



Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Further Mathematics (9FM0)
Paper 3B: Further Statistics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

AL FM Stats 1 1906 Mark Scheme Final

Qu	Scheme	Marks	AO
1(a)	[Let X = no. of prizes Andreia wins] $X \sim B(40, 0.02)$	M1	3.3
	[Require $P(X \geq 3) = 1 - P(X \leq 2)$] = 0.04567... awrt <u>0.0457</u>	A1	1.1b
(b)	[Let Y = no. of the bar when Barney wins] $Y \sim \text{NegBin}(3, 0.02)$	(2) M1	3.3
	$[P(Y = 40) =] \binom{39}{2} \times 0.02^2 \times 0.98^{37} \times 0.02$	M1	3.4
	= 0.0028071... awrt <u>0.00281</u>	A1	1.1b
		(3)	
(c)	$E(Y) = \frac{3}{0.02} = \underline{\underline{150}}$	B1	1.1b
		(1)	
		(6 marks)	
Notes			
(a)	M1 for selecting a suitable model i.e. $B(40, p)$ where p is any probability Written or used, may be implied by a correct ans or 0.037429... from $P(X = 3)$		
	A1 for awrt 0.0457 (correct answer only 2/2)		
(b)	1 st M1 for selecting a suitable model ($NB(3, 0.02)$) May be implied by a correct expression 2 nd M1 for use of model to form a correct expression		
SC	$p \neq 0.02$ Allow prob of the form $\binom{39}{2} p^3 (1 - p)^{37}$ where $0 < p < 1$ scores M0M1		
	A1 for awrt 0.00281 (accept awrt 2.81×10^{-3}) [correct answer with no working scores 3/3]		
(c)	B1 for 150		

Qu	Scheme	Marks	AO
2(a)	{Let C = no of calls in a 20 min period} $C \sim \text{Po}(\dots)$	M1	3.3
	80 calls per 4-hour period gives $\frac{20}{3}$ per 20 mins i.e. $C \sim \text{Po}(\frac{20}{3})$	M1	3.4
	$[P(C > 4)] = 1 - P(C \leq 4)$ $= 0.79437\dots$ awrt 0.794	A1 (3)	1.1b
(b)	{ X = no. of 5 min periods with no calls } $X \sim B(4, e^{-\frac{5}{3}})$	M1	3.3
	$P(X = 3) = 0.02186125\dots$ awrt 0.0219	A1 (2)	1.1b
(c)	P(exactly one call) $e^{-\frac{5}{3}} \times \frac{5}{3}$ or $e^{-5} \times 5$	M1	2.1
	$P(\text{exactly one call in each break}) = \left(e^{-\frac{5}{3}} \times \frac{5}{3} \right) \times (e^{-5} \times 5)$	M1	1.1b
	$= 0.0106052\dots$ awrt 0.0106	A1 (3)	1.1b
(8 marks)			
Notes			
(a)	1 st M1 for selecting a Poisson model – written or used. May be implied by 2 nd M1 or a correct Answer.		
	2 nd M1 for the correct Poisson $\text{Po}(\frac{20}{3})$ or $\text{Po}(6.67)$ or better seen		
	and writing or using $1 - P(C \leq 4)$		
	A1 for awrt 0.794 (correct ans with no incorrect working scores 3/3)		
(b)	M1 for selecting a correct model $B(4, 0.189)$ or better (calc: 0.188875...)		
	A1 for using the model to get awrt 0.0219 (correct ans with no incorrect working scores 2/2)		
(c)	1 st M1 for a correct prob of 1 call (expressions in e or values) (allow 0.31479... or awrt 0.315 or 0.033689... or awrt 0.0337)		
	2 nd M1 for a correct probability statement or expression. E.g. $P(S = 1 S \sim \text{Po}(\frac{5}{3})) \times P(T = 1 T \sim \text{Po}(5))$		
SC	e.g. $F \sim \text{Po}(\lambda)$ used in (b) to find $P(F = 0)$		
	Then if we see $Y \sim \text{Po}(3\lambda)$ and statement $P(F = 1) \times P(Y = 1)$ award M0M1		
	A1 for awrt 0.0106 (correct ans with no incorrect working scores 3/3)		

