Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3D**

Further Mathematics

Advanced
PAPER 3D: Decision Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 2 0 9 4 A

Pearson

1. A gardener needs the following lengths of string. All lengths are in metres.

4.3 6.1 5.1 4.7 2.5 5.9 3.4 1.7 2.1 0.4 1.3

She cuts the lengths from balls of string. Each ball contains 10 m of string.

(a) Calculate a lower bound for the number of balls of string the gardener needs.
You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to determine how the lengths could be cut from the balls of string.

(3)

(Total for Question 1 is 5 marks)

2.

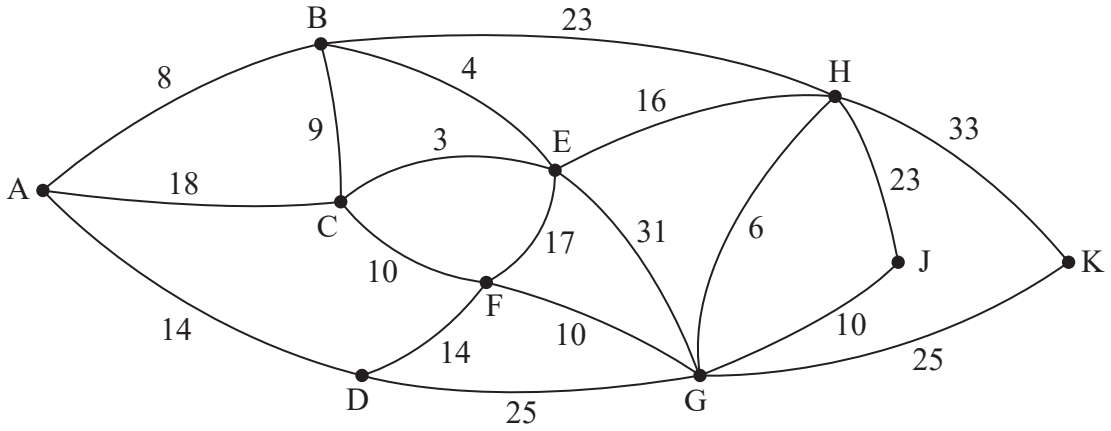


Figure 1

[*The total weight of the network is* 299]

Figure 1 represents a network of cycle tracks between 10 landmarks, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding track.

One day, Blanche wishes to cycle from A to K. She wishes to minimise the distance she travels.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to K.

(ii) State the length of the shortest path from A to K.

(6)

The cycle tracks between the landmarks now need to be inspected. Blanche must travel along each track at least once. She wishes to minimise the length of her inspection route. Blanche will start her inspection route at D and finish at E.

(b) (i) State the edges that will need to be traversed twice.

(ii) Find the length of Blanche's route.

(2)

It is now decided to start the inspection route at A and finish at K. Blanche must minimise the length of her route and travel along each track at least once.

(c) By considering the pairings of all relevant nodes, find the length of Blanche's new route. You must make your method and working clear.

(5)

(Total for Question 2 is 13 marks)

3. The initial distance matrix (Table 1) shows the lengths, in metres, of the corridors connecting six classrooms, A, B, C, D, E and F, in a school. For safety reasons, some of the corridors are one-way only.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	32	24	29	11
B	12	–	17	8	∞	∞
C	32	17	–	4	12	∞
D	24	∞	4	–	∞	13
E	∞	∞	12	18	–	12
F	11	∞	∞	13	12	–

Table 1

(a) By adding the arcs from vertex A, along with their weights, complete the drawing of this network on Diagram 1 in the answer book.

(2)

Floyd's algorithm is to be used to find the complete network of shortest distances between the six classrooms.

The distance matrix after **two** iterations of Floyd's algorithm is shown in Table 2.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	29	20	29	11
B	12	–	17	8	41	23
C	29	17	–	4	12	40
D	24	36	4	–	53	13
E	∞	∞	12	18	–	12
F	11	23	40	13	12	–

Table 2

(b) Perform the next two iterations of Floyd's algorithm that follow from Table 2. You should show the distance matrix after each iteration.

(4)

The final distance matrix after completion of Floyd's algorithm is shown in Table 3.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	24	20	23	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	23	29	12	16	–	12
F	11	23	17	13	12	–

Table 3

Yinka must visit each classroom. He will start and finish at E and wishes to minimise the total distance travelled.

