# Pearson Edexcel Level 3 GCE

# Tuesday 25 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4D**

# Further Mathematics

## Advanced

## Paper 4D: Decision Mathematics 2

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read **each** question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**Answer ALL questions. Write your answers in the answer book provided.**

**1.** Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to each of four demand points, P, Q, R and S. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

| | P | Q | R | S | Supply |
|---|---|---|---|---|---|
| A | 15 | 14 | 17 | 11 | 23 |
| B | 10 | 9 | 16 | 12 | 42 |
| C | 11 | 13 | 8 | 10 | 18 |
| D | 15 | 13 | 16 | 17 | 19 |
| Demand | 25 | 45 | 12 | 20 | |

**Table 1**

Table 2 shows an initial solution given by the north-west corner method.

| | P | Q | R | S |
|---|---|---|---|---|
| A | 23 | | | |
| B | 2 | 40 | | |
| C | | 5 | 12 | 1 |
| D | | | | 19 |

**Table 2**

(a) Taking DQ as the entering cell, use the stepping-stone method to find an improved solution. Make your method clear.

**(2)**

(b) Perform one further iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by stating the

- shadow costs
- improvement indices
- route
- entering cell and exiting cell.

**(4)**

(c) Determine whether the solution obtained from this second iteration is optimal, giving a reason for your answer.

**(3)**

(d) State the cost of the solution found in (b).

**(1)**

**(Total for Question 1 is 10 marks)**



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**2.** Four workers, Ted (T), Harold (H), James (J) and Margaret (M), are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker.

The profit, in pounds, resulting from allocating each worker to each task, is shown in the table below. The profit is to be maximised.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | 103 | 97 | 74 | 80 |
| H | 201 | 155 | 145 | 155 |
| J | 111 | 80 | 77 | 92 |
| M | 203 | 188 | 137 | 184 |

(a) Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total profit. You must make your method clear and show the table after each stage.

**(6)**

(b) Determine the resulting total profit.

