Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Tuesday 25 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4C**

Further Mathematics

Advanced

Paper 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1.

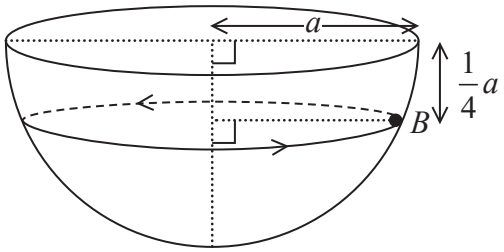


Figure 1

A hemispherical shell of radius a is fixed with its rim uppermost and horizontal. A small bead, B, is moving with constant angular speed, ω, in a horizontal circle on the smooth inner surface of the shell. The centre of the path of B is at a distance $\frac{1}{4}a$ vertically below the level of the rim of the hemisphere, as shown in Figure 1.

Find the magnitude of ω, giving your answer in terms of a and g.

(6)

Reaction R acts along the normal, through the centre O of the hemisphere, at angle θ to the vertical. Let r be the radius of the circle of B.

$\cos\theta = \frac{a/4}{a} = \frac{1}{4} \Rightarrow \sin\theta = \frac{\sqrt{15}}{4}$, $r = a\sin\theta = \frac{\sqrt{15}}{4}a$

$R(\uparrow)$: $R\cos\theta = mg$

$R(\rightarrow$, towards centre of circle$)$: $R\sin\theta = mr\omega^2$

$R\sin\theta = m(a\sin\theta)\omega^2$

$\Rightarrow R = ma\omega^2$

From vertical: $R = \frac{mg}{\cos\theta} = 4mg$

$\Rightarrow ma\omega^2 = 4mg$

$\omega^2 = \frac{4g}{a}$

$\omega = 2\sqrt{\frac{g}{a}}$

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Question 1 continued

(Total for Question 1 is 6 marks)

2. A particle, P, of mass 0.4 kg is moving along the positive x-axis, in the positive x direction under the action of a single force. At time t seconds, $t > 0$, P is x metres from the origin O and the speed of P is v m s^{-1}. The force is acting in the direction of x increasing and has magnitude $\frac{k}{v}$ newtons, where k is a constant.

At $x = 3$, $v = 2$ and at $x = 6$, $v = 2.5$

(a) Show that $v^3 = \frac{61x + 9}{24}$

(6)

The time taken for the speed of P to increase from 2 m s^{-1} to 2.5 m s^{-1} is T seconds.

(b) Use algebraic integration to show that $T = \frac{81}{61}$

(4)

(a)

$F = ma$: $0.4v\frac{dv}{dx} = \frac{k}{v}$

$\int 0.4v^2\, dv = \int k\, dx$

$\frac{0.4}{3}v^3 = kx + C$ i.e. $\frac{2}{15}v^3 = kx + C$

$x = 3$, $v = 2$: $\frac{2}{15}(8) = 3k + C \Rightarrow \frac{16}{15} = 3k + C$

$x = 6$, $v = 2.5$: $\frac{2}{15}(15.625) = 6k + C \Rightarrow \frac{25}{12} = 6k + C$

Subtract: $3k = \frac{25}{12} - \frac{16}{15} = \frac{61}{60} \Rightarrow k = \frac{61}{180}$

$C = \frac{16}{15} - \frac{61}{60} = \frac{1}{20}$

$v^3 = \frac{15}{2}\left(\frac{61x}{180} + \frac{1}{20}\right) = \frac{61x}{24} + \frac{9}{24}$

$\therefore\ v^3 = \frac{61x + 9}{24}$ as required

(b)

$F = ma$: $0.4\frac{dv}{dt} = \frac{k}{v} = \frac{61}{180v}$

$\int_2^{2.5} 0.4v\, dv = \int_0^T \frac{61}{180}\, dt$

$[0.2v^2]_2^{2.5} = \frac{61}{180}T$

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Question 2 continued

$0.2(2.5^2 - 2^2) = \frac{61}{180}T \quad \Rightarrow \quad 0.45 = \frac{61}{180}T$

$T = 0.45 \times \frac{180}{61} = \frac{81}{61}$ as required

Question 2 continued

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Question 2 continued

(Total for Question 2 is 10 marks)

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3. Numerical (calculator) integration is not acceptable in this question.