(c) (i) Use the nearest neighbour algorithm, starting at E, to find two Hamiltonian cycles in the completed network of shortest distances.

(ii) Find the length of each of the two cycles.

(iii) State, with a reason, which of the two cycles provides the better upper bound for the length of Yinka's route.

(4)

(Total for Question 3 is 10 marks)

4. A linear programming problem in x, y and z is to be solved using the big-M method. The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

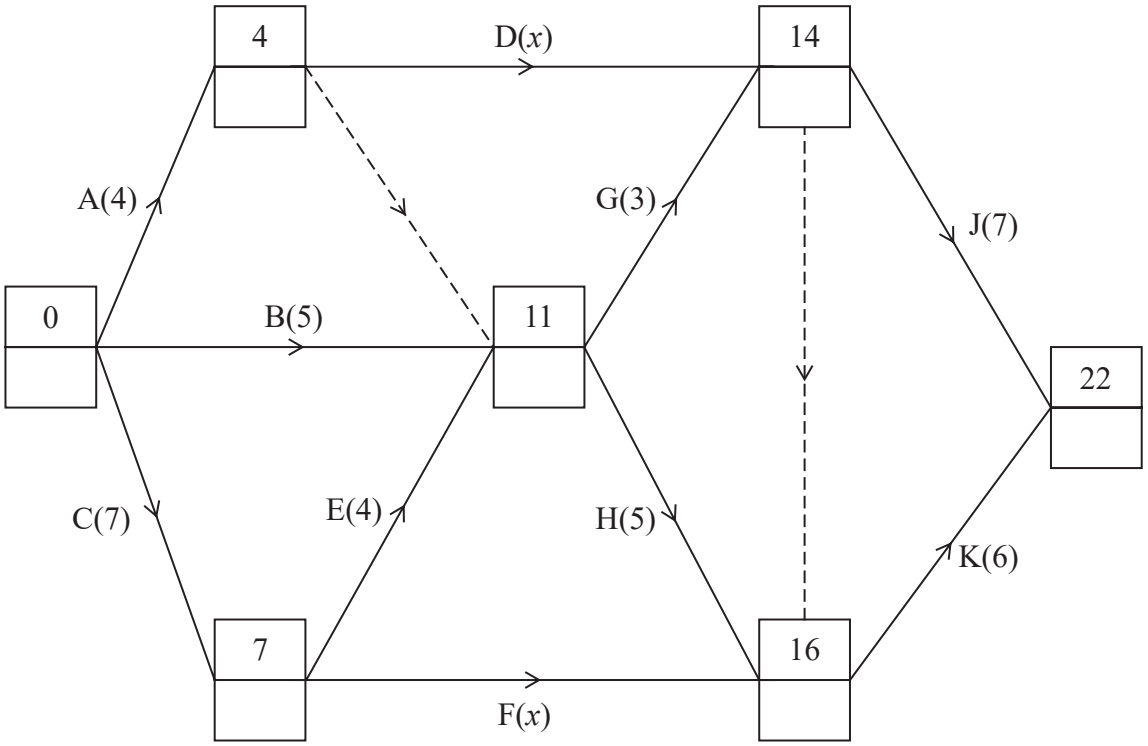
(b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total for Question 4 is 6 marks)

Please turn over for Question 5

5.



Key:

Early event time
Late event time

Figure 2

The network in Figure 2 shows the activities that need to be completed for a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times are shown in Figure 2.

(a) Complete Table 1 in the answer book to show the immediately preceding activities for each activity.

(2)

It is given that $4 < x \leqslant m$

(b) State the largest possible integer value of m.

(1)

(c) (i) Complete Diagram 1 in the answer book to show the late event times.

(ii) State the activities that must be critical.

(3)

(d) Calculate the total float for activity G.

(1)

The resource histogram in Figure 3 shows the number of workers required when each activity starts at its earliest possible time. The histogram also shows which activities happen at each time.

