Pearson Edexcel Level 3 GCE

Monday 24 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3D**

Further Mathematics

Advanced

Paper 3D: Decision Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the answer book provided.

1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(2)

The list of numbers is now to be sorted into descending order.

(b) Perform a quick sort on the original list to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

For a list of n numbers, the quick sort algorithm has, on average, order $n \log n$.

Given that it takes 2.32 seconds to run the algorithm when $n = 450$

(c) calculate approximately how long it will take, to the nearest tenth of a second, to run the algorithm when $n = 11\,250$. You should make your method and working clear.

(2)

(Total for Question 1 is 8 marks)

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2.

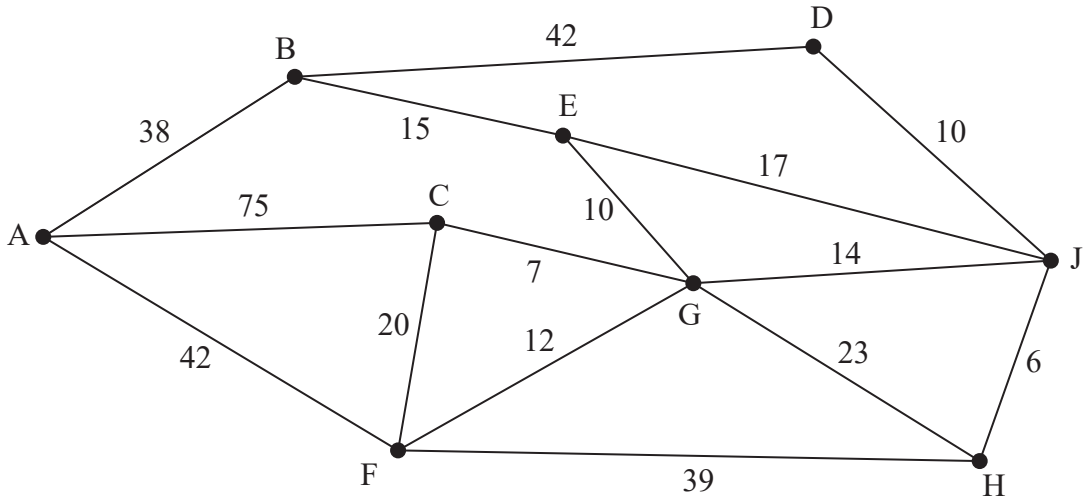


Figure 1

[*The total weight of the network is* 370]

Figure 1 represents a network of corridors in a building. The number on each arc represents the length, in metres, of the corresponding corridor.

(a) Use Dijkstra's algorithm to find the shortest path from A to D, stating the path and its length.

(6)

On a particular day, Naasir needs to check the paintwork along each corridor. Naasir must find a route of minimum length. It must traverse each corridor at least once, starting at B and finishing at G.

(b) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

(4)

(c) Find the length of Naasir's route.

(1)

On a different day, all the corridors that start or finish at B are closed for redecorating. Naasir needs to check all the remaining corridors and may now start at any vertex and finish at any vertex. A route is required that excludes all those corridors that start or finish at B.

(d) (i) Determine the possible starting and finishing points so that the length of Naasir's route is minimised. You must give reasons for your answer.

(ii) Find the length of Naasir's new route.

(3)

(Total for Question 2 is 14 marks)

3.

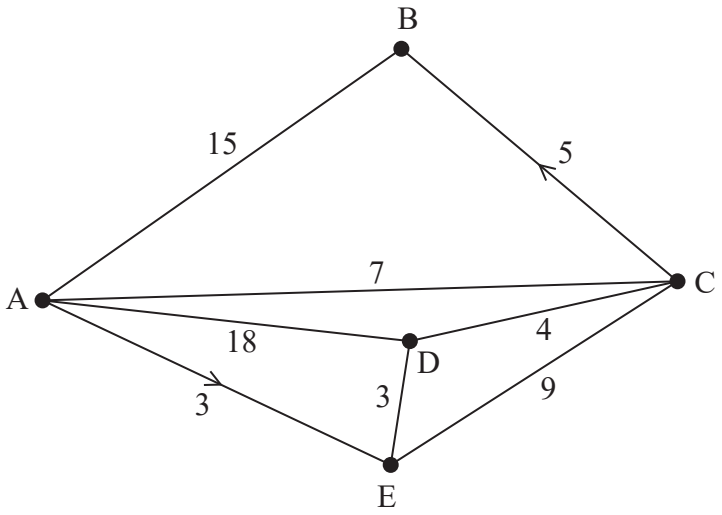


Figure 2

The network in Figure 2 shows the direct roads linking five villages, A, B, C, D and E. The number on each arc represents the length, in miles, of the corresponding road. The roads from A to E and from C to B are one-way, as indicated by the arrows.

(a) Complete the initial distance and route tables for the network provided in the answer book.

(2)

(b) Perform the first three iterations of Floyd's algorithm. You should show the distance table and the route table after each of the three iterations.

(5)

After five iterations of Floyd's algorithm the final distance table and partially completed final route table are shown below.

Distance table

	A	**B**	**C**	**D**	**E**
A	–	12	7	6	3
B	15	–	22	21	18
C	7	5	–	4	7
D	11	9	4	–	3
E	14	12	7	3	–

Route table

	A	**B**	**C**	**D**	**E**
A	A				
B	A	B			
C	A	B	C		
D	C	C	C	D	
E	D	D	D	D	E

(c) (i) Explain how the partially completed final route table can be used to find the shortest route from E to A.