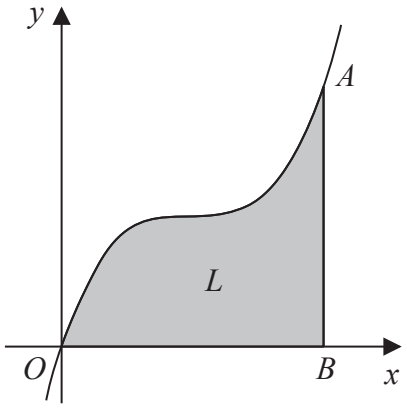


Figure 2

The shaded region OAB in Figure 2 is bounded by the x-axis, the line with equation $x = 4$ and the curve with equation $y = \frac{1}{4}(x - 2)^3 + 2$. The point A has coordinates (4, 4) and the point B has coordinates (4, 0).

A uniform lamina L has the shape of OAB. The unit of length on both axes is one centimetre. The centre of mass of L is at the point with coordinates $(\bar{x}, \bar{y})$.

Given that the area of L is $8\,\text{cm}^2$,

(a) show that $\bar{y} = \frac{8}{7}$

(4)

The lamina is freely suspended from A and hangs in equilibrium with AB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(7)

(a) Moments about the x-axis (area 8):

$8\bar{y} = \int_0^4 \frac{1}{2}y^2\,dx$, $\quad y^2 = \frac{(x-2)^6}{16} + (x-2)^3 + 4$

$8\bar{y} = \frac{1}{2}\left[\frac{(x-2)^7}{7 \times 16} + \frac{(x-2)^4}{4} + 4x\right]_0^4$

$8\bar{y} = \frac{1}{2}\left[\left(\frac{8}{7} + 4 + 16\right) - \left(\frac{-8}{7} + 4 + 0\right)\right] = \frac{1}{2}\left(\frac{128}{7}\right) = \frac{64}{7}$

$\bar{y} = \frac{64}{7} \div 8 = \frac{8}{7}$ as required

(b) Moments about the y-axis:

$8\bar{x} = \int_0^4 xy\,dx = \int_0^4 \left(\frac{x(x-2)^3}{4} + 2x\right) dx$

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Question 3 continued

$= \int_0^4 \left(\frac{x^4}{4} - \frac{3x^3}{2} + 3x^2 - 2x + 2x\right) dx$

$= \left[\frac{x^5}{20} - \frac{3x^4}{8} + x^3\right]_0^4 = 51.2 - 96 + 64 = 19.2$

$\bar{x} = 19.2 \div 8 = 2.4$

Suspended from A, so AG is vertical, where G is (2.4, 8/7).

AB is the line $x = 4$, so the angle between AB and the vertical is angle GAB.

Horizontal distance of G from AB: $4 - 2.4 = 1.6$

Vertical distance of G below A: $4 - \frac{8}{7} = \frac{20}{7}$

$\tan\theta = \frac{1.6}{20/7} = \frac{14}{25}$

$\theta = \tan^{-1}(0.56) = 29.248...$

$\theta = 29.2$

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

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4. A flagpole, AB, is 4 m long. The flagpole is modelled as a non-uniform rod so that, at a distance x metres from A, the mass per unit length of the flagpole, m kg m^{-1}, is given by $m = 18 - 3x$.

(a) Show that the mass of the flagpole is 48 kg.