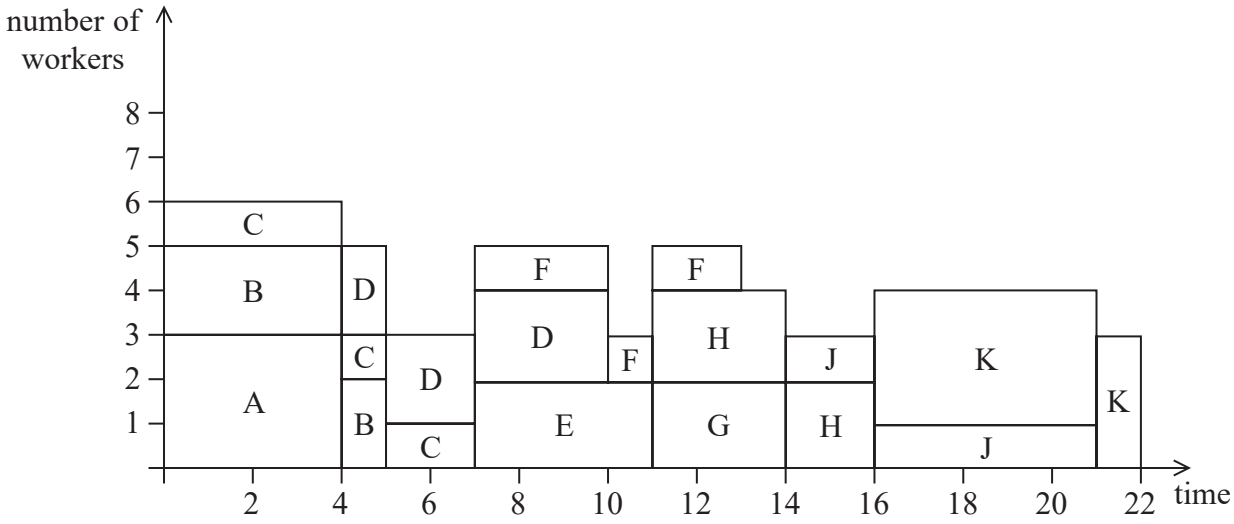


Figure 3

(e) Complete Table 2 in the answer book to show the number of workers required for each activity of the project.

(2)

(f) Draw a Gantt chart on Grid 1 in the answer book to represent the activity network.

(5)

(Total for Question 5 is 14 marks)

6. The following algorithm determines the number of comparisons made when Prim's algorithm is applied to K_n

Step 1	Start
Step 2	Input the value of n
Step 3	Let $a = 1$
Step 4	Let $b = n - 2$
Step 5	Let $c = b$
Step 6	Let $a = a + 1$
Step 7	Let $b = b - 1$
Step 8	Let $c = c + (a \times b) + (a - 1)$
Step 9	If $b > 0$ go to Step 6
Step 10	Output c
Step 11	Stop

(a) For K_5, complete the table in the answer book to show the results obtained at each step of the algorithm.

(3)

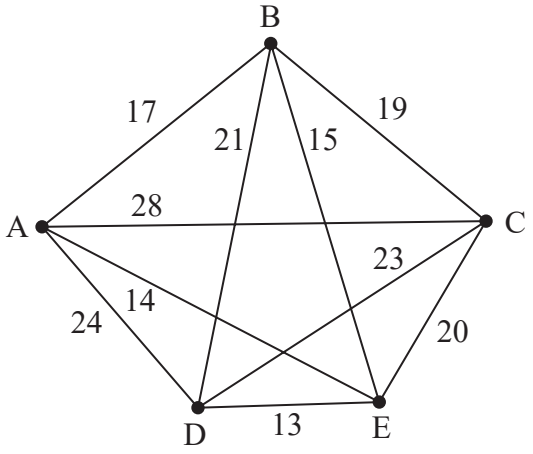


Figure 4

The weights of the ten arcs in Figure 4 are

17 21 24 14 23 13 15 19 28 20

(b) (i) Starting at the left-hand end of the above list, sort the list into ascending order using bubble sort. You need only write down the state of the list at the end of each pass.

(ii) Find the total number of comparisons performed during the sort.

(5)

(c) Find the maximum total number of comparisons required to sort the weights of the 10 arcs of K_5 into ascending order using bubble sort.

(1)

It is given that the maximum total number of comparisons required to sort the weights of the arcs of K_n into ascending order using bubble sort is

$$\lambda n(n-1)(n+1)(n-2)$$

where λ is a constant.

(d) Determine the maximum total number of comparisons required to sort the weights of the arcs of K_{50} into ascending order using bubble sort. You must make your method and working clear.

(3)

(Total for Question 6 is 12 marks)

7.