(ii) State this route.

(3)

Mabintou decides to use the distance table to try to find the shortest cycle that passes through each vertex. Starting at D, she applies the nearest neighbour algorithm to the final distance table.

(d) (i) State the cycle obtained using the nearest neighbour algorithm.

(ii) State the length of this cycle.

(iii) Interpret the cycle in terms of the actual villages visited.

(iv) Prove that Mabintou's cycle is not optimal.

(4)

(Total for Question 3 is 14 marks)

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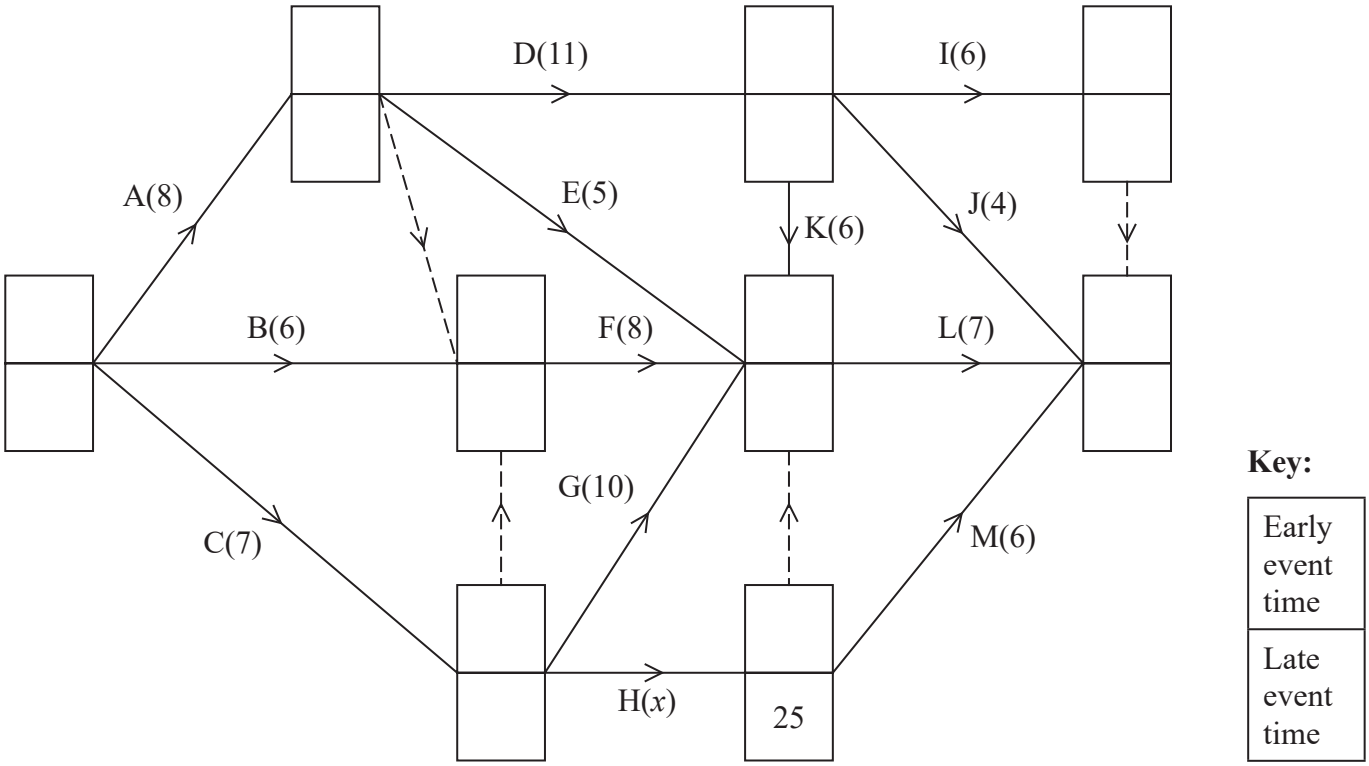


Figure 3

The network in Figure 3 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and one late event time has been completed for you.

The total float of activity H is 7 days.

(a) Explain, with detailed reasoning, why $x = 11$

(2)

(b) Determine the missing early event times and late event times, and hence complete Diagram 1 in your answer book.

(3)

Each activity requires one worker and the project must be completed in the shortest possible time using as few workers as possible.

(c) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time.

(1)

(d) Schedule the activities using Grid 1 in the answer book.

(3)

(Total for Question 4 is 9 marks)

5.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	C
F	B, C, D
G	A
H	B, C, D
I	B, C, D, G
J	B, C, D, G
K	E, H

(a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain only the minimum number of dummies.

(5)

Given that all the activities shown in the precedence table have the same duration,

(b) state the critical path for the network.

(1)

(Total for Question 5 is 6 marks)

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6. A linear programming problem in x, y and z is described as follows.

Maximise $P = 2x + 2y - z$

subject to $3x + y + 2z \leqslant 30$

$x - y + z \geqslant 8$

$4y + 2z \geqslant 15$

$x, y, z \geqslant 0$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

(7)

After a first iteration of the big-M method, the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	0	1.5	1	0	0.25	0	−0.25	26.25
a_1	1	0	1.5	0	−1	−0.25	1	0.25	11.75
y	0	1	0.5	0	0	−0.25	0	0.25	3.75
P	$-(2 + M)$	0	$2 - 1.5M$	0	M	$-0.5 + 0.25M$	0	$0.5 + 0.75M$	$7.5 - 11.75M$

(c) State the value of each variable after the first iteration.