**(1)**

**(Total for Question 2 is 7 marks)**

**3.**

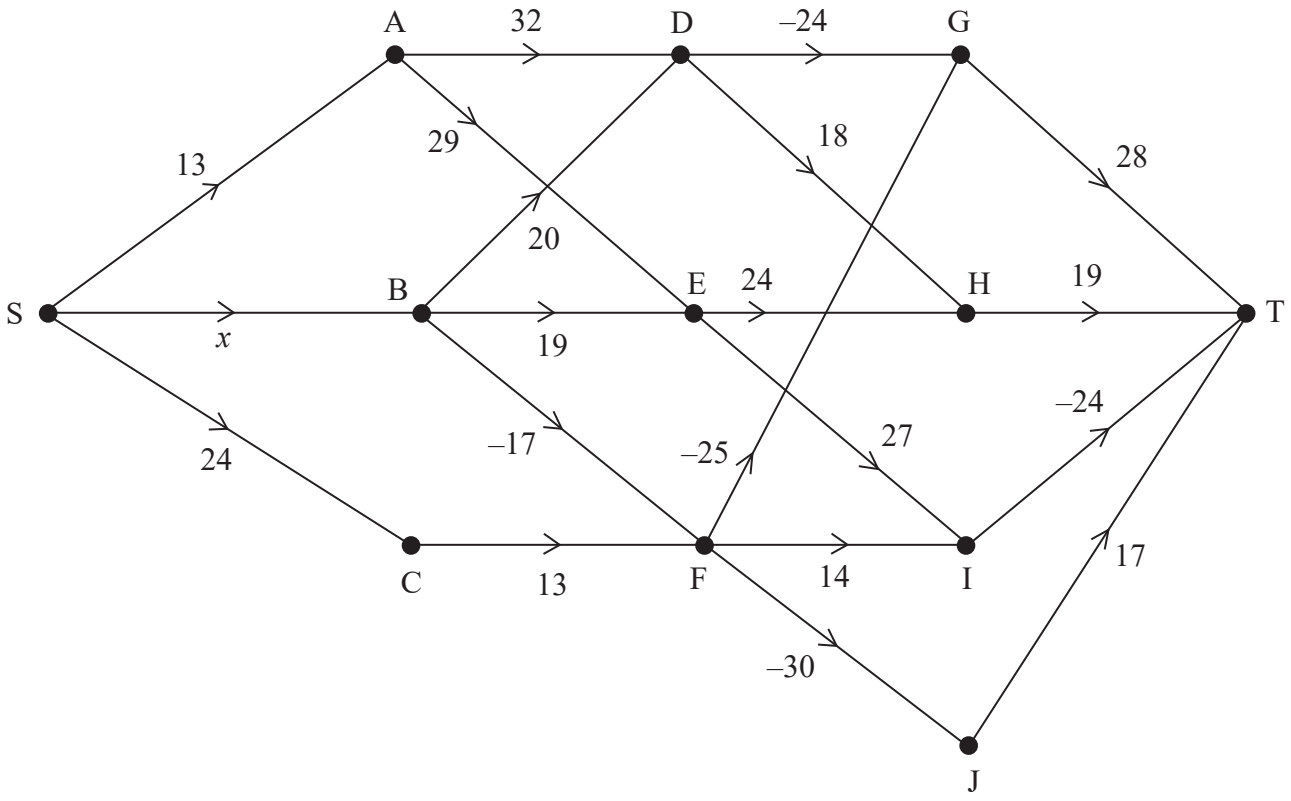



**Figure 1**

In Figure 1 the weight of arc SB is denoted by $x$ where $x \geqslant 0$

(a) Explain why Dijkstra's algorithm cannot be used on the directed network in Figure 1.

**(1)**

It is given that the minimum weight route from S to T passes through B.

(b) Use dynamic programming to find

(i) the range of possible values of $x$

(ii) the minimum weight route from S to T.

**(12)**

**(Total for Question 3 is 13 marks)**





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**4.**

| | | Player B | | |
|---|---|---|---|---|
| | | Option X | Option Y | Option Z |
| Player A | Option P | 3 | –2 | 0 |
| | Option Q | –4 | 4 | –2 |
| | Option R | 1 | 2 | –1 |

A two person zero-sum game is represented by the pay-off matrix for player A shown above.

(a) Verify that there is no stable solution to this game.

**(2)**

Player A intends to make a random choice between options P, Q and R, choosing option P with probability $p_1$, option Q with probability $p_2$ and option R with probability $p_3$

Player A wants to find the optimal values of $p_1$, $p_2$ and $p_3$ using the Simplex algorithm. Player A formulates the following linear programming problem for the game, writing the constraints as inequalities.

Maximise $P = V$

subject to $V \geqslant 3p_1 - 4p_2 + p_3$
$V \geqslant -2p_1 + 4p_2 + 2p_3$
$V \geqslant -2p_2 - p_3$
$p_1 + p_2 + p_3 \leqslant 1$
$p_1 \geqslant 0, p_2 \geqslant 0, p_3 \geqslant 0, V \geqslant 0$

(b) Correct the errors made by player A in the linear programming formulation of the game, giving reasons for your answer.

**(3)**

(c) Write down an initial Simplex tableau for the corrected linear programming problem.

**(3)**

The Simplex algorithm is used to solve the corrected linear programming problem.

The optimal values are $p_1 = 0.6$, $p_2 = 0$ and $p_3 = 0.4$

(d) Calculate the value of the game to player A.

**(2)**

(e) Determine the optimal strategy for player B, making your reasoning clear.

**(4)**

**(Total for Question 4 is 14 marks)**

**5.** An increasing sequence $\{u_n\}$ for $n \in \mathbb{N}$ is such that the difference between the $n$th term of $\{u_n\}$ and the mean of the previous two terms of $\{u_n\}$ is always 6

(a) Show that, for $n \geqslant 3$

$$2u_n - u_{n-1} - u_{n-2} = 12$$

**(2)**

Given that $u_1 = 2$ and $u_2 = 8$

(b) find the solution of this second order recurrence relation to obtain an expression for $u_n$ in terms of $n$.

**(7)**

(c) Show that as $n \to \infty$, $u_n \to kn$ where $k$ is a constant to be determined. You must give reasons for your answer.