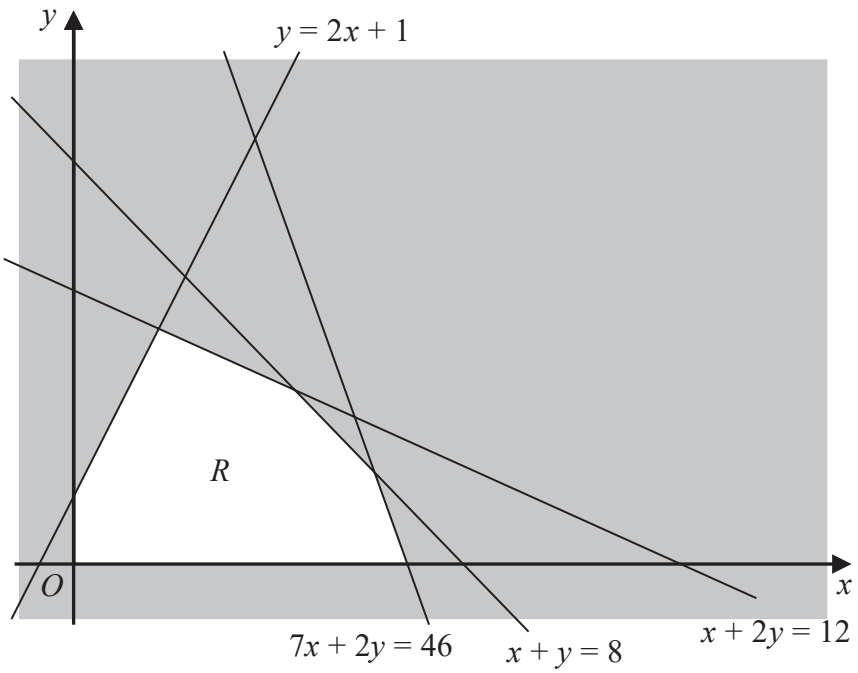


Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y where R is the feasible region.

The objective is to maximise $P = x + ky$, where k is a positive constant.

The optimal vertex of R is to be found using the Simplex algorithm.

(a) Set up an initial tableau for solving this linear programming problem using the Simplex algorithm.

(5)

After two iterations of the Simplex algorithm a possible tableau T is

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5
P	0	0	0	$\frac{1}{5}+\frac{2}{5}k$	0	$-\frac{2}{5}+\frac{1}{5}k$	$5k+2$

(b) State the value of each variable after the second iteration.

(1)

It is given that T does not give an optimal solution to the linear programming problem.

After a third iteration of the Simplex algorithm the resulting tableau does give an optimal solution to the problem.

(c) Perform the third iteration of the Simplex algorithm and hence determine the range of possible values for P. You should state the row operations you use and make your method and working clear.

(9)

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3D**

Further Mathematics

Advanced

PAPER 3D: Decision Mathematics 1

Answer Book

Do not return the question paper with the answer book.

Total Marks

Turn over ▶

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P 7 2 0 9 4 A 0 1 2 0

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1.

4.3 6.1 5.1 4.7 2.5 5.9 3.4 1.7 2.1 0.4 1.3

(a) Total = 4.3 + 6.1 + 5.1 + 4.7 + 2.5 + 5.9 + 3.4 + 1.7 + 2.1 + 0.4 + 1.3 = 37.5

37.5 ÷ 10 = 3.75

∴ lower bound = 4 balls of string

(b) First-fit:

Ball 1: 4.3 5.1 0.4 (9.8)

Ball 2: 6.1 2.5 1.3 (9.9)

Ball 3: 4.7 3.4 1.7 (9.8)

