Please check the examination details below before entering your candidate information

Candidate surname	Other names
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Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. A particle A of mass $3m$ and a particle B of mass m are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of A is ku and the speed of B is u.
Immediately after the collision, the speed of A is v and the speed of B is $2v$.

The magnitude of the impulse received by B in the collision is $\frac{3}{2}mu$.

(a) Find v in terms of u only.

(3)

(b) Find the two possible values of k.

(5)

(a) Taking the direction of A's initial motion as positive:

Impulse on B = change in momentum of B

$3/2\ mu = m(2v - (-u))$

$3/2\ u = 2v + u \Rightarrow 2v = u/2$

$v = u/4$

(b) Case 1: A continues in its original direction

CLM: $3m(ku) - mu = 3mv + m(2v) = 5mv = 5mu/4$

$3ku - u = 5u/4 \Rightarrow 3k = 9/4$

$k = 3/4$

Case 2: A reverses its direction of motion

CLM: $3m(ku) - mu = -3mv + m(2v) = -mv = -mu/4$

$3ku - u = -u/4 \Rightarrow 3k = 3/4$

$k = 1/4$

$\therefore\ k = 3/4$ or $k = 1/4$

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Question 1 continued

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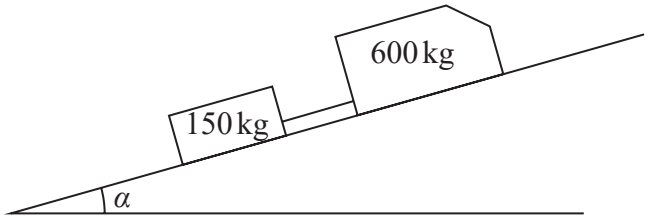


Figure 1

A van of mass 600 kg is moving up a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$. The van is towing a trailer of mass 150 kg. The van is attached to the trailer by a towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 1.

The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 200 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 100 N.

The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 12 kW.

Find the tension in the towbar at the instant when the speed of the van is 9 m s^{-1}

(8)

Driving force $F = P/v = 12000/9 = 4000/3$ N $(= 1333.33...)$

$\sin\alpha = 1/15$. Up the slope, whole system (van + trailer), mass 750 kg:

$F - 200 - 100 - 750g\sin\alpha = 750a$

$4000/3 - 300 - 750 \times 9.8 \times 1/15 = 750a$

$4000/3 - 300 - 490 = 750a$

$a = 543.33.../750 = 0.72444...$ m s^{-2}

Trailer alone:

$T - 100 - 150g\sin\alpha = 150a$

$T = 100 + 150 \times 9.8 \times 1/15 + 150 \times 0.72444...$

$T = 100 + 98 + 108.666...$

$T = 306.666... = 307$ N (3 s.f.)

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Question 2 continued

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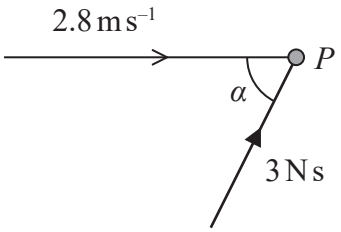


Figure 2

A particle P of mass 0.5 kg is moving in a straight line with speed 2.8 m s^{-1} when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of P immediately before receiving the impulse and the line of action of the impulse is α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of P immediately after receiving the impulse.

(5)

Take i in the original direction of motion and j perpendicular to it.

$\tan\alpha = 4/3 \Rightarrow \cos\alpha = 3/5, \ \sin\alpha = 4/5$

Impulse $= 3\cos\alpha\, i + 3\sin\alpha\, j = 1.8i + 2.4j$ N s

$I = mv - mu$, with velocity after $= v_x i + v_y j$:

i: $3 \times 3/5 = 0.5(v_x - 2.8) \Rightarrow v_x = 6.4 \ (= 32/5)$

j: $3 \times 4/5 = 0.5(v_y - 0) \Rightarrow v_y = 4.8 \ (= 24/5)$

Speed $= \sqrt{(6.4^2 + 4.8^2)} = \sqrt{64}$

Speed $= 8$ m s^{-1}

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Question 3 continued

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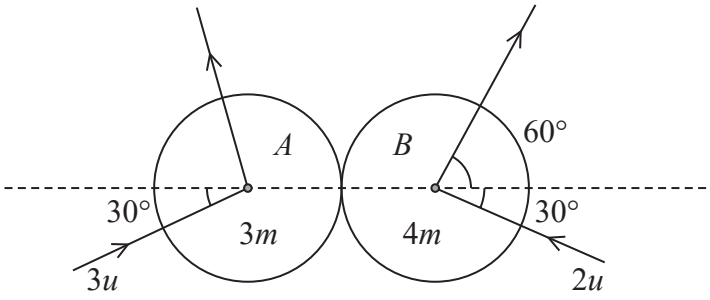


Figure 3

Two smooth uniform spheres, A and B, have equal radii. The mass of A is $3m$ and the mass of B is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, A is moving with speed $3u$ at 30° to the line of centres of the spheres and B is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of B is turned through an angle of 90° by the collision, as shown in Figure 3.