Qu	Scheme	Marks	AO
3.	{ Let X = the number when the spinner is spun} $\mu = \underline{3}$ $[E(X^2) =] 0.3 + 4 \times 0.1 + 9 \times 0.2 + 16 \times 0.1 + 25 \times 0.3 [= 11.6 \text{ or } \frac{58}{5}]$ $\sigma^2 [= 11.6 - 3^2 =] \underline{2.6}$ $\bar{X} \approx \sim N \left("3", \sqrt{\frac{"2.6"}{80}}^2 \right)$ $P(\bar{X} > 3.25) = [P(Z > 1.3867...) =] 0.0827589... (\text{calc})$ awrt <u>0.0828</u>	B1 M1 A1 M1 A1ft A1	1.1b 1.1b 1.1b 2.1 1.1b 3.4
(6 marks)			
Notes			
ALT	B1 for stating or using mean = 3 1st M1 for using the given model to attempt $E(X^2)$ with at least 3 correct products seen 1st A1 for $\text{Var}(X) = 2.6$ or $\sigma = \sqrt{2.6} = 1.6124... (\text{awrt } 1.61)$ Use of pgf (B1 when mean = 3 seen) (M1 when correct $G''(t)$ seen with attempt at $G''(1)$) $G(t) = 0.3t + 0.1t^2 + 0.2t^3 + 0.1t^4 + 0.3t^5$ $G'(t) = 0.3 + 0.2t + 0.6t^2 + 0.4t^3 + 1.5t^4$ $G''(t) = 0.2 + 1.2t + 1.2t^2 + 6t^3$ leading to $G''(1) = 8.6$ 2nd M1 for use of CLT – must use $\bar{X}$ and normal <u>or</u> sight of $N \left("3", \sqrt{\frac{"2.6"}{80}}^2 \right)$ with any letter 2nd A1ft for a correct mean and variance, ft their 3 and their 2.6 This M1A1ft may be implied by sight of correct st. dev. used in a standardisation leading to $P(Z > 1.39)$ Must see correct use of Z NB $\frac{2.6}{80} = 0.0325$ and $\sqrt{\frac{2.6}{80}} = 0.18027... \text{ so allow e.g. } N(3, \text{awrt } (0.180)^2)$ 3rd A1 for using the normal model to find probability awrt 0.0828		
ALT	Use of $\sum X$ (If see clear attempt at $P(\sum X > 260)$ condone $P(\sum X > 260.5)$ then: 2nd M1 for $\sum X \sim N(...)$ <u>or</u> any letter $\sim N("240", \sqrt{"2.6" \times 80}^2)$ 2nd A1ft for mean = "3" $\times 80 = 240$ <u>and</u> variance = "2.6" $\times 80 = 208$ May see $P(\sum X > 260.5) = 0.077597... \text{ but it will only score } 2^{\text{nd}} \text{ M1 } 2^{\text{nd}} \text{ A1ft and } \mathbf{3^{\text{rd}} A0}$		