(3)

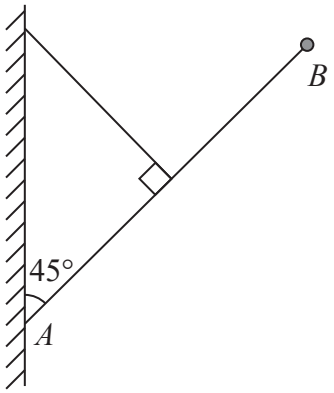


Figure 3

The end A of the flagpole is fixed to a point on a vertical wall. A cable has one end attached to the midpoint of the flagpole and the other end attached to a point on the wall that is vertically above A. The cable is perpendicular to the flagpole. The flagpole and the cable lie in the same vertical plane that is perpendicular to the wall. A small ball of mass 4 kg is attached to the flagpole at B. The cable holds the flagpole and ball in equilibrium, with the flagpole at 45° to the wall, as shown in Figure 3.

The tension in the cable is T newtons.

The cable is modelled as a light inextensible string and the ball is modelled as a particle.

(b) Using the model, find the value of T.

(8)

(c) Give a reason why the answer to part (b) is not likely to be the true value of T.

(1)

(a) Mass $= \int_0^4 (18 - 3x)\,dx$

$= \left[18x - \frac{3x^2}{2}\right]_0^4 = 72 - \frac{3 \times 16}{2} = 72 - 24$

$= 48$ kg as required

(b) Let the centre of mass of the pole be d m from A.

Moments about A: $48d = \int_0^4 x(18 - 3x)\,dx = \left[9x^2 - x^3\right]_0^4$

$48d = 144 - 64 = 80$

$d = \frac{80}{48} = \frac{5}{3}$ m

Pole at 45° to the wall: a point s m from A is $s \sin 45°$ horizontally from A.

Cable is perpendicular to the pole, at the midpoint (2 m from A).

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Question 4 continued

M(A): $T \times 2 = 48g \times \frac{5}{3} \sin 45° + 4g \times 4 \sin 45°$

$2T = 80g \sin 45° + 16g \sin 45° = \frac{96g}{\sqrt{2}}$

$T = \frac{48g}{\sqrt{2}} = 24\sqrt{2}\, g = 332.6...$

$T = 333$

(c) The ball is not really a particle: it has size, so its centre is further from A than B, giving a larger moment (also the cable is not really light).



Question 4 continued

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Question 4 continued

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5.

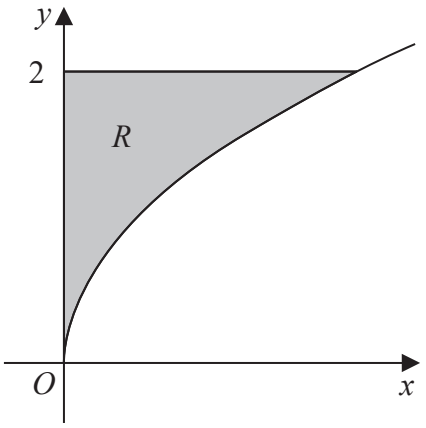


Figure 4

The region R, shown shaded in Figure 4, is bounded by part of the curve with equation $y^2 = 2x$, the line with equation $y = 2$ and the y-axis. The unit of length on both axes is one centimetre. A uniform solid, S, is formed by rotating R through 360° about the y-axis.

Given that the volume of S is $\frac{8}{5}\pi\,\text{cm}^3$,

(a) show that the centre of mass of S is $\frac{1}{3}$ cm from its plane face.

(4)

A uniform solid cylinder, C, has base radius 2 cm and height 4 cm. The cylinder C is attached to S so that the plane face of S coincides with a plane face of C, to form the paperweight P, shown in Figure 5. The density of the material used to make S is three times the density of the material used to make C.

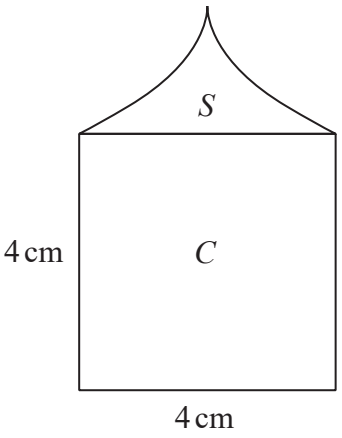


Figure 5

The plane face of P rests in equilibrium on a desk lid that is inclined at an angle $\theta°$ to the horizontal. The lid is sufficiently rough to prevent P from slipping. Given that P is on the point of toppling,

(b) find the value of θ.