(i) Find the size of the angle through which the direction of motion of A is turned as a result of the collision.

(ii) Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(9)

(i) Along the line of centres take A → B as positive. Components perpendicular to the line of centres are unchanged (smooth spheres).

Let A have components w_x (along) and w_y (perpendicular) after; B speed v at 60°.

Perpendicular: A: $w_y = 3u \sin 30° = 3u/2$

B: $v \sin 60° = 2u \sin 30° = u \Rightarrow v = 2u/\sqrt{3}$

so B's component along the line of centres $= v \cos 60° = u/\sqrt{3}$

CLM along the line of centres:

$3m(3u \cos 30°) - 4m(2u \cos 30°) = 4m(v \cos 60°) + 3m\, w_x$

$9\sqrt{3}u/2 - 4\sqrt{3}u = 4u/\sqrt{3} + 3w_x$

$\sqrt{3}u/2 - 4\sqrt{3}u/3 = 3w_x$

$w_x = -5\sqrt{3}u/18$ (A now moves away from B along the line of centres)

Angle θ between A's new direction and the line of centres:

$\tan\theta = (3u/2) \div (5\sqrt{3}u/18) = 9\sqrt{3}/5$

$\theta = 72.18...°$

A originally moved at 30° to AB; it now moves at θ to BA, so it is turned through

$180° - 30° - 72.18...° = 77.8°$ (3 s.f.)

(ii) Impulse on B = change in momentum of B along the line of centres

$I = 4m(v \cos 60° - (-2u \cos 30°))$

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Question 4 continued

$I = 4m(u/\sqrt{3} + \sqrt{3}u)$

$I = 4m \times 4\sqrt{3}u/3$

$I = (16\sqrt{3}/3)\ mu \quad (= 9.24mu)$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

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5. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of P is $3m$ and the mass of Q is $4m$.
Immediately before the collision the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{7}(9e + 2)$

(6)

After the collision with P, particle Q collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of Q.

The coefficient of restitution between Q and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of e for which there is a second collision between P and Q.

(4)

(a) Take P's initial direction (towards Q) as positive.

After the collision: P moves with speed v back (←), Q with speed w (→).

CLM: $3m(2u) - 4m(u) = -3mv + 4mw$

$2u = -3v + 4w$ ... (1)

NLR: $w + v = e(2u + u)$

$w + v = 3eu$ ... (2)

3 × (2): $3w + 3v = 9eu$

add (1): $7w = 9eu + 2u$

$w = u(9e + 2)/7$

∴ speed of Q immediately after the collision is $(u/7)(9e + 2)$ as required

(b) Q rebounds from the wall with speed $w' = \frac{1}{2} \times u(9e + 2)/7 = u(9e + 2)/14$

From (2): $v = 3eu - u(9e + 2)/7 = u(12e - 2)/7$ (velocity of P away from Q)

Both now move in the same direction (away from the wall); for a second collision Q must be faster than P: $w' > v$

$u(9e + 2)/14 > u(12e - 2)/7$

$9e + 2 > 24e - 4$

$6 > 15e \Rightarrow e < 2/5$

(if $v \leq 0$, P moves towards the wall, so they certainly collide again)