Qu	Scheme	Marks	AO												
4(a)	[T = no. of oak trees in a square] $T \sim \text{Binomial}$ $T \sim B(6, p)$	M1 A1 (2)	3.3 1.1b												
(b)	Expected frequency for 6 is less than 5 so pool: new $E_i = 13.08$ <table><tr><td>$\frac{(O_i - E_i)^2}{E_i}$</td><td>0.051</td><td>2.51</td><td>0.0654</td><td>3.84</td><td>1.85</td></tr><tr><td>$\frac{O_i^2}{E_i}$</td><td>4.521</td><td>29.617</td><td>21.805</td><td>7.599</td><td>24.771</td></tr></table> $\sum \frac{(O_i - E_i)^2}{E_i} = 8.313$ p needed estimating ($\hat{p} = 0.55$) so $\nu = 5 - 2 = 3$; cv 7.815 Significant result, so Liam's <u>model is not suitable</u>	$\frac{(O_i - E_i)^2}{E_i}$	0.051	2.51	0.0654	3.84	1.85	$\frac{O_i^2}{E_i}$	4.521	29.617	21.805	7.599	24.771	M1 M1,A1 B1,B1ft M1,A1 (7)	2.1 1.1b x2 1.1b x2 1.1b2.2b
$\frac{(O_i - E_i)^2}{E_i}$	0.051	2.51	0.0654	3.84	1.85										
$\frac{O_i^2}{E_i}$	4.521	29.617	21.805	7.599	24.771										
(c)	[R = no. of oak trees in a square for Simone's model] $R \sim \text{Po}(3.3)$ Correct expression for s or t using Poisson $s = \underline{17.67}$ and $t = \underline{9.62}$	M1 M1 A1,A1 (4)	3.3 3.4 1.1b x2												
(d)	H_0 : Poisson is a good fit (for no. of oak trees per square) H_1 : Poisson is not a good fit (for no. of oak trees per square)	B1 (1)	2.5												
(e)	No pooling needed so degrees of freedom is $6 - 2 = 4$ Critical value is 9.488 (accept 9.49) Not significant so Poisson (or Simone's) model is suitable	B1 B1 B1 (3)	1.1b 1.1a 2.2b												
(f)	Poisson model has better fit so suggests that oak trees occur at random <u>Or</u> binomial suggests deliberately planted or cultivated Therefore the forest is likely to be wild not cultivated	B1 B1 (2)	2.2b 3.5a												
		(19 marks)													
Notes															
(a)	M1 for choosing binomial A1 for $B(6, p)$ can be in words and allow $B(6, 0.55)$														
(b)	1 st M1 for pooling last 2 classes ($E_i = 13.08$ but accept 13.1) 2 nd M1 for at least 3 correct values or expressions. Either row to at least 2 sf 1 st A1 for awrt 8.31 (8.31 gets 3/3) [NB no pooling gives awrt 16.8458.. and implies M0M1A0] 1 st B1 for 3 degrees of freedom 2 nd B1ft for critical value of 7.815 (e.g. $\nu = 4$ use 9.488) 3 rd M1 for a correct conclusion (non-contextual ignore any contradictory contextual comments for this mark) based on their cv and their test statistic This mark can be implied by a fully correct solution ending with correct contextual conclusion 2 nd A1 for correct conclusion in context with all other marks scored														
(c)	1 st M1 for selecting a correct model $\text{Po}(3.3)$ [Allow $\text{Po}(\text{awrt } 3.3)$] 2 nd M1 for use of the model with an expression or correct value for s or t 1 st A1 for one correct 2 nd A1 for both correct (allow awrt 2dp)														
(d)	B1 for correct hypotheses must mention Poisson: use of $\text{Po}(3.3)$ is B0														
(e)	1 st B1 for correct degrees of freedom $\nu = 4$ only 2 nd B1 for selecting correct critical value (9.488 only) 3 rd B1 for <u>not significant</u> conclusion based on 8.749 vs their cv (condone use of $\text{Po}(3.3)$ here)														
(f)	1 st B1 for choosing Poisson as better <u>or</u> stating Poisson implies wild <u>or</u> bino'l implies cultivated 2 nd B1 (dep on rejecting bin and accepting Poisson) for clearly stating woodland is wild If the tests give the same results then 2 nd B0 automatically														

