><br/> Or<br/> <math>a - b + c = 0</math>, <math>b = a + c</math> so <math>2b + 1</math> is a multiple of 9, <math>2b + 1 = 9</math> or <math>18</math><br/> leading to <math>b = \dots (4)</math></p> | M1       | 2.1          |
|                                                                                                                                                                                                                                                                                                                                                | $N = 143, 242, 341, 440$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | M1<br>A1 | 1.1b<br>2.2a |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | (4)      |              |
|                                                                                                                                                                                                                                                                                                                                                | <b>Alternative</b><br>$(N) = 11n \equiv 8 \pmod{9}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | B1       | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                | Finds multiplicative inverse of 11 using $5 \times 9 - 4 \times 11 = 1$<br>$-4 \times 11n \equiv -4 \times 8 \pmod{9} \Rightarrow n \equiv -32 \pmod{9} \Rightarrow n \equiv 4 \pmod{9}$<br>So $N = 11(9n + 4)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | M1       | 2.1          |
|                                                                                                                                                                                                                                                                                                                                                | $N = 143, 242, 341, 440$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | M1       | 1.1b         |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | A1       | 2.2a         |
|                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | (4)      |              |
| <b>(8 marks)</b>                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |
| <b>(i)(a)</b><br><b>B1:</b> See scheme                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |
| <b>(i)(b)</b><br><b>B1:</b> See scheme                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |
| <b>(ii) (a)</b><br><b>M1:</b> Uses the divisibility rule for 11 and the restrictions on the values $a$ , $b$ and $c$ leading to $a - b + c = 0$ or $11$ only<br><b>A1*:</b> Uses the information that the sum of the digits is even and that $a - b + c = 0$ or $11$ from the divisibility rule for 11 to <b>show</b> that $a - b + c = 0$ cso |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |
| <b>(ii) (b)</b><br><b>B1:</b> States $a + b + c \equiv 8 \pmod{9}$ or $a + b + c + 1 \equiv 0 \pmod{9}$                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |          |              |

**M1:** Uses their equations to find a value for  $a + c = \dots$  or a value for  $b$

**M1:** Uses their values to find at least two correct values for  $N$

**A1:** Deduces all four correct values of  $N$  and no extra values

### **Alternative**

**B1:** States  $(N) = 11n \equiv 8 \pmod{9}$

**M1:** Finds the multiplicative inverse of 11 and uses this to find an expression for  $N$

**M1:** Uses their expression for  $N$  to find at least two values for  $N$

**A1:** Deduces all four correct values of  $N$  and no extra values

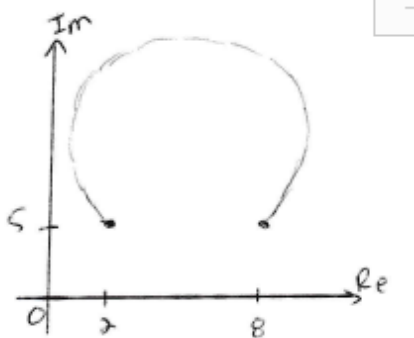
### **Trial and error**

**B1:** For identifying  $a + c = b$

**M1:** For one correct value, this implies B1

**M1:** For two correct values

**A1:** Deduces all four correct values of  $N$  and no extra values

| Question | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Marks | AOs  |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 8 (a)    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | B1    | 1.1b |
|          | The end points of their arc (2, 5) and (8, 5) and the arc drawn above the coordinates                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | B1    | 1.1b |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | (2)   |      |
| (b)      | The centre lies on the <b>perpendicular bisector/midpoint/equidistant</b> of 2 and 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | B1    | 2.4  |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | (1)   |      |
| (c)      | <p>A complete method to find the radius of the circle</p> $\sin\left(\frac{\pi}{3}\right) = \frac{3}{r} \Rightarrow r = \dots$ <p>Or</p> $6^2 = r^2 + r^2 - 2 \times r \times r \times \cos\left(\frac{2\pi}{3}\right) \Rightarrow r = \dots$ <p>Or</p> $h = \frac{3}{\tan\left(\frac{\pi}{3}\right)} = \sqrt{3} \Rightarrow r = \sqrt{(\sqrt{3})^2 + 3^2} = \dots$ <p>or</p> $\begin{aligned} \tan[\arg(x + yi - (8 + 5i)) - \arg(x + yi - (2 + 5i))] \\ = \tan\left(\frac{\pi}{3}\right) \\ \frac{\tan[\arg(x + yi - (8 + 5i))] - \tan[\arg(x + yi - (2 + 5i))]}{1 + \tan[\arg(x + yi - (8 + 5i))] \tan[\arg(x + yi - (2 + 5i))]} \\ = \sqrt{3} \\ \frac{\frac{y-5}{x-8} - \frac{y-8}{x-2}}{1 + \frac{y-5}{x-8} \times \frac{y-8}{x-2}} = \sqrt{3} \end{aligned}$ <p>Leading to an equation of a circle by completing the square<br/> <math>(x - a)^2 + (y - b)^2 = r^2</math> leading to <math>r = \dots</math></p> | M1    | 3.1a |
|          | $r = \frac{6}{\sqrt{3}}$ or $2\sqrt{3}$ o.e.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    | 1.1b |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | (2)   |      |
| (d)      | $y = 5 + h$<br>where $h = \frac{3}{\tan\left(\frac{\pi}{3}\right)}$ or $h = '2\sqrt{3}' \cos\left(\frac{\pi}{3}\right)$ or $h = \sqrt{('2\sqrt{3}')^2 - 3^2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | M1    | 3.1a |