(1)

(d) Explain why the solution given by the first iteration is not feasible.

(1)

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

(2)

(Total for Question 6 is 12 marks)

7. A shop sells two types of watch, analogue watches and digital watches.

The shop manager knows that, each month, she should order at least 60 watches in total. In addition, at most 80% of the watches she orders must be digital.

Let x be the number of analogue watches ordered and let y be the number of digital watches ordered.

(a) Write down inequalities, in terms of x and y, to model these constraints.

(2)

Two further constraints are

$$y + 3x \geqslant 140$$

$$4y + x \geqslant 80$$

(b) Represent all these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

The cost to the shop of ordering an analogue watch is five times the cost of ordering a digital watch. The shop manager wishes to minimise the total cost.

(c) Determine the number of each type of watch the shop manager should order. You must make your method clear.

(3)

Given that the minimum total cost of ordering the watches is £4455

(d) determine the cost of ordering one analogue watch and the cost of ordering one digital watch. You must make your method clear.

(3)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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Please check the examination details below before entering your candidate information

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Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 24 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3D**

Further Mathematics

Advanced

Paper 3D: Decision Mathematics 1

Answer Book
Do not return the question paper with the answer book.

Total Marks

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1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

(a) Bin 1: 2.1 1.7 1.2

Bin 2: 3.0 1.9

Bin 3: 3.2 1.4 0.2

Bin 4: 3.3 1.5

(b) Quick sort, pivot = middle right item of each sublist (pivots ringed):

2.1	1.7	3.0	1.9	3.2	(1.2)	3.3	1.4	1.5	0.2	Pivot: 1.2
2.1	1.7	3.0	1.9	(3.2)	3.3	1.4	1.5	1.2	(0.2)	Pivots: 3.2, 0.2
(3.3)	3.2	2.1	1.7	3.0	(1.9)	1.4	1.5	1.2	0.2	Pivots: 3.3, 1.9
3.3	3.2	2.1	(3.0)	1.9	1.7	(1.4)	1.5	1.2	0.2	Pivots: 3.0, 1.4
3.3	3.2	3.0	(2.1)	1.9	1.7	(1.5)	1.4	1.2	0.2	Pivots: 2.1, 1.5
3.3	3.2	3.0	2.1	1.9	1.7	1.5	1.4	1.2	0.2	Sort complete

(1.7 is the only item left unsorted, so the list is now sorted)

(c) Time is proportional to n log n

$t = 2.32 \times (11\,250 \log 11\,250) \div (450 \log 450)$

$= 2.32 \times 38.172...$

$= 88.559...$

$t = 88.6$ seconds

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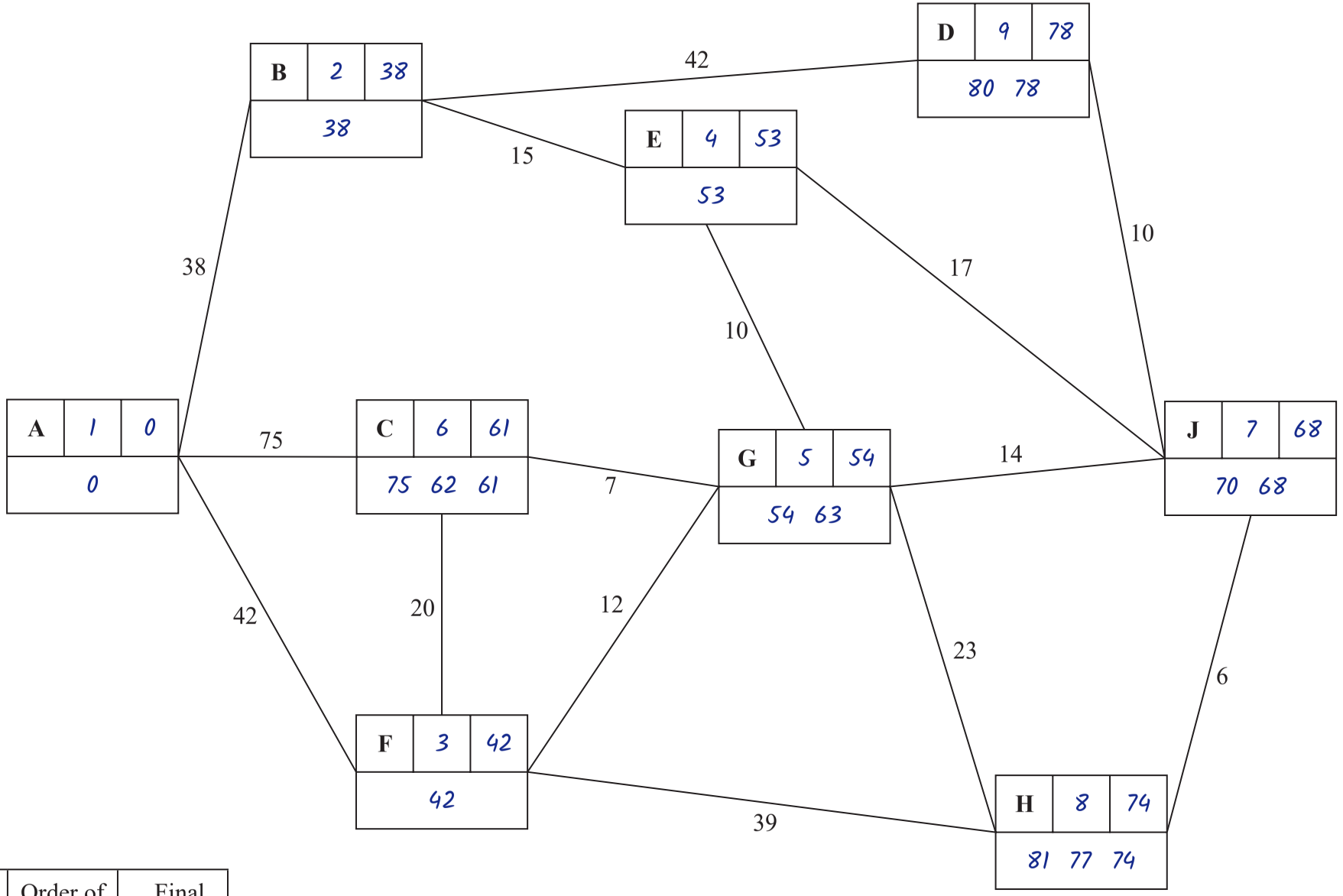
Question 1 continued