(7)

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Question 5 continued

(a) Rotating about the y-axis: $x = \frac{1}{2}y^2$, so $x^2 = \frac{1}{4}y^4$

Moment about the x-axis: $\int_0^2 \pi x^2 y \, dy = \pi \int_0^2 \frac{y^5}{4} \, dy = \pi \left[\frac{y^6}{24}\right]_0^2$

$$= \frac{64\pi}{24} = \frac{8\pi}{3}$$

$\frac{8\pi}{5}\bar{y} = \frac{8\pi}{3} \quad \Rightarrow \quad \bar{y} = \frac{8\pi}{3} \times \frac{5}{8\pi} = \frac{5}{3}$

The plane face is the line $y = 2$, so the distance is $2 - \frac{5}{3} = \frac{1}{3}$ cm as required

(b) Density of C = ρ, density of S = 3ρ. Distances from the base of P:

S: mass $3 \times \frac{8\pi}{5}\rho = \frac{24\pi}{5}\rho$, c.o.m. $4 + \frac{1}{3} = \frac{13}{3}$ cm

C: mass $\pi \times 2^2 \times 4 \times \rho = 16\pi\rho$, c.o.m. 2 cm

P: mass $\frac{24\pi}{5}\rho + 16\pi\rho = \frac{104\pi}{5}\rho$, c.o.m. d cm

Moments about the base: $\frac{104\pi}{5}\rho \, d = \frac{24\pi}{5}\rho \times \frac{13}{3} + 16\pi\rho \times 2$

$$\frac{104}{5}d = \frac{104}{5} + 32 = \frac{264}{5}$$

$$d = \frac{264}{104} = \frac{33}{13} = 2.538\ldots \text{ cm}$$

On the point of toppling, the centre of mass G is vertically above the lowest point of the rim of the base (radius 2 cm):

$\tan\theta = \frac{2}{d} = 2 \div \frac{33}{13} = \frac{26}{33}$

$\theta = \tan^{-1}(26/33) = 38.233\ldots$

$\theta = 38.2$



Question 5 continued

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Question 5 continued

(Total for Question 5 is 11 marks)

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6. The points A and B lie on a smooth horizontal surface with $AB = 4.5\,\text{m}$.

A light elastic string has natural length 1.5 m and modulus of elasticity 15 N. One end of the string is attached to A and the other end of the string is attached to B. A particle, P, of mass 0.2 kg, is attached to the stretched string so that APB is a straight line and $AP = 1.5\,\text{m}$. The particle rests in equilibrium on the surface.

The particle is now moved directly towards A and is held on the surface so APB is a straight line with $AP = 1\,\text{m}$.

The particle is released from rest.

(a) Prove that P moves with simple harmonic motion.

(5)

(b) Find

(i) the maximum speed of P during the motion,

(ii) the maximum acceleration of P during the motion.

(3)

(c) Find the total time, in each complete oscillation of P, for which the speed of P is greater than $5\,\text{m s}^{-1}$.

(5)

(a) The string (natural length 1.5 m) is stretched uniformly to 4.5 m, so AP has natural length 0.5 m and PB has natural length 1 m.

Hooke's law $T = \frac{\lambda x}{l}$: at equilibrium $T_{AP} = \frac{15 \times 1}{0.5} = 30 = \frac{15 \times 2}{1} = T_{PB}$

Let x m be the displacement of P from equilibrium, in the direction AB.