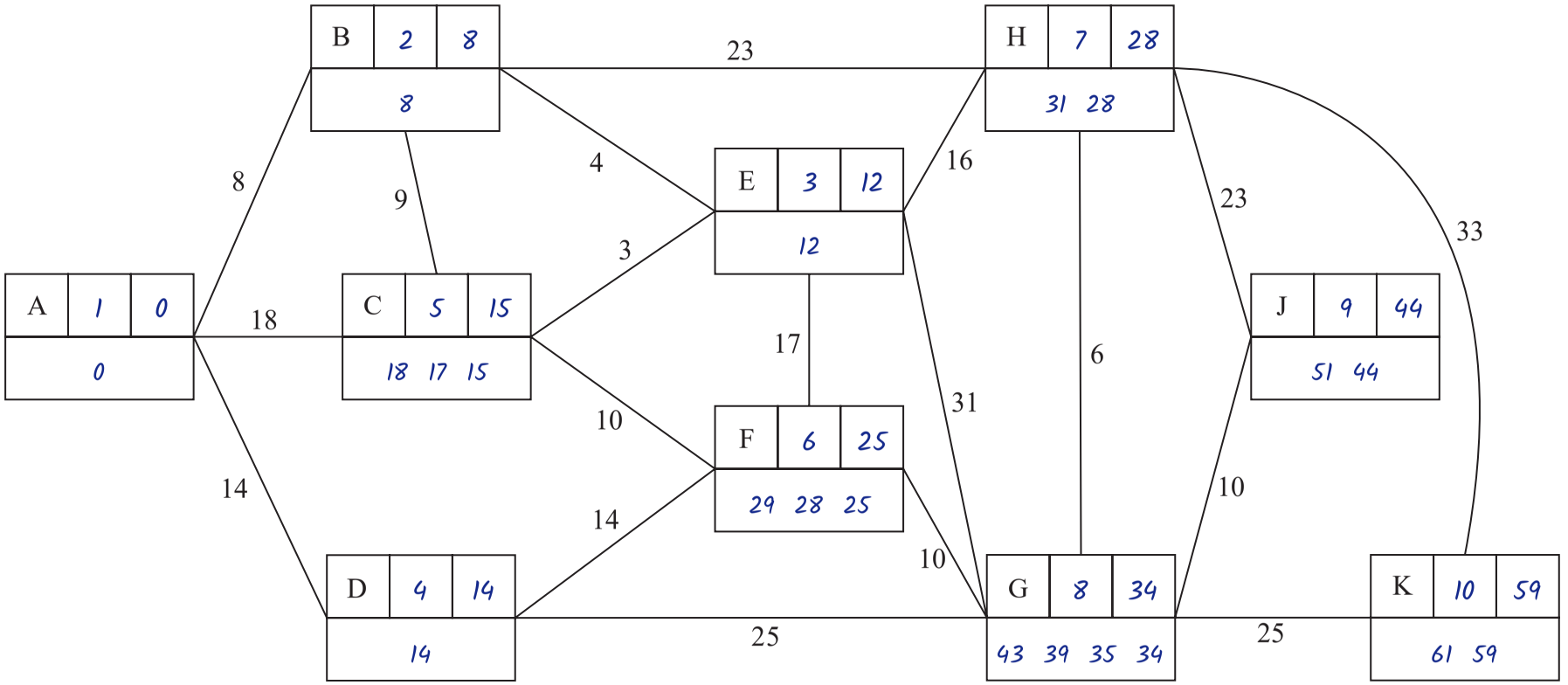
Ball 4: 5.9 2.1 (8.0)

(Total for Question 1 is 5 marks)

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2.



Key:

Vertex	Order of labelling	Final value
Working value		

Shortest path from A to K: A B E H G K

Length of shortest path from A to K: 59 km

P 7 2 0 9 4 A 0 3 2 0

Question 2 continued

(b)(i) Odd nodes: A, D, E and H

Route starts at D and finishes at E, so A and H must be paired

Shortest A to H: A B E H = 8 + 4 + 16 = 28

Edges traversed twice: AB, BE and EH

(ii) Length of route = 299 + 28 = 327 km

(c) Route starts at A and finishes at K, so pair the nodes D, E, H and K

DE + HK = 26 (DABE) + 31 (HGK) = 57

DH + EK = 30 (DFGH) + 47 (EHGK) = 77

DK + EH = 49 (DFGK) + 16 (EH) = 65

Minimum pairing is DE + HK = 57

Length of new route = 299 + 57 = 356 km

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Question 2 continued

(Total for Question 2 is 13 marks)

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3.

(a)

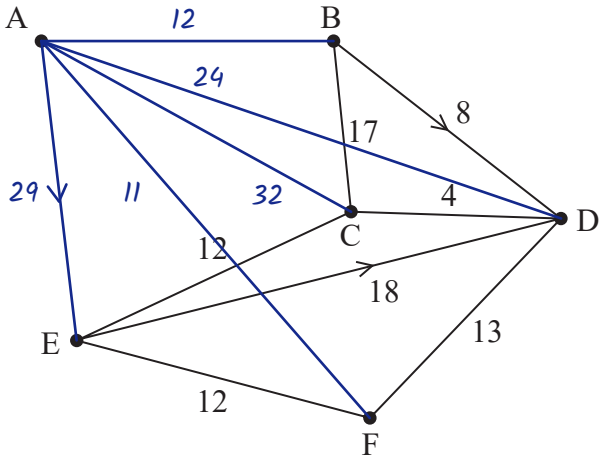


Diagram 1

(b)

There are spare copies of these tables, if required, on Page 9.