2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

(Total for Question 1 is 8 marks)

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2.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to D: AFGJD

Length of shortest path from A to D: 78 metres

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Question 2 continued

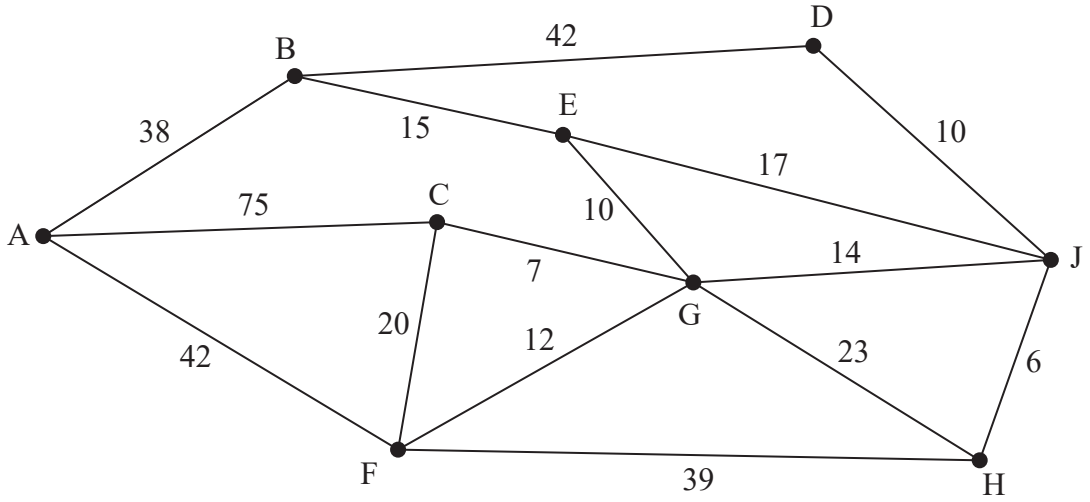


Figure 1

(b) Odd vertices: A, B, C, E, G, H. The route starts at B and ends at G, so B and G may stay odd. Pair up A, C, E and H:

AC + EH = 61 + 23 = 84 (AFGC, EJH)

AE + CH = 53 + 27 = 80 (ABE, CGJH)

AH + CE = 74 + 17 = 91 (AFGJH, CGE)

Least is AE + CH = 80

Arcs traversed twice: AB, BE, CG, GJ and JH

(c) Length of route = 370 + 80 = 450 metres

(d)(i) Removing AB, BD and BE changes the degrees of A, D and E, so the odd vertices are now C, D, G and H only.

Shortest paths between them:

CD = 31, CG = 7, CH = 27, DG = 24, DH = 16, GH = 20

The shortest of these is CG = 7, so CG is repeated and the route must start and finish at the other two odd vertices:

start at D and finish at H (or start at H and finish at D).

(ii) Length = 370 − 38 − 42 − 15 + 7 = 282 metres

Question 2 continued

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Question 2 continued

(Total for Question 2 is 14 marks)

3. (a)

Initial distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	∞	∞	∞
C	7	5	–	4	9
D	18	∞	4	–	3
E	∞	∞	9	3	–

Initial route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b) 1st iteration

Distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	22	33	18
C	7	5	–	4	9
D	18	33	4	–	3
E	∞	∞	9	3	–

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

2nd iteration

Distance table

	A	B	C	D	E
A	–	15	7	18	3
B	15	–	22	33	18
C	7	5	–	4	9
D	18	33	4	–	3
E	∞	∞	9	3	–

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

3rd iteration

Distance table

	A	B	C	D	E
A	–	12	7	11	3
B	15	–	22	26	18
C	7	5	–	4	9
D	11	9	4	–	3
E	16	14	9	3	–

Route table

	A	B	C	D	E
A	A	C	C	C	E
B	A	B	A	C	A
C	A	B	C	D	E
D	C	C	C	D	E
E	C	C	C	D	E

Turn to page 11 for spare copies of these tables if you need to correct your work.

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Question 3 continued

(c)(i) In the final route table, row E column A is D, so the shortest route from E to A goes from E to D first.

Row E column D is D, so E to D is the direct road ED.

Row D column A is C, so from D the route goes to C next.

Row C column A is A, so C to A is the direct road CA.

(ii) Route: E D C A

(d)(i) From D nearest is E (3); from E nearest unvisited is C (7); from C nearest unvisited is B (5); then A (15); return A to D (6).

Cycle: D - E - C - B - A - D

(ii) Length = 3 + 7 + 5 + 15 + 6 = 36 miles

(iii) E to C is actually via D and A to D is actually via E, so the villages visited are D - E - D - C - B - A - E - D

(D is visited three times and E twice).