∴ $0 < e < 2/5$

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Question 5 continued

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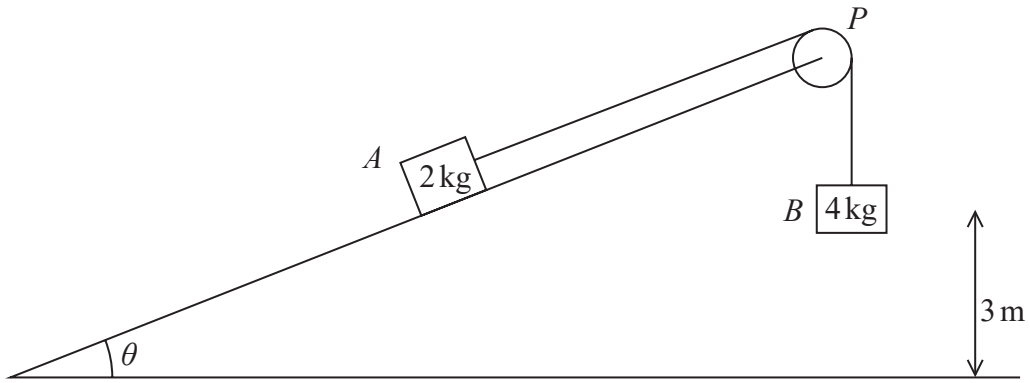


Figure 4

Two blocks, A and B, of masses 2 kg and 4 kg respectively are attached to the ends of a light inextensible string.

Initially A is held on a fixed rough plane. The plane is inclined to horizontal ground at an angle θ, where $\tan\theta = \frac{3}{4}$

The string passes over a small smooth light pulley P that is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane.

Block A is held on the plane with the distance AP greater than 3 m.
Block B hangs freely below P at a distance of 3 m above the ground, as shown in Figure 4.

The coefficient of friction between A and the plane is μ

Block A is released from rest with the string taut.

By modelling the blocks as particles,

(a) find the potential energy lost by the whole system as a result of B falling 3 m.

(3)

Given that the speed of B at the instant it hits the ground is $4.5\,\text{m s}^{-1}$ and ignoring air resistance,

(b) use the work-energy principle to find the value of μ

(6)

After B hits the ground, A continues to move up the plane but does not reach the pulley in the subsequent motion.
Block A comes to instantaneous rest after moving a total distance of $(3 + d)$ m from its point of release.

Ignoring air resistance,

(c) use the work-energy principle to find the value of d

(4)

(a) $\tan\theta = 3/4 \Rightarrow \sin\theta = 3/5, \cos\theta = 4/5$. PE lost $= 4g \times 3 - 2g \times 3\sin\theta$

$= 117.6 - 35.28 = 82.32$

PE lost $= 82.3$ J (3 s.f.)

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Question 6 continued

(b) KE gained by system = $\frac{1}{2} \times (2 + 4) \times 4.5^2 = 60.75$ J

Resolving perpendicular to the plane for A: $R = 2g \cos \theta = 15.68$ N

$F_{max} = \mu R = 15.68\mu$ N

Work done against friction = $15.68\mu \times 3 = 47.04\mu$ J

Work-energy principle: PE lost = KE gained + work done against friction

$82.32 = 60.75 + 47.04\mu$

$\mu = 21.57/47.04 = 0.45854...$

$\mu = 0.459$ (3 s.f.)

(c) After B hits the ground the string goes slack. A moves a further d m up the plane, from 4.5 m s^{-1} to rest. Work-energy principle for A:

KE lost = PE gained + work done against friction

$\frac{1}{2} \times 2 \times 4.5^2 = 2g \sin \theta \times d + 2g \cos \theta \times \mu \times d$

$20.25 = 19.6 \times 3/5 \times d + 19.6 \times 4/5 \times 0.45854... \times d$

$20.25 = 11.76d + 7.19d = 18.95d$

$d = 1.0686... = 1.07$ (3 s.f.)

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Question 6 continued

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Question 6 continued

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7. A spring of natural length a has one end attached to a fixed point A. The other end of the spring is attached to a package P of mass m.
The package P is held at rest at the point B, which is vertically below A such that $AB = 3a$.
After being released from rest at B, the package P first comes to instantaneous rest at A.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

(5)

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g.

(6)

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

(1)

(a) At B the extension is $3a - a = 2a$; at A the spring is compressed by a.
EPE at B $= \lambda(2a)^2/(2a) = 2\lambda a$; EPE at A $= \lambda a^2/(2a) = \lambda a/2$
P is at rest at B and at A, so by the work-energy principle:
EPE at B = EPE at A + GPE gained
$\lambda(2a)^2/(2a) = \lambda a^2/(2a) + mg \times 3a$
$2\lambda a = \lambda a/2 + 3mga$
$3\lambda a/2 = 3mga \Rightarrow \lambda = 2mg$ as required

(b)(i) Maximum speed when the acceleration is zero, i.e. tension = weight:
$T = \lambda x/a = 2mgx/a = mg$
$x = a/2$, so P has maximum speed when the extension is $\frac{1}{2}a$