**(2)**

**(Total for Question 5 is 11 marks)**



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**6.**

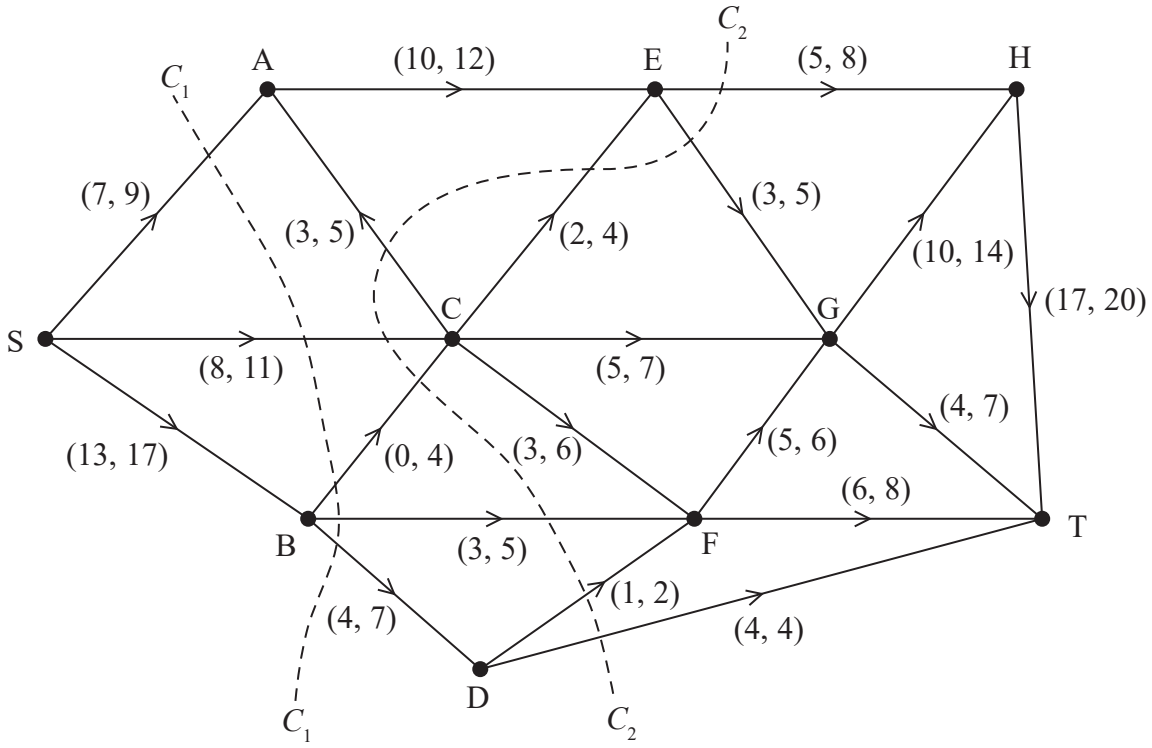



**Figure 2**

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid flows from a source, S, to a sink, T.

The numbers $(l, u)$ on each arc represent, in litres per second, the lower capacity, $l$, and the upper capacity, $u$, of the corresponding pipe.

Two cuts $C_1$ and $C_2$ are shown.

(a) Find the capacity of

(i) cut $C_1$

(ii) cut $C_2$

**(2)**

(b) Explain why the arcs AE and CE cannot be at their upper capacities simultaneously.

**(1)**

(c) Explain why a flow of 31 litres per second through the system is not possible.

**(1)**

(d) Hence determine a minimum feasible flow and a maximum feasible flow through the system. You must draw these feasible flows on the diagrams in the answer book and give reasons to justify your answer. **You should not apply the labelling procedure to find these flows.**

**(4)**

**(Total for Question 6 is 8 marks)**



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**7.** Aisha is deciding whether or not to play a game.

The game involves rolling three fair six-sided dice, which have faces numbered from 1 to 6

If the total score on the three dice is 16 or more then she wins a prize. If the total score is 15 or less then she loses and will have to pay the person running the game £3

(a) Given that the prize is £15

(i) draw a decision tree to model Aisha's possible decisions and the possible outcomes

(ii) determine Aisha's optimal EMV and state the optimal strategy indicated by the decision tree.

**(6)**

The utility function of the game to Aisha is $u(m) = 1 - e^{-\frac{m}{500}}$ where $m$ is the amount of money that Aisha has available. Given that Aisha has exactly £3 and that the prize is now £$x$

(b) find the expected utility to Aisha of playing the game in the form

$$\frac{a}{b}\left(1 - e^{-\frac{(x+c)}{500}}\right)$$

where $a$, $b$ and $c$ are integers to be found.

**(2)**

Aisha decides to use the expected utilities to determine whether she should play the game or not.

(c) Find the minimum prize for which Aisha would consider playing the game.