(iv) e.g. the cycle A - E - D - C - B - A has length 3 + 3 + 4 + 5 + 15 = 30 miles, and 30 < 36, so Mabintou's cycle is not optimal.

Question 3 continued

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Question 3 continued

Only use these tables if you need to correct your work.

(a)

Initial distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Initial route table

	A	B	C	D	E
A					
B					
C					
D					
E					

(b) 1st iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

2nd iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

3rd iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

(Total for Question 3 is 14 marks)

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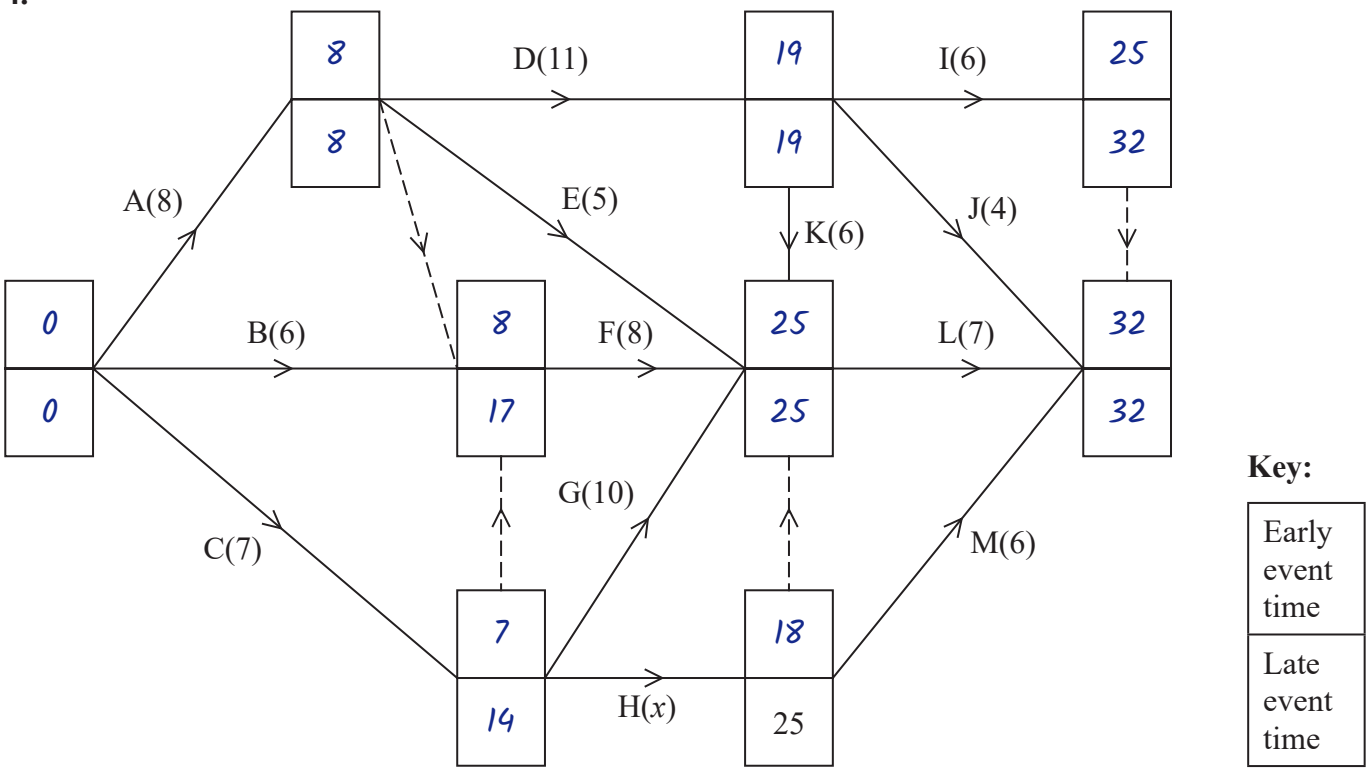


Diagram 1

(a) C is the only activity leading into the event at the end of C,
so the early event time at the start of H is 7.
Total float of H = late time at end − early time at start − duration
$= 25 - 7 - x$
Total float = 7, so $25 - 7 - x = 7 \Rightarrow 18 - x = 7 \Rightarrow x = 11$

(b) See Diagram 1.

(c) Total of durations:
$8 + 6 + 7 + 11 + 5 + 8 + 10 + 11 + 6 + 4 + 6 + 7 + 6 = 95$
Lower bound $= 95 \div 32 = 2.968...$
so at least 3 workers are needed.