3rd iteration

	A	B	C	D	E	F
A	–	12	29	20	29	11
B	12	–	17	8	29	23
C	29	17	–	4	12	40
D	24	21	4	–	16	13
E	41	29	12	16	–	12
F	11	23	40	13	12	–

4th iteration

	A	B	C	D	E	F
A	–	12	24	20	29	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	40	29	12	16	–	12
F	11	23	17	13	12	–

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Question 3 continued

(c)(i) From E the nearest vertices are C (12) and F (12), giving two cycles:

Cycle 1: E – C – D – F – A – B – E

Cycle 2: E – F – A – B – D – C – E

(ii) Cycle 1: 12 + 4 + 13 + 11 + 12 + 24 = 76 m

Cycle 2: 12 + 11 + 12 + 8 + 4 + 12 = 59 m

(iii) Cycle 2 (E – F – A – B – D – C – E) gives the better upper bound, since 59 < 76 and the smaller value is the better upper bound.

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Question 3 continued

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Question 3 continued

(b)

Spare copy for 3rd iteration

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

Spare copy for 4th iteration

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

(Total for Question 3 is 10 marks)

4.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Constraints:

$2x + 3y + 4z \leq 13$

$x - 2y + 2z \geq 8$

$3x - 4z \geq 12$

$x, y, z \geq 0$

Objectives: Maximise $P = -2x + 3y + z$

or Minimise $Q = 2x - 3y - z$

(b) As M is large, $2 - 4M$ is the only negative value in the objective row, so the pivot is in the x column.

θ values: s_1: $13 \div 2 = 6.5$, a_1: $8 \div 1 = 8$, a_2: $12 \div 3 = 4$

4 is the least θ value ($4 < 6.5$ and $4 < 8$), so the pivot is the 3 in the x column of the a_2 row.

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Question 4 continued

(Total for Question 4 is 6 marks)

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5.

Activity	Immediately preceding activity
A	–
B	–
C	–
D	A
E	C

Activity	Immediately preceding activity
F	C
G	A, B, E
H	A, B, E
J	D, G
K	D, F, G, H