**(4)**

**(Total for Question 7 is 12 marks)**

**TOTAL FOR PAPER IS 75 MARKS**



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Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

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## Paper 4D: Decision Mathematics 2

**Answer Book**
Do not return the question paper with the answer book

Total Marks

*Turn over* ▶

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**1.**

| | P | Q | R | S | Supply |
|---|---|---|---|---|---|
| A | 15 | 14 | 17 | 11 | 23 |
| B | 10 | 9 | 16 | 12 | 42 |
| C | 11 | 13 | 8 | 10 | 18 |
| D | 15 | 13 | 16 | 17 | 19 |
| Demand | 25 | 45 | 12 | 20 | |

**Table 1**

| | P | Q | R | S |
|---|---|---|---|---|
| A | 23 | | | |
| B | 2 | 40 | | |
| C | | 5 − θ | 12 | 1 + θ |
| D | | θ | | 19 − θ |

(a) Entering cell DQ (put θ in DQ)

Route: DQ(+), CQ(−), CS(+), DS(−)

Max θ = 5 (CQ empties), so CQ exits

| | P | Q | R | S |
|---|---|---|---|---|
| A | 23 | | | |
| B | 2 | 40 | | |
| C | | | 12 | 6 |
| D | | 5 | | 14 |

Improved solution:

AP 23, BP 2, BQ 40, CR 12, CS 6,

DQ 5, DS 14

| | P | Q | R | S |
|---|---|---|---|---|
| A | X | 0 | 1 | −7 |
| B | X | X | 5 | −1 |
| C | 4 | 7 | X | X |
| D | 1 | X | 1 | X |

(b) Shadow costs (cost = R + K, occupied cells):

R: A = 0, B = −5, C = −8, D = −1

K: P = 15, Q = 14, R = 16, S = 18

Improvement indices I = cost − R − K

(X = occupied cell) shown in table

Most negative II is AS (−7)

| | P | Q | R | S |
|---|---|---|---|---|
| A | 23 − θ | | | θ |
| B | 2 + θ | 40 − θ | | |
| C | | | 12 | 6 |
| D | | 5 + θ | | 14 − θ |

Entering cell AS

Route: AS(+), DS(−), DQ(+), BQ(−),

BP(+), AP(−)

θ = min(14, 40, 23) = 14

Exiting cell DS

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**Question 1 continued**

| | P | Q | R | S |
|---|---|---|---|---|
| A | 9 | | | 14 |
| B | 16 | 26 | | |
| C | | | 12 | 6 |
| D | | 19 | | |

| | P | Q | R | S |
|---|---|---|---|---|
| A | X | 0 | 8 | X |
| B | X | X | 12 | 6 |
| C | −3 | 0 | X | X |
| D | 1 | X | 8 | 7 |

| | P | Q | R | S |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |

| | P | Q | R | S |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |

| | P | Q | R | S |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |

*Improved solution after (b):*

*AP 9, AS 14, BP 16, BQ 26,*

*CR 12, CS 6, DQ 19*

*(c) Shadow costs:*

*R: A = 0, B = −5, C = −1, D = −1*

*K: P = 15, Q = 14, R = 9, S = 11*

*Improvement indices shown in table*

*e.g. CP: 11 − (−1) − 15 = −3*

*There is a negative improvement index*

*(CP = −3) so the solution is not optimal.*

*(d) Cost = 9×15 + 14×11 + 16×10 + 26×9*

*+ 12×8 + 6×10 + 19×13*

*= 135 + 154 + 160 + 234 + 96 + 60 + 247*

*= £1086*

**(Total for Question 1 is 10 marks)**

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**2.**

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | 103 | 97 | 74 | 80 |
| H | 201 | 155 | 145 | 155 |
| J | 111 | 80 | 77 | 92 |
| M | 203 | 188 | 137 | 184 |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | 100 | 106 | 129 | 123 |
| H | 2 | 48 | 58 | 48 |
| J | 92 | 123 | 126 | 111 |
| M | 0 | 15 | 66 | 19 |

(a) Subtract every entry from 203
(converts to a minimising problem)

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | 0 | 6 | 29 | 23 |
| H | 0 | 46 | 56 | 46 |
| J | 0 | 31 | 34 | 19 |
| M | 0 | 15 | 66 | 19 |

Reduce rows: subtract 100, 2, 92, 0

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | 0 | 0 | 0 | 4 |
| H | 0 | 40 | 27 | 27 |
| J | 0 | 25 | 5 | 0 |
| M | 0 | 9 | 37 | 0 |