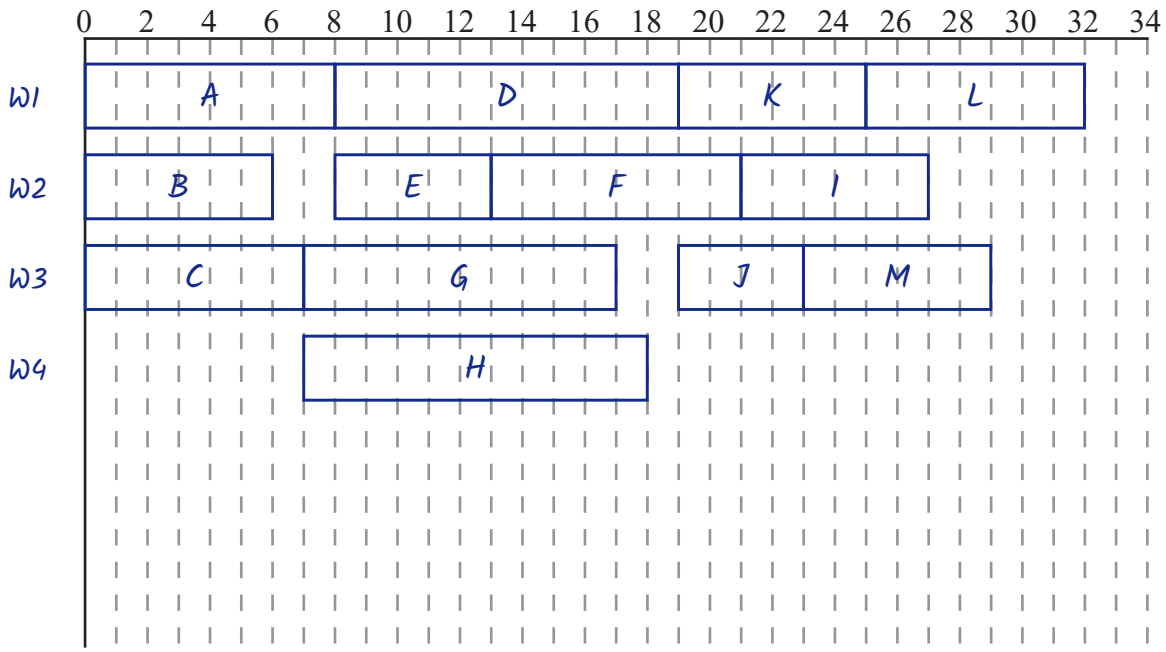
(d) See Grid 1.

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Question 4 continued

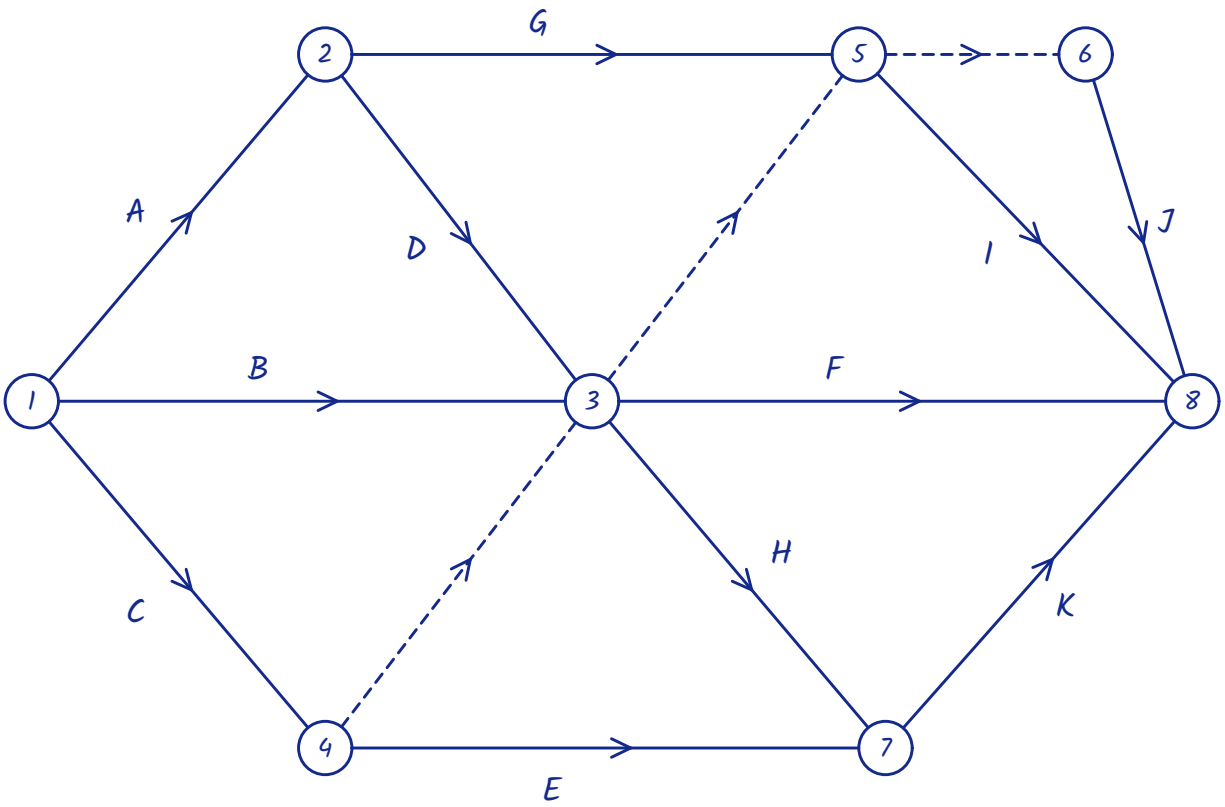


Grid 1

(Total for Question 4 is 9 marks)

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5.



(dummies shown dashed)

(b) Critical path: A D H K

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Question 5 continued

(Total for Question 5 is 6 marks)

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6.

(a) The constraints are not all of the form ≤ (two of them are ≥), so the origin is not a feasible solution and there is no initial basic feasible solution from which to start the Simplex algorithm.