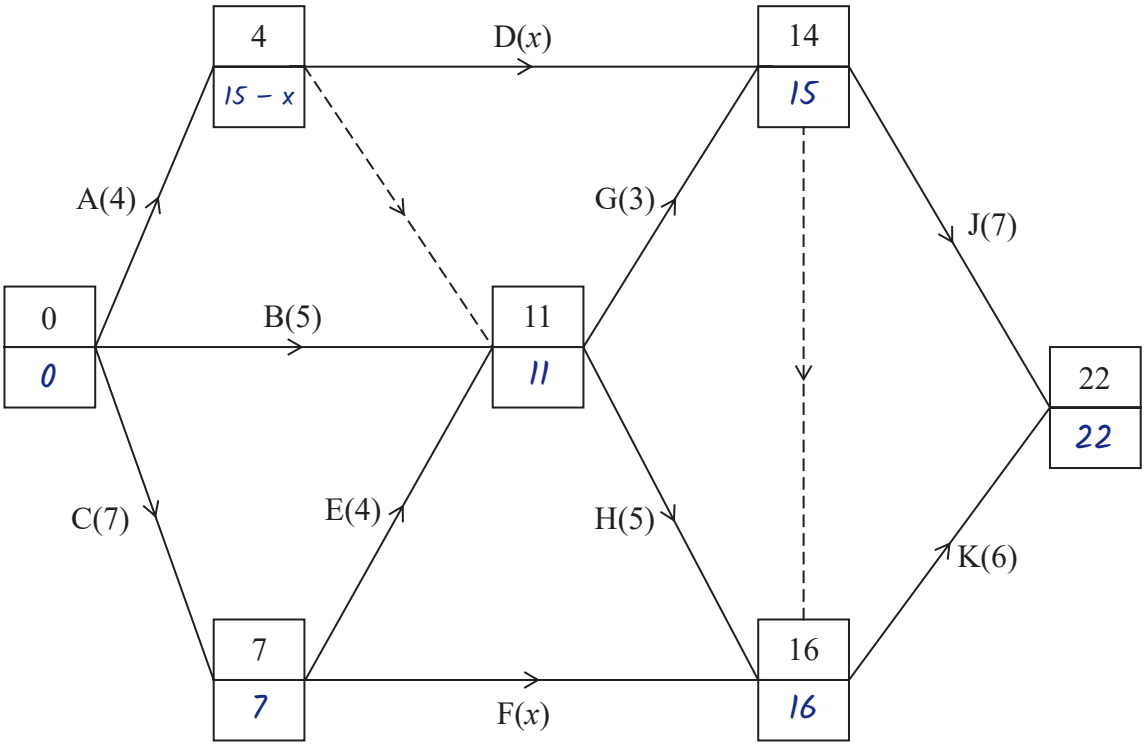
Table 1

(b)

Event with early time 16: $7 + x \leq 16 \Rightarrow x \leq 9$ (also $4 + x \leq 14 \Rightarrow x \leq 10$)

$m = 9$

(c)(i)



Key:

Early event time
Late event time

Diagram 1

(c)(ii)

C, E, H and K

P 7 2 0 9 4 A 0 1 2 2 0

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(d)

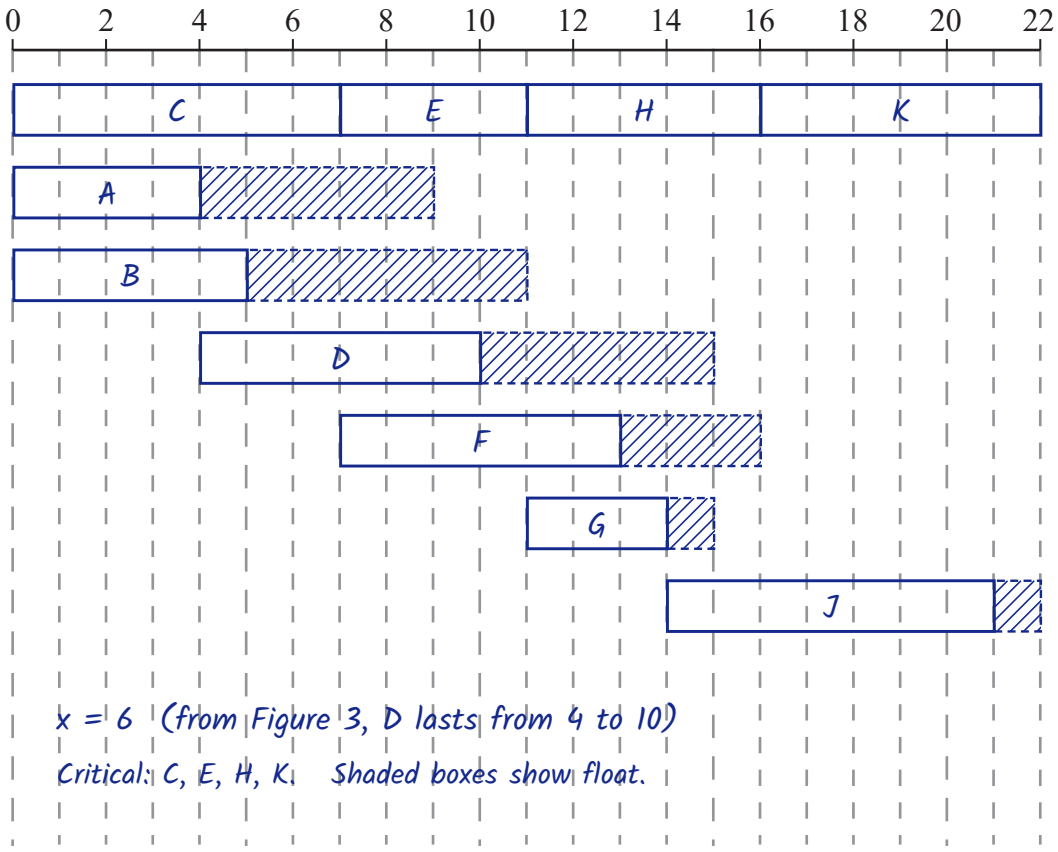
Total float for G = 15 − 11 − 3 = 1 day

(e)

Activity	Number of workers
A	3
B	2
C	1
D	2
E	2

Activity	Number of workers
F	1
G	2
H	2
J	1
K	3

Table 2



x = 6 (from Figure 3, D lasts from 4 to 10)

Critical: C, E, H, K. Shaded boxes show float.