Reduce columns: subtract 0, 6, 29, 19
Zeros covered by 3 lines (column 1,
row T, column 4): 3 < 4, not optimal
Smallest uncovered element = 5

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | 5 | 0 | 0 | 9 |
| H | 0 | 35 | 22 | 27 |
| J | 0 | 20 | 0 | 0 |
| M | 0 | 4 | 32 | 0 |

Subtract 5 from uncovered elements,
add 5 to elements covered twice
(T1 and T4), others unchanged
Now 4 lines needed, so optimal

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**Question 2 continued**

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | | | | |
| H | | | | |
| J | | | | |
| M | | | | |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | | | | |
| H | | | | |
| J | | | | |
| M | | | | |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | | | | |
| H | | | | |
| J | | | | |
| M | | | | |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| T | | | | |
| H | | | | |
| J | | | | |
| M | | | | |

*Allocation:*

*T - 2, H - 1, J - 3, M - 4*

*(b) Profit = 97 + 201 + 77 + 184*

*= £559*

**(Total for Question 2 is 7 marks)**

**3.**

(a) Dijkstra's algorithm cannot be used because the network has negative weights (arcs with negative values).

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| 0 | G | GT | T | 28* |
| | H | HT | T | 19* |
| | I | IT | T | −24* |
| | J | JT | T | 17* |
| 1 | D | DG | G | −24 + 28 = 4* |
| | | DH | H | 18 + 19 = 37 |
| | E | EH | H | 24 + 19 = 43 |
| | | EI | I | 27 − 24 = 3* |
| | F | FG | G | −25 + 28 = 3 |
| | | FI | I | 14 − 24 = −10 |
| | | FJ | J | −30 + 17 = −13* |
| 2 | A | AD | D | 32 + 4 = 36 |
| | | AE | E | 29 + 3 = 32* |
| | B | BD | D | 20 + 4 = 24 |
| | | BE | E | 19 + 3 = 22 |
| | | BF | F | −17 − 13 = −30* |
| | C | CF | F | 13 − 13 = 0* |
| 3 | S | SA | A | 13 + 32 = 45 |
| | | SB | B | x − 30 |
| | | SC | C | 24 + 0 = 24 |
| | | | | |
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**Question 3 continued**

| Stage | State | Action | Destination | Value |
|---|---|---|---|---|
| | | | | |
| | | | | |
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| | | | | |

(b)(i) The minimum weight route passes through B, so

value via B < value via A and value via C:

$x - 30 < 24$ and $x - 30 < 45$

$\Rightarrow x < 54$, so the range is $0 \leqslant x < 54$

(ii) Minimum weight route: S - B - F - J - T

(weight $x - 30$)

**(Total for Question 3 is 13 marks)**

**4.**

| | | Player B | | |
|---|---|---|---|---|
| | | Option X | Option Y | Option Z |
| Player A | Option P | 3 | –2 | 0 |
| | Option Q | –4 | 4 | –2 |
| | Option R | 1 | 2 | –1 |

(a) Row minima: P −2, Q −4, R −1; max of row minima = −1

Column maxima: X 3, Y 4, Z 0; min of column maxima = 0

Row maximin (−1) ≠ column minimax (0), so the game has no stable solution.

(b) V must be non-negative (V ≥ 0), but the pay-offs include negative values, so the coefficients in the three inequalities involving V must be made non-negative: add 5 to every entry.

New matrix: P: 8, 3, 5 Q: 1, 9, 3 R: 6, 7, 4

Also V is the minimum that A can expect to win (the expected winnings must be at least V), so the inequalities should be V ≤ ... not V ≥ ... Corrected constraints:

$V \leq 8p_1 + p_2 + 6p_3$

$V \leq 3p_1 + 9p_2 + 7p_3$

$V \leq 5p_1 + 3p_2 + 4p_3$

$p_1 + p_2 + p_3 \leq 1, \quad p_1, p_2, p_3 \geq 0, \quad V \geq 0$

(where V = value of the game to A + 5)

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**Question 4 continued**

| b.v. | $V$ | $p_1$ | $p_2$ | $p_3$ | $r$ | $s$ | $t$ | $u$ | Value |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | 1 | −8 | −1 | −6 | 1 | 0 | 0 | 0 | 0 |
| $s$ | 1 | −3 | −9 | −7 | 0 | 1 | 0 | 0 | 0 |
| $t$ | 1 | −5 | −3 | −4 | 0 | 0 | 1 | 0 | 0 |
| $u$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| $P$ | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Only use the tableau below if you need to correct your initial tableau.