(b) $3x + y + 2z \leqslant 30 \Rightarrow 3x + y + 2z + s_1 = 30$

$x - y + z \geqslant 8 \Rightarrow x - y + z - s_2 + a_1 = 8$

$4y + 2z \geqslant 15 \Rightarrow 4y + 2z - s_3 + a_2 = 15$

$P = 2x + 2y - z - M(a_1 + a_2)$

$a_1 + a_2 = 23 - x - 3y - 3z + s_2 + s_3$

$P = 2x + 2y - z - M(23 - x - 3y - 3z + s_2 + s_3)$

$P = (2 + M)x + (2 + 3M)y + (3M - 1)z - Ms_2 - Ms_3 - 23M$

$P - (2 + M)x - (2 + 3M)y - (3M - 1)z + Ms_2 + Ms_3 = -23M$

Initial tableau: see (b) on the next page.

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Question 6 continued

(b)

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	1	2	1	0	0	0	0	30
a_1	1	−1	1	0	−1	0	1	0	8
a_2	0	4	2	0	0	−1	0	1	15
P	$-(2 + M)$	$-(2 + 3M)$	$-(3M - 1)$	0	M	M	0	0	$-23M$

Only use the tableau below if you need to correct the above tableau.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
									30
									8
									15
P									

Question 6 continued

(c) $s_1 = 26.25$, $a_1 = 11.75$, $y = 3.75$,

$x = 0$, $z = 0$, $s_2 = 0$, $s_3 = 0$, $a_2 = 0$

(d) $a_1 = 11.75$, which is not zero. a_1 is an artificial variable and must be zero in a feasible solution, so the solution is not feasible.

(e) The most negative entry in the profit row is $2 - 1.5M$ (for large M, $2 - 1.5M < -(2 + M)$), so the pivot is in the z column.

Ratios (value ÷ z entry):

s_1 row: $26.25 \div 1.5 = 17.5$; a_1 row: $11.75 \div 1.5 = 7.833...$;

y row: $3.75 \div 0.5 = 7.5$

7.5 is the smallest ratio (less than 17.5 and 7.833...), so the pivot is the 0.5 in the y row (z column).

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Question 6 continued

(Total for Question 6 is 12 marks)

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7.

(a) $x + y \geqslant 60$

$y \leqslant 0.8(x + y)$ i.e. $y \leqslant 4x$

(b) See Diagram 1.

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Question 7 continued

Turn to page 24 for a spare copy of these axes if you need to correct your work.

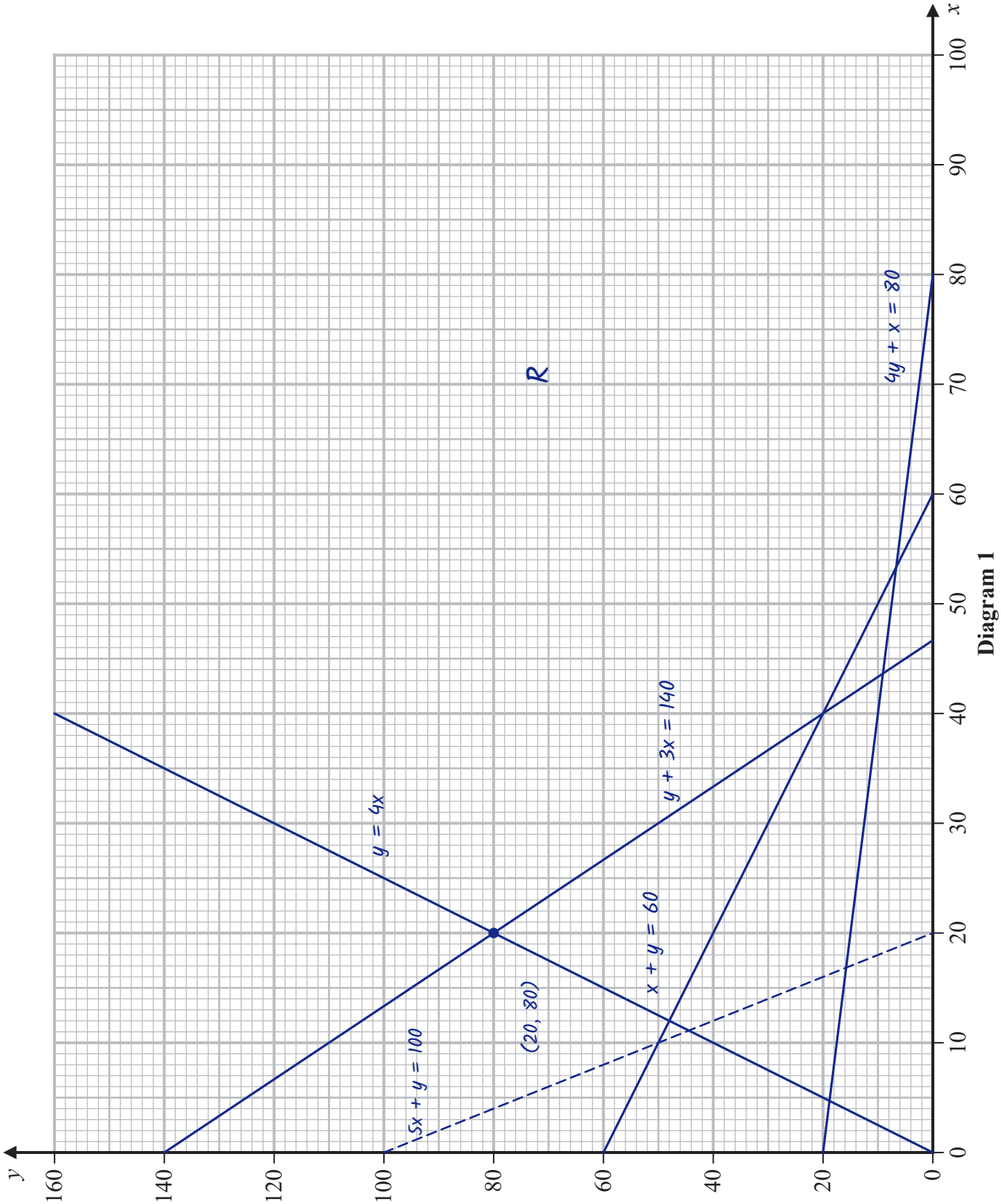


Diagram 1

P 6 1 1 8 2 A 0 2 1 2 4
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Question 7 continued

(c) Let a digital watch cost £d, so an analogue watch costs £5d.