|                                                                                                                                                                                                                                                                              |                    |     |      |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|-----|------|
|                                                                                                                                                                                                                                                                              | $y = 5 + \sqrt{3}$ | A1  | 2.2a |
|                                                                                                                                                                                                                                                                              |                    | (2) |      |
| (7 Marks)                                                                                                                                                                                                                                                                    |                    |     |      |
| <b>Notes:</b>                                                                                                                                                                                                                                                                |                    |     |      |
| <p><b>(a)</b><br/> <b>B1:</b> Major arc drawn anywhere<br/> <b>B1:</b> Correct end points for their arc drawn above the end points, condone written as complex numbers</p>                                                                                                   |                    |     |      |
| <p><b>(b)</b><br/> <b>B1:</b> States perpendicular bisector or midpoint of 2 and 8. Condone “in between 2 and 8” if they write <math>\frac{2+8}{2} = 5</math><br/> <b>Note:</b> <math>\frac{2+8}{2} = 5</math> on its own is B0<br/> In between 2 and 8 on its own is B0</p> |                    |     |      |
| <p><b>(c)</b><br/> <b>M1:</b> A complete method to find the radius of the circle<br/> <b>A1:</b> Correct radius</p>                                                                                                                                                          |                    |     |      |
| <p><b>(d)</b><br/> <b>M1:</b> Any correct complete strategy. If they attempt to find the height (even if incorrect method) in part (c) then <math>y = 5 +</math> their height<br/> <b>A1:</b> Correct answer</p>                                                             |                    |     |      |