| b.v. | | | | | | | | | Value |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| $P$ | | | | | | | | | |

(c) r, s, t, u are slack variables (tableau above)

(d) $V \leq 8(0.6) + 1(0) + 6(0.4) = 7.2$

$V \leq 3(0.6) + 9(0) + 7(0.4) = 4.6$

$V \leq 5(0.6) + 3(0) + 4(0.4) = 4.6$

so $V = 4.6$

Value of the game to player A = 4.6 − 5 = −0.4

(e) Against X, A expects 7.2 − 5 = 2.2 > −0.4, so B should never play option X: B plays only options Y and Z.

Let B play Y with probability $q$ and Z with probability $1 - q$.

Value of the game to B = 0.4

If A plays P: B gains $2q + 0(1 - q) = 0.4 \Rightarrow q = 0.2$

(check, A plays R: $-2(0.2) + 1(0.8) = 0.4$)

Player B should never play option X, should play option Y with probability 0.2 and option Z with probability 0.8

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**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 14 marks)**





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**5.**

(a) $u_n - \frac{1}{2}(u_{n-1} + u_{n-2}) = 6$

Multiply by 2: $2u_n - (u_{n-1} + u_{n-2}) = 12$

$\therefore 2u_n - u_{n-1} - u_{n-2} = 12$ for $n \geq 3$

(b) Auxiliary equation: $2m^2 - m - 1 = 0$

$(2m + 1)(m - 1) = 0 \Rightarrow m = 1, m = -\frac{1}{2}$

Complementary function: $u_n = A + B(-\frac{1}{2})^n$

A constant is already in the CF, so try $u_n = \lambda n$:

$2\lambda n - \lambda(n - 1) - \lambda(n - 2) = 12$

$2\lambda n - \lambda n + \lambda - \lambda n + 2\lambda = 12 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$

General solution: $u_n = A + B(-\frac{1}{2})^n + 4n$

$u_1 = 2$: $A - \frac{1}{2}B + 4 = 2 \Rightarrow 2A - B = -4$

$u_2 = 8$: $A + \frac{1}{4}B + 8 = 8 \Rightarrow 4A + B = 0$

Adding: $6A = -4 \Rightarrow A = -2/3$, $B = -4A = 8/3$

$$u_n = -\frac{2}{3} + \frac{8}{3}\left(-\frac{1}{2}\right)^n + 4n$$

(c) As $n \to \infty$, $(-\frac{1}{2})^n \to 0$, so $(8/3)(-\frac{1}{2})^n \to 0$, and the constant $-2/3$ becomes negligible compared with $4n$.

So $u_n \to 4n$ as $n \to \infty$, i.e. $k = 4$

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**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 11 marks)**





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**6.**

(a) (i) Capacity of cut $C_1$ = 9 + 11 + 4 + 5 + 7 = 36

(ii) Capacity of cut $C_2$ = 8 + 5 − 2 − 3 + 11 + 4 + 5 + 2 + 4 = 34

(b) If AE and CE were both at upper capacity, the flow into E would be 12 + 4 = 16 litres per second, but the maximum flow out of E (along EH and EG) is only 8 + 5 = 13 < 16, so AE and CE cannot both be at their upper capacities.

(c) The cut through AE, CE, CG, FG, FT and DT separates S from T and all its arcs go from the S side to the T side. The sum of their lower capacities is 10 + 2 + 5 + 5 + 6 + 4 = 32, so at least 32 litres per second must flow; a flow of 31 is not possible.

(d) Minimum: from (c) any feasible flow is at least 32. The flow of 32 shown is feasible (every arc within its bounds, flow conserved at every node), so minimum feasible flow = 32.
Maximum: flow ≤ capacity of the smallest cut found in (a), $C_2$ = 34. The flow of 34 shown saturates $C_2$ (forward arcs at upper capacity, CA and CE at lower capacity) and is feasible, so maximum feasible flow = 34 litres per second.