$T_{AP} = \frac{15(1 + x)}{0.5} = 30(1 + x)$, $T_{PB} = \frac{15(2 - x)}{1} = 15(2 - x)$

N2L in the direction AB: $0.2\ddot{x} = 15(2 - x) - 30(1 + x)$

$$0.2\ddot{x} = -45x$$

$$\ddot{x} = -225x = -15^2x$$

$|x| \leq 0.5$, so both parts of the string stay taut throughout the motion.

This is of the form $\ddot{x} = -\omega^2 x$ with $\omega = 15$, so P moves with SHM.

(b) Amplitude = $1.5 - 1 = 0.5$ m, $\omega = 15$

(i) Max speed = $a\omega = 0.5 \times 15 = 7.5\ \text{m s}^{-1}$

(ii) Max acceleration = $a\omega^2 = 0.5 \times 15^2 = 112.5\ \text{m s}^{-2}$

(c) $t = 0$ at release: $x = -0.5 \cos 15t$, $\dot{x} = 7.5 \sin 15t$

Speed 5: $7.5 \sin 15t = 5 \Rightarrow \sin 15t = \frac{2}{3}$

$15t = 0.72972... \Rightarrow t = 0.048648...$ s

So the speed increases from 0 to 5 m s^{-1} in 0.048648... s. By symmetry,

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Question 6 continued

in each oscillation the speed is less than 5 for 4 such intervals.

Period = $\frac{2\pi}{15}$ = 0.41887... s

Time with speed > 5 = $\frac{2\pi}{15}$ − 4 × 0.048648... = 0.22428...

= 0.224 s

Question 6 continued

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Question 6 continued

(Total for Question 6 is 13 marks)

7. A particle, P, of mass m is attached to one end of a light rod of length L. The other end of the rod is attached to a fixed point O so that the rod is free to rotate in a vertical plane about O. The particle is held with the rod horizontal and is then projected vertically downwards with speed u. The particle first comes to instantaneous rest at the point A.

(a) Explain why the acceleration of P at A is perpendicular to OA.

(1)

At the instant when P is at the point A the acceleration of P is in a direction making an angle θ with the horizontal. Given that $u^2 = \frac{2gL}{3}$,

(b) find

(i) the magnitude of the acceleration of P at the point A,

(ii) the size of θ.

(6)

(c) Find, in terms of m and g, the magnitude of the tension in the rod at the instant when P is at its lowest point.

(5)

(a) At A, P is instantaneously at rest, v = 0, so $v^2/L = 0$: there is no acceleration component towards O. So the acceleration of P is entirely tangential, i.e. perpendicular to OA.

(b) P comes to rest at A above the level of O. Let OA make angle α with the horizontal, so A is L sin α above the starting level.

Conservation of energy (start to A):

$\frac{1}{2}mu^2 = mgL\sin\alpha \Rightarrow \frac{1}{2} \times \frac{2gL}{3} = gL\sin\alpha \Rightarrow \sin\alpha = \frac{1}{3}$

At A: the acceleration is the tangential component of g (along the tangent):

acceleration $= g\cos\alpha$, $\cos\alpha = \sqrt{\left(1 - \frac{1}{9}\right)} = \frac{2\sqrt{2}}{3}$

(i) magnitude $= \frac{2\sqrt{2}}{3}g = 9.239\ldots = 9.24\ \text{m s}^{-2}$

(ii) The acceleration is perpendicular to OA and OA makes angle α with the horizontal, so the acceleration makes angle 90° − α with the horizontal.

$\alpha = \sin^{-1}\left(\frac{1}{3}\right) = 19.47\ldots° \Rightarrow \theta = 90° - 19.47\ldots° = 70.5°$

(c) At the lowest point P is L below the starting level.

Energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgL \Rightarrow v^2 = \frac{2gL}{3} + 2gL = \frac{8gL}{3}$

Towards O: $T - mg = \frac{mv^2}{L} = \frac{m}{L} \times \frac{8gL}{3} = \frac{8mg}{3}$

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Question 7 continued

$T = mg + \frac{8mg}{3} = \frac{11mg}{3}$

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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