Minimise $C = 5dx + dy$, i.e. minimise $5x + y$.

Objective line $5x + y = 100$ drawn (dashed) on Diagram 1 and moved away from the origin: the first vertex of R reached is (20, 80).

Check vertices: (20, 80): $5(20) + 80 = 180$

(40, 20): $5(40) + 20 = 220$

(80, 0): $5(80) + 0 = 400$

Minimum at (20, 80):

order 20 analogue watches and 80 digital watches.

(d) $20a + 80d = 4455$ and $a = 5d$

$20(5d) + 80d = 4455 \Rightarrow 180d = 4455 \Rightarrow d = 4455 \div 180 = 24.75$

$a = 5 \times 24.75 = 123.75$

An analogue watch costs £123.75 and a digital watch costs £24.75.

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Question 7 continued

Question 7 continued

Only use these axes if you need to correct your work.

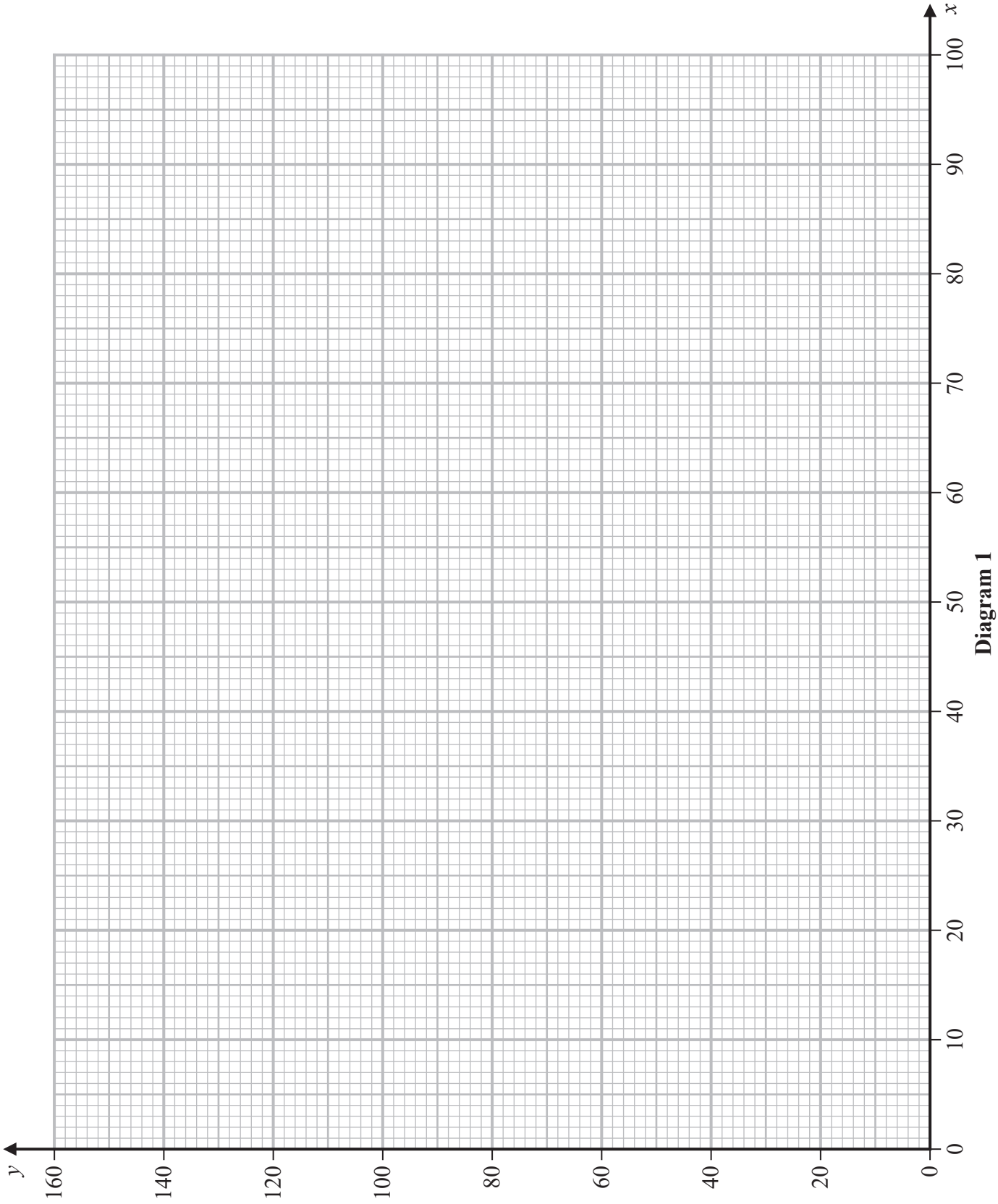


Diagram 1

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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