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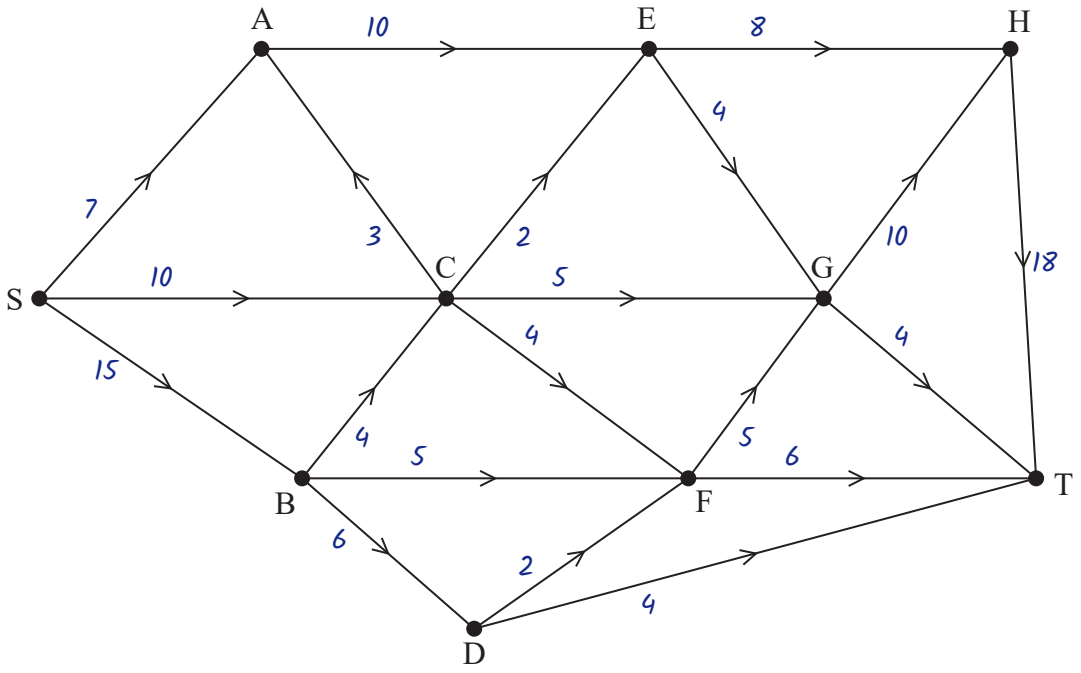
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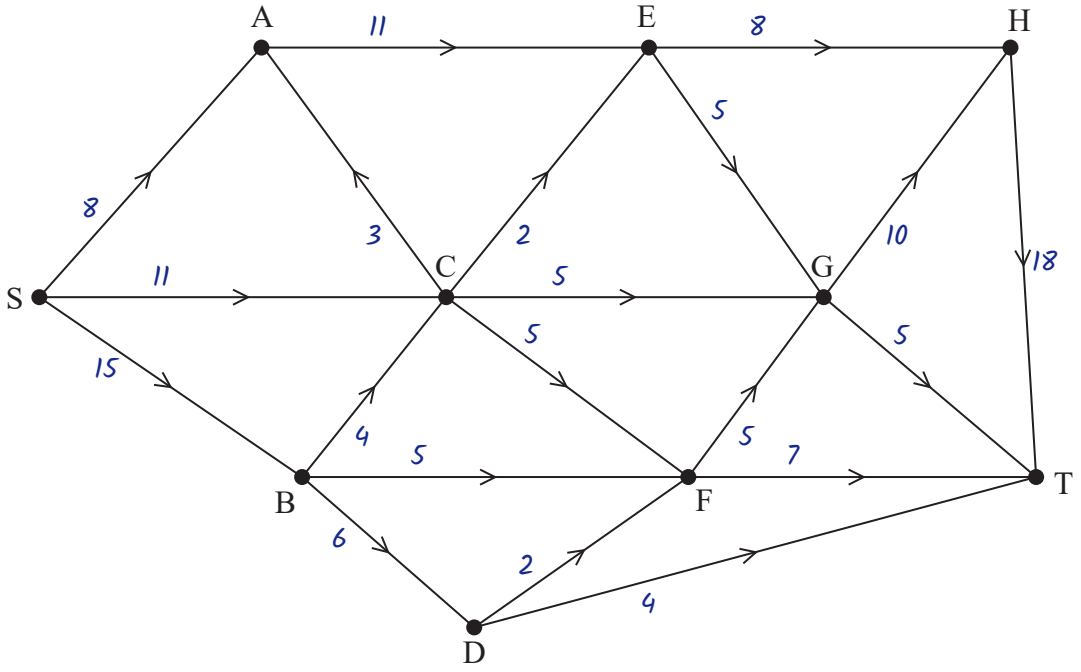
**Question 6 continued**