| Question  | Scheme                                                                                                                                                                                                                                                                                                                 | Marks | AOs  |
|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 9 (a)     | $I_n = \int \sin^n 2x \, dx = \int \sin^{n-1} 2x \sin 2x \, dx$ <p>Leading to <math>I_n = \left[ \lambda \sin^{n-1} 2x \cos 2x \right] - \mu \int \sin^{n-2} 2x \cos^2 2x \, dx</math></p>                                                                                                                             | M1    | 2.1  |
|           | $I_n = \left[ -\frac{1}{2} \sin^{n-1} 2x \cos 2x \right] + \int (n-1) \sin^{n-2} 2x \cos^2 2x \, dx$                                                                                                                                                                                                                   | A1    | 1.1b |
|           | $I_n = 0 + (n-1) \int \sin^{n-2} 2x (1 - \sin^2 2x) \, dx$ $= (n-1) \int \sin^{n-2} 2x \, dx - (n-1) \int \sin^n 2x \, dx$                                                                                                                                                                                             | dM1   | 1.1b |
|           | $nI_n = (n-1)I_{n-2} \text{ \& } I_n = \frac{n-1}{n} I_{n-2}^*$                                                                                                                                                                                                                                                        | A1*   | 2.1  |
|           |                                                                                                                                                                                                                                                                                                                        | (4)   |      |
|           | <p><b>Alternative: On open MAMA make sure marks are recorded in the correct place</b></p> $I_n = \int \sin^n 2x \, dx = \int \sin^{n-2} 2x \sin^2 2x \, dx$ $= \int \sin^{n-2} 2x (1 - \cos^2 2x) \, dx$ $= \int \sin^{n-2} 2x \, dx - \int \sin^{n-2} 2x \cos^2 2x \, dx$ <p>Leading to an attempt at integration</p> | M1    | 1.1b |
|           | $I_n = I_{n-2} - \int (\sin^{n-2} 2x \cos 2x)(\cos 2x) \, dx$ $I_n = I_{n-2} - \left[ \lambda \sin^{n-1} 2x \cos 2x \right] + \mu \int \sin 2x \sin^{n-1} 2x \, dx$                                                                                                                                                    | dM1   | 2.1  |
|           | $I_n = I_{n-2} - \left[ \frac{1}{2(n-1)} \sin^{n-1} 2x \cos 2x \right] - \frac{1}{n-1} \int \sin^n 2x \, dx$                                                                                                                                                                                                           | A1    | 1.1b |
|           | $I_n = I_{n-2} - \frac{1}{n-1} I_n \Rightarrow (n-1)I_n = (n-1)I_{n-2} - I_n \Rightarrow I_n = \frac{n-1}{n} I_{n-2}^*$                                                                                                                                                                                                | A1*   | 2.1  |
|           |                                                                                                                                                                                                                                                                                                                        | (4)   |      |
| (b)       | $\int 64 \sin^5 x \cos^5 x \, dx = \int A \sin^5 2x \, dx$ <p><b>Note</b> <math>A = 2</math></p>                                                                                                                                                                                                                       | M1    | 2.1  |
|           | $I_5 = \frac{4}{5} I_3, I_3 = \frac{2}{3} I_1 \text{ and } I_1 = \int_0^{\frac{\pi}{2}} \sin 2x \, dx = [\alpha \cos 2x]_0^{\frac{\pi}{2}} = \dots$                                                                                                                                                                    | M1    | 1.1b |
|           | $= 2 \times \left(\frac{4}{5}\right) \times \left(\frac{2}{3}\right) \times 1 = \frac{16}{15}$                                                                                                                                                                                                                         | A1    | 1.1b |
|           |                                                                                                                                                                                                                                                                                                                        | (3)   |      |
| (7 marks) |                                                                                                                                                                                                                                                                                                                        |       |      |

**Notes:****(a)****M1:** Writes  $\sin^n 2x$  as  $\sin^{n-1} 2x \sin 2x$  and integrates using by parts to the form

$$I_n = \left[ \lambda \sin^{n-1} 2x \cos 2x \right] - \mu \int \sin^{n-2} 2x \cos^2 2x \, dx$$

**A1:** Correct integration, may be unsimplified**dM1:** Substitutes the limits of 0 and  $\frac{\pi}{2}$  into 'uv', this may be implied by 0. Replaces  $\cos^2 2x = 1 - \sin^2 2x$  and multiplies out into separate integrals.**A1\*:** Achieves the printed answer following a correct intermediate line and no errors. Cso**Alternative****M1:** Writes  $\sin^n 2x$  as  $\sin^{n-2} 2x \sin^2 2x = \sin^{n-2} 2x (1 - \cos^2 2x)$ , writes as

$$= \int \sin^{n-2} 2x \, dx - \int \sin^{n-2} 2x \cos^2 2x \, dx \text{ and attempts to integrate}$$

**dM1:** Integrates using by parts to the form  $I_n = I_{n-2} - \left[ \lambda \sin^{n-1} 2x \cos 2x \right] + \mu \int \sin 2x \sin^{n-1} 2x \, dx$ **A1:** Correct integration, may be unsimplified**A1\*:** Achieves the printed answer following a correct intermediate line and no errors. cso**(b)****M1:** Uses the identity  $\sin 2x = 2 \sin x \cos x$  in an attempt to write the integral as  $\int A \sin^5 2x \, dx$ **M1:** Uses the answer to part (a) to find a value for  $I_5$  and  $I_3$  and finds  $I_1 = \int_0^{\frac{\pi}{2}} \sin 2x \, dx = \dots$ **A1:**  $\frac{16}{15}$  cso