Qu	Scheme	Marks	AO
5(a)	$H_0: \lambda = 2.5$ (or $\mu = 7.5$) $H_1: \lambda \neq 2.5$ (or $\mu \neq 7.5$)	B1	2.5
	[X = no. of accidents in a 3-month period] $X \sim \text{Po}(7.5)$	M1	3.3
	$P(X \leq 2) = 0.0203$ (calc: 0.020256...) { or $P(X \leq 3) = 0.0591$ }		
	$P(X \leq 13) = 0.9784$ so $P(X \geq 14) = 0.0216$ (calc: 0.0215646...)	M1	3.4
	{or $P(X \geq 15) = 0.0103$ }		
	Giving Critical region of: $X \leq 2$	A1	1.1b
	$X \geq 14$	A1	1.1b
		(5)	
	(b) [0.0203 + 0.0216] = awrt 0.0419 or (calc: 0.041821366... awrt 0.0418)	B1ft	1.2
		(1)	
(c)	[Let M = no of 3-month periods with a significant result]		
	$M \sim B(8, "0.0419")$	M1	3.3
	$[P(M \geq 2)] = 1 - P(M \leq 1)$	M1	1.1b
	$[= 1 - 0.9584...]$ $= 0.04153...$ (calc: 0.041394...) [0.04139~0.04154]	A1cso	1.1b
(d)		(3)	
	$Y \sim \text{Po}(6.3)$	M1	3.3
	$P(\text{Type II error}) = P(3 \leq Y \leq 13)$ or $P(Y \leq 13) - P(Y \leq 2)$	M1	3.4
	$[= 0.9945147... - 0.049846...]$ $= 0.9446...$ awrt 0.945	A1	1.1b
		(3)	
(12 marks)			
Notes			
(a)	B1 for both hypotheses in terms of λ or μ (either way around)		
	1 st M1 for selecting the correct Po model. Sight or use of Po(7.5) may be implied by 2 nd M1		
	2 nd M1 for using the correct model to find one of these probs with correct label (2sf or better)		
	1 st A1 for one end correct Allow any letter, even CR ≤ 2 or set notation but not $P(X \leq 2)$		
(b)	2 nd A1 for a fully correct CR Can have $X < 3$ and $X > 13$ etc		
	B1ft for awrt 0.0419 or awrt 0.0418		
(c)	or ft addition of their two probs provided both are $0 < \text{prob} < 0.025$ (awrt 3sf)		
	1 st M1 for selecting a correct binomial model, ft their answer to part (b)		
	2 nd M1 for a correct probability statement of $1 - P(M \leq 1)$ dep on a binomial selected		
(d)	A1cso for answer in range [0.04139, 0.04154] dep on use of B(8, "0.0419") or better		
	1 st M1 for selecting a Po(6.3) model		
	2 nd M1 for a correct probability statement using their Poisson model and their CR in (a) which may have just one tail.		
(e)	A1 for awrt 0.945		

Qu	Scheme	Marks	AO
6 (a)	$G(1) = 1 \Rightarrow k \ln 2 = 1 \text{ so } k = \frac{1}{\ln 2}$	B1	2.1
(b)	$\left\{ G(t) = \frac{1}{\ln 2} [\ln 2 - \ln(2-t)] \right\} \Rightarrow G'(t) = \frac{1}{\ln 2} \left[\frac{1}{2-t} \right] \text{ or } \frac{1}{\ln 2} (2-t)^{-1}$ $[E(X) =] G'(1) = \frac{1}{\ln 2}$ $G''(t) = \frac{1}{\ln 2} \times \left[\frac{1}{(2-t)^2} \right]$ $\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2 = \frac{1}{\ln 2} + \frac{1}{\ln 2} - \left(\frac{1}{\ln 2} \right)^2$ $= \frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$	(1) M1 A1 A1 M1 A1 M1 A1	2.1 1.1b 1.1b 2.1 1.1b 2.1 1.1b
(c)	$P(X = 3) = \text{coefficient of } t^3 \text{ by Maclaurin need } G'''(0)$ $G'''(t) = \frac{1}{\ln 2} \frac{2}{(2-t)^3}$ $P(X = 3) = \frac{G'''(0)}{3!}$ $= \frac{\frac{1}{4\ln 2}}{6} = \frac{1}{24\ln 2} = 0.0601122... \text{ awrt } \underline{\underline{0.0601}}$	(7) M1 A1ft M1 A1 (4)	3.1a 1.1b 3.2a 1.1b
(12 marks)			
Notes			
(a)	B1 for finding k (must be exact)		
(b)	1 st M1 for an attempt to differentiate $G(t)$ e.g. $A(2-t)^{-1}$ (o.e.) 1 st A1 for a correct first derivative (condone k or use of $\frac{1}{\ln 2} = \text{awrt } 1.44$) 2 nd A1 for correct $E(X)$ or $G'(1)$ (allow awrt 1.44 calc: 1.442695...but not k) seen anywhere 2 nd M1 for attempting second derivative (ft their $G'(t)$) 3 rd A1 for a correct 2 nd derivative (condone k or use of $\frac{1}{\ln 2} = \text{awrt } 1.44$) 3 rd M1 for a correct method for $\text{Var}(X)$ (some substitution into the correct formula) 4 th A1 for $\frac{1}{\ln 2} \left(2 - \frac{1}{\ln 2} \right)$ o.e. but must simplify i.e. collect like terms [Mark final answer – penalise incorrect log work etc] NB 0.8040211.. is A0 unless exact answer seen		
(c)	1 st M1 for a suitable strategy to solve the problem (finding link with Maclaurin) Need mention of coefficient of t^3 and $[G'''(t) \text{ or } G'''(0)]$ (condone $G'''(1)$) 1 st A1ft for 3 rd derivative, ft their 2 nd derivative in (b) (provided $G''(t)$ not const) Correct $G'''(t)$ or $G'''(0)$ scores 1 st M1 1 st A1ft 2 nd M1 for translating Maclaurin to probability (a correct expression) 2 nd A1 for $\frac{1}{24\ln 2}$ or awrt 0.0601		