*(d) Flows (litres per second) written on each arc:*

Minimum feasible flow




Maximum feasible flow




*Minimum feasible flow = 32 litres per second*

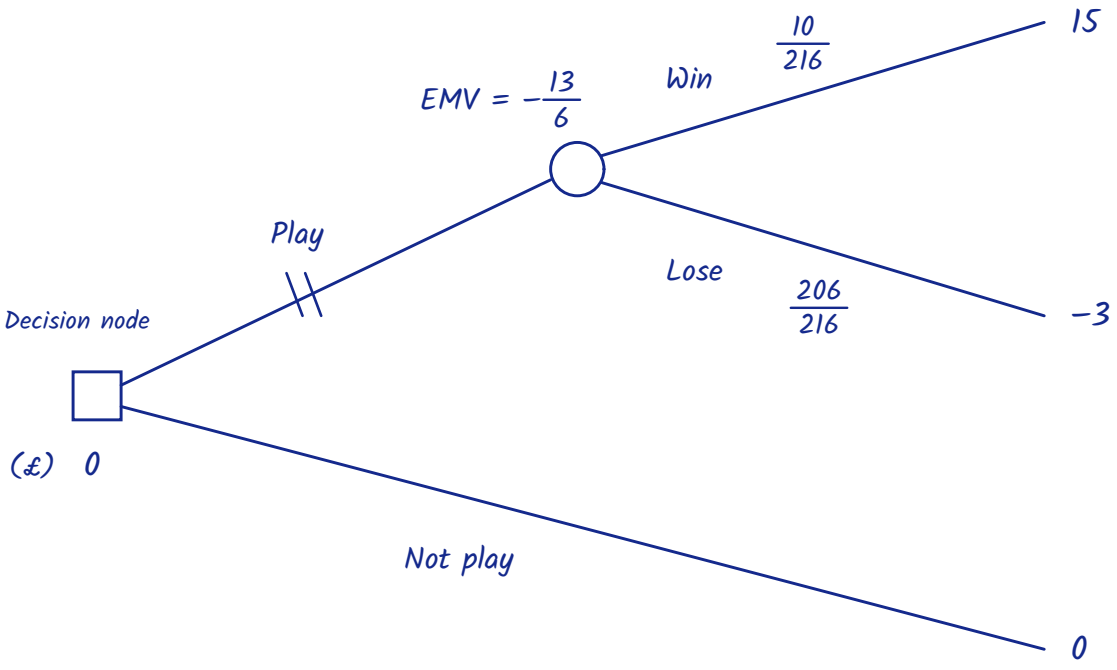
*Maximum feasible flow = 34 litres per second*

**(Total for Question 6 is 8 marks)**



P 6 1 1 8 6 A 0 1 7 2 0
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7.




(a)(i) P(total ≥ 16): 16 in 6 ways, 17 in 3 ways, 18 in 1 way

P(win) = 10/216 = 5/108, P(lose) = 206/216

EMV(play) = 15 × 10/216 + (−3) × 206/216

= (150 − 618)/216 = −468/216 = −13/6 (≈ −£2.17)

EMV(not play) = 0, so the decision node value is 0

(ii) Optimal EMV = £0, so Aisha should not play the game.

(b) If she wins she has £(x + 3); if she loses she has £0

$u(0) = 1 - e^{0} = 0$

Expected utility = $(10/216)(1 - e^{-(x+3)/500}) + (206/216) \times 0$

$$= \frac{10}{216}(1 - e^{-(x+3)/500})$$

a = 10, b = 216, c = 3

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**Question 7 continued**

(c) If she does not play she keeps £3:

utility = $u(3) = 1 - e^{-3/500} = 0.005982...$

She would consider playing if

$(10/216)(1 - e^{-(x+3)/500}) > 1 - e^{-3/500}$

$1 - e^{-(x+3)/500} > 21.6(1 - e^{-3/500}) = 0.129211...$

$e^{-(x+3)/500} < 0.870788...$

$(x + 3)/500 > -\ln(0.870788...) = 0.138356...$

$x + 3 > 69.178... \Rightarrow x > 66.178...$

Minimum prize = £66.18

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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