| Question | Scheme                                                                                                                                                                                                                                                            | Marks    | AOs         |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|-------------|
| 10(a)    | $\text{Surface of revolution} = 2\pi \int_a^b x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ $= 2\pi \int (10 + 15t - 5t^3) \sqrt{(15 - 15t^2)^2 + (30t)^2} dt$                                                                         | M1<br>A1 | 3.4<br>1.1b |
|          | $\text{Surface of revolution} =$ $\{2\pi\} \int \dots \sqrt{225 - 450t^2 + 225t^4 + 900t^2} dt$ $= \{2\pi\} \int \dots \sqrt{225 + 450t^2 + 225t^4} dt$                                                                                                           | M1       | 1.1b        |
|          | $= \{2\pi\} \int \dots \sqrt{(15 + 15t^2)^2} dt$ $= \{2\pi\} \int \dots (15 + 15t^2) dt$                                                                                                                                                                          | M1       | 2.1         |
|          | $= 2\pi \int (10 + 15t - 5t^3)(15 + 15t^2) dt$ $\left\{ 2\pi \int (150 + 150t^2 + 225t + 225t^3 - 75t^3 - 75t^5) dt \right\}$ $150\pi \int_0^1 (2 + 3t + 2t^2 + 2t^3 - t^5) dt *$                                                                                 | A1*      | 1.1b        |
|          |                                                                                                                                                                                                                                                                   | (5)      |             |
| (b)      | $\text{Surface of revolution} = 150\pi \left[ 2t + \frac{3}{2}t^2 + \frac{2}{3}t^3 + \frac{2}{4}t^4 - \frac{1}{6}t^6 \right]_0^1$ $= 150\pi \left[ \left( 2(1) + \frac{3}{2}(1)^2 + \frac{2}{3}(1)^3 + \frac{2}{4}(1)^4 - \frac{1}{6}(1)^6 \right) - (0) \right]$ | M1       | 1.1b        |
|          | Surface of revolution = $675\pi$ = awrt 2120                                                                                                                                                                                                                      | A1       | 1.1b        |
|          | Adds their surface of revolution to their area of the <b>circular</b> base<br>= ' $675\pi$ ' + $\pi \times 10^2$ = ..                                                                                                                                             | M1       | 3.4         |
|          | Total surface area of the inner surface of the pot is<br>$775\pi$ or awrt 2430                                                                                                                                                                                    | A1       | 1.1b        |
|          |                                                                                                                                                                                                                                                                   | (4)      |             |
| (c)      | Number of plant pots = $\frac{120\,000}{2 \times \text{their total surface area}}$<br>or<br>Number of plant pots = $\frac{12}{2 \times \frac{\text{their total surface area}}{100^2}}$                                                                            | M1       | 3.4         |
|          | 24 (complete plant pots)                                                                                                                                                                                                                                          | A1       | 1.1b        |
|          |                                                                                                                                                                                                                                                                   | (2)      |             |
| (d)      | For example:<br>The surface area of the outside of the plant pot will be more than the inside.<br>There will be a (small) rim on the plant pot which is not considered<br>There is thickness to the pot                                                           | B1       | 3.5b        |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                      |     |  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|-----|--|
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | The surface area of the pot may not perfectly fit the curve<br>Pot may not be smooth |     |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                      | (1) |  |
| (12 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                      |     |  |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                      |     |  |
| <p><b>(a)</b></p> <p><b>M1:</b> Uses the formula surface area <math>2\pi \int_0^1 x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt</math> with their model.</p> <p><b>A1:</b> Correct formula for the inner surface area.</p> <p><b>M1:</b> Starts the process of manipulating into an integrable form by squaring and simplifying the expression under the square root.</p> <p><b>M1:</b> Completes the process by writing as something squared and cancelling.</p> <p><b>A1*:</b> Achieves the printed answer with no errors seen. Evidence of 75 taken out as a factor (it may be in stages) Limits just stated.</p> <p><b>Note</b> working with <math>2\pi \int_0^1 y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt</math> the maximum is M0A0M1M1A0</p> |                                                                                      |     |  |
| <p><b>(b)</b></p> <p><b>M1:</b> Integrates the polynomial and uses the limits of <math>t = 0</math> and <math>t = 1</math> the correct way around and subtracts. The limit of <math>t = 0</math> can be implied.</p> <p><b>A1:</b> Correct value for the inner surface area of revolution.</p> <p><b>M1:</b> Adds their area of the inner surface to their area of the <b>circular</b> base</p> <p><b>A1:</b> Correct total inner surface area for the plant pot.</p> <p><b>Note</b> Correct answer for the integration <math>675\pi = 2120.6</math> seen scores M1 A1</p>                                                                                                                                                                                                                                  |                                                                                      |     |  |
| <p><b>(c)</b></p> <p><b>M1:</b> Finds the total number of plant pots by dividing 120 000 by 2 times their total surface area or divides 12 by 2 times their surface area/100<sup>2</sup></p> <p><b>A1:</b> 24 (complete plant pots).</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                      |     |  |
| <p><b>(d)</b></p> <p><b>B1:</b> Gives a correct limitation of the model.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                      |     |  |