(ii) From B (extension $2a$) to extension $a/2$, P rises $2a - a/2 = 3a/2$.
Conservation of mechanical energy (V = maximum speed):
$2mg(2a)^2/(2a) = 2mg(a/2)^2/(2a) + mg(3a/2) + \frac{1}{2}mV^2$
$4mga = mga/4 + 3mga/2 + \frac{1}{2}mV^2$
$\frac{1}{2}V^2 = 9ag/4 \Rightarrow V^2 = 9ag/2$
$V = 3\sqrt{(ag/2)}$ $(= 2.12\sqrt{(ag)})$

(c) The spring would have mass, so its kinetic energy (and its gravitational potential energy) would also need to be included in the energy equation.

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Question 7 continued

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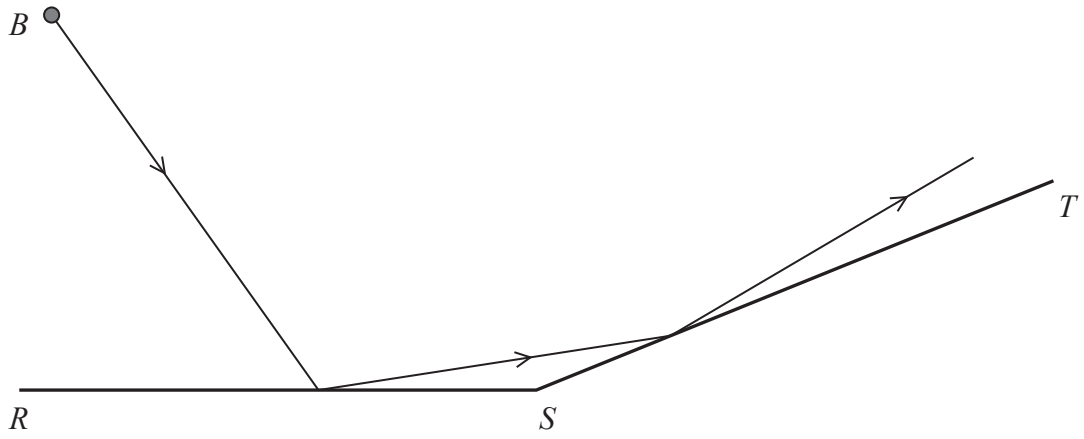


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where RS and ST are smooth fixed vertical walls. The vector $\overrightarrow{RS}$ is in the direction of $\mathbf{i}$ and the vector $\overrightarrow{ST}$ is in the direction of $(2\mathbf{i} + \mathbf{j})$.

A small ball B is projected across the floor towards RS. Immediately before the impact with RS, the velocity of B is $(6\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$. The ball bounces off RS and then hits ST.

The ball is modelled as a particle.

Given that the coefficient of restitution between B and RS is e,

(a) find the full range of possible values of e.

(3)

It is now given that $e = \frac{1}{4}$ and that the coefficient of restitution between B and ST is $\frac{1}{2}$

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of B immediately after its impact with ST.

(7)

(a) Component parallel to RS (i) is unchanged: 6i

Perpendicular component −8j becomes +8e j, so velocity after = 6i + 8e j

To go on to hit ST, the path must be less steep than ST (gradient 1/2):

8e/6 < 1/2 ⇒ e < 3/8

0 < e < 3/8

(b) e = ¼: velocity after hitting RS, v = 6i + 2j

Unit vector along ST: (2i + j)/√5; perpendicular to ST: (−i + 2j)/√5

Parallel component (unchanged): (6i + 2j)·(2i + j)/√5 = (12 + 2)/√5 = 14/√5

Perpendicular component: (6i + 2j)·(−i + 2j)/√5 = (−6 + 4)/√5 = −2/√5

Reversed and halved by the impact: ½ × 2/√5 = 1/√5 (towards (−i + 2j))

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Question 8 continued

$w = (14/\sqrt{5}) \times (2i + j)/\sqrt{5} + (1/\sqrt{5}) \times (-i + 2j)/\sqrt{5}$

$w = (14/5)(2i + j) + (1/5)(-i + 2j)$

$w = (28/5 - 1/5)i + (14/5 + 2/5)j$

$w = (27/5\ i + 16/5\ j)\ m\ s^{-1} \quad (= 5.4i + 3.2j)$

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Question 8 continued

Question 8 continued

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TOTAL FOR PAPER IS 75 MARKS

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