ALT	Log series 1 st M1 attempt to write $G(t)$ in suitable form as far as: $k[\ln 2 - \ln(2[1 - \frac{t}{2}])]$
	1 st A1 reaching $-k \ln(1 - \frac{t}{2})$ 2 nd M1 use of $-\ln(1 - x)$ series (<u>some</u> correct substitution) NB $G(t) = \frac{1}{\ln 2} \left(\frac{t}{2} + \frac{t^2}{8} + \frac{t^3}{24} + \dots \right)$

Qu	Scheme	Marks	AO
7(a)(i)	$[B \sim \text{Geo}(\frac{1}{3})] \quad P(B = 4) = \left(\frac{2}{3}\right)^3 \times \frac{1}{3}$ $= \frac{8}{81}$	M1 A1	3.3 1.1b
(ii)	$P(B \leq 5) = 1 - P(B > 5) \quad \text{or} \quad 1 - \left(\frac{2}{3}\right)^5$ $= \frac{211}{243}$	M1 A1 (4)	2.1 1.1b
(b)	$E(B^2) = \text{Var}(B) + [E(B)]^2$ From formula booklet: $E(B) = \frac{1}{\frac{1}{3}} = 3$ and $\text{Var}(B) = \frac{1 - \frac{1}{3}}{\left(\frac{1}{3}\right)^2} = 6$ So $E(B^2) = 6 + 9 = \underline{15}$	M1 B1 A1 (3)	2.1 1.1b 1.1b
(c)	[Let R = no. of the spin when it first lands on red] $X = R \sim \text{Geo}(\frac{2}{3})$ Require $E(e^X) = \sum_{x=1}^{\infty} e^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$ $= \frac{2e}{3} \sum_{x=1}^{\infty} \left(\frac{e}{3}\right)^{x-1}$ $= \frac{2e}{3} \times \frac{1}{1 - \frac{e}{3}} \quad \text{or} \quad \frac{2e}{3 - e}$ $E(e^X) = 19.297 \dots \{ > 15 = E(B^2) \}$ so Tamara should choose red since it has the greater expected score	M1 M1 M1 A1 A1 (5)	3.3 3.1a 2.1 1.1b 2.2a
Notes			
(a)(i)	M1 for selecting the correct model i.e. $\text{Geo}(p)$ (May be implied by a correct expression) A1 for $\frac{8}{81}$ (= 0.098765... accept awrt 0.0988)		
(ii)	M1 for a suitable strategy to use the geometric model to find a correct expression A1 for $\frac{211}{243}$ (= 0.868312... accept awrt 0.868)		
(b)	M1 for a suitable strategy to find $E(B^2)$ [allow $G''(1) + G'(1)$] B1 for use of the correct formulae to find $E(B) = 3$ <u>and</u> $\text{Var}(B) = 6$ <u>or</u> $G''(1) = 12$ A1 for 15		
SC	Formula for $E(B^2)$ Allow M1B1A0 for $E(B^2) = \frac{2-p}{p^2}$ (o.e.)		

Qu7	Notes
(c)	1st M1 for choosing a suitable geometric model (sight of $\text{Geo}(\frac{2}{3})$ or at least 3 correct probabilities) 2nd M1 for realising the need for appropriate expected value and using $E(g(X))$ [Need sum and $f(x)$] NB simply finding $e^{E(X)} = e^{1.5} = \text{awrt } 4.48$ is M0 and probably no more marks. 3rd M1 for a suitable strategy to turn the expression into a sum that can be found 1st A1 for correct use of sum to infinity of geometric series 2nd A1 for interpreting the outcome of the calculations in terms of a solution to the problem must choose red and see the awrt 19.3 (and allow ft of their $E(B^2) < 19$)