Grid 1

(Total for Question 5 is 14 marks)

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6. *You may not need to use all the rows in this table.*

It may not be necessary to complete all the boxes in each row.

n	*a*	*b*	*c*
5	1	3	3
	2	2	8
	3	1	13
	4	0	16

Output: 16

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Question 6 continued

17 21 24 14 23 13 15 19 28 20

(b)(i)

Pass 1: 17 21 14 23 13 15 19 24 20 28

Pass 2: 17 14 21 13 15 19 23 20 24 28

Pass 3: 14 17 13 15 19 21 20 23 24 28

Pass 4: 14 13 15 17 19 20 21 23 24 28

Pass 5: 13 14 15 17 19 20 21 23 24 28

Pass 6: 13 14 15 17 19 20 21 23 24 28

No swaps in pass 6, so the list is sorted.

(ii) Comparisons = 9 + 8 + 7 + 6 + 5 + 4 = 39

(c) Maximum = 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45

(d) For K_5 the maximum number of comparisons is 45, so

$\lambda(5)(5-1)(5+1)(5-2) = 45$

$360\lambda = 45 \Rightarrow \lambda = 1/8$

For K_{50}: $(1/8)(50)(50-1)(50+1)(50-2)$

$= (1/8)(50)(49)(51)(48)$

= 749 700 comparisons

Question 6 continued

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Question 6 continued

(Total for Question 6 is 12 marks)

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7.

(a)

$x + y \leq 8 \quad \Rightarrow \quad x + y + s_1 = 8$

$x + 2y \leq 12 \quad \Rightarrow \quad x + 2y + s_2 = 12$

$7x + 2y \leq 46 \quad \Rightarrow \quad 7x + 2y + s_3 = 46$

$y \leq 2x + 1 \quad \Rightarrow \quad -2x + y + s_4 = 1$

$P = x + ky \quad \Rightarrow \quad P - x - ky = 0$

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	1	1	1	0	0	0	8
s_2	1	2	0	1	0	0	12
s_3	7	2	0	0	1	0	46
s_4	−2	1	0	0	0	1	1
P	−1	−k	0	0	0	0	0

(b)

$x = 2, \quad y = 5, \quad s_1 = 1, \quad s_2 = 0, \quad s_3 = 22, \quad s_4 = 0$

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Question 7 continued

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5
P	0	0	0	$\frac{1}{5}+\frac{2}{5}k$	0	$-\frac{2}{5}+\frac{1}{5}k$	$5k+2$

b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops
s_4	0	0	5	−3	0	1	5	R1 = 5r1
x	1	0	2	−1	0	0	4	r2 + 0.4R1
s_3	0	0	−12	5	1	0	10	r3 − 2.4R1
y	0	1	−1	1	0	0	4	r4 − 0.2R1
P	0	0	2 − k	k − 1	0	0	4k + 4	r5 − (0.2k−0.4)R1

Spare copy

b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops
P								

Question 7 continued

(c) T is not optimal and $1/5 + (2/5)k > 0$ (as $k > 0$), so the s_4 entry is negative:

$-2/5 + (1/5)k < 0 \Rightarrow k < 2$

Pivot column s_4. θ values: s_1: $1 \div 1/5 = 5$, s_3: $22 \div 12/5 = 9.17$,

y: $5 \div 1/5 = 25$ (x row negative, ignore)

Least $\theta = 5$, so pivot on the $1/5$ in the s_1 row (s_4 enters, s_1 leaves).

Third iteration: see the completed tableau on page 19 (row ops shown).

Optimal value of $P = 4k + 4$ (at $x = 4$, $y = 4$)

Tableau optimal $\Rightarrow 2 - k \geq 0$ and $k - 1 \geq 0 \Rightarrow 1 \leq k \leq 2$

Together with $k < 2$: $1 \leq k < 2$

$P = 4k + 4 \Rightarrow 4(1) + 4 \leq P < 4(2) + 4$

$8 \leq P < 12$

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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