Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

# Thursday 13 June 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3A**

# Further Mathematics

## Advanced

## Paper 3A: Further Pure Mathematics 1

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**Answer ALL questions. Write your answers in the spaces provided.**

**1.** Use Simpson's rule with 4 intervals to estimate

$$\int_{0.4}^{2} e^{x^2} dx$$

**(5)**

$h = \frac{2 - 0.4}{4} = 0.4$

$x_0 = 0.4$: $y_0 = e^{0.16} = 1.173510...$

$x_1 = 0.8$: $y_1 = e^{0.64} = 1.896480...$

$x_2 = 1.2$: $y_2 = e^{1.44} = 4.220695...$

$x_3 = 1.6$: $y_3 = e^{2.56} = 12.93581...$

$x_4 = 2$: $y_4 = e^{4} = 54.59815...$

$\int_{0.4}^{2} e^{x^2} dx \approx \frac{h}{3}[y_0 + y_4 + 4(y_1 + y_3) + 2y_2]$

$= \frac{0.4}{3}[1.173510 + 54.59815 + 4(1.896480 + 12.93581) + 2(4.220695)]$

$= \frac{0.4}{3} \times 123.5422...$

$= 16.47229...$

$\int_{0.4}^{2} e^{x^2} dx \approx 16.5$

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**Question 1 continued**

**(Total for Question 1 is 5 marks)**

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**2.** Given that $k$ is a real non-zero constant and that

$$y = x^3 \sin kx$$

use Leibnitz's theorem to show that

$$\frac{\mathrm{d}^5 y}{\mathrm{d}x^5} = (k^2x^2 + A)k^3 x \cos kx + B(k^2x^2 + C)k^2 \sin kx$$

where $A$, $B$ and $C$ are integers to be determined.

**(4)**

$u = x^3$: $u' = 3x^2$, $u'' = 6x$, $u''' = 6$, $u^{(4)} = u^{(5)} = 0$

$v = \sin kx$: $v' = k\cos kx$, $v'' = -k^2 \sin kx$, $v''' = -k^3 \cos kx$,

$v^{(4)} = k^4 \sin kx$, $v^{(5)} = k^5 \cos kx$

By Leibnitz's theorem:

$$\frac{d^5y}{dx^5} = uv^{(5)} + 5u'v^{(4)} + \frac{5 \times 4}{2!}u''v''' + \frac{5 \times 4 \times 3}{3!}u'''v'' \quad (+ 0)$$

$= x^3(k^5 \cos kx) + 5(3x^2)(k^4 \sin kx) + 10(6x)(-k^3 \cos kx)$

$\quad + 10(6)(-k^2 \sin kx)$

$= k^5x^3 \cos kx + 15k^4x^2 \sin kx - 60k^3x \cos kx - 60k^2 \sin kx$

$= (k^2x^2 - 60)k^3x \cos kx + 15(k^2x^2 - 4)k^2 \sin kx$

$\therefore \; A = -60, \quad B = 15, \quad C = -4$

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**Question 2 continued**

**(Total for Question 2 is 4 marks)**

**3.**

$$\frac{dy}{dx} = x - y^2 \qquad \text{(I)}$$

(a) Show that

$$\frac{d^5y}{dx^5} = ay\frac{d^4y}{dx^4} + b\frac{dy}{dx}\frac{d^3y}{dx^3} + c\left(\frac{d^2y}{dx^2}\right)^2$$

where $a$, $b$ and $c$ are integers to be determined.

**(4)**

(b) Hence find a series solution, in ascending powers of $x$ as far as the term in $x^5$, of the differential equation (I), given that $y = 1$ at $x = 0$

**(5)**

(a)

$\frac{dy}{dx} = x - y^2$

$\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$

$\frac{d^3y}{dx^3} = -2y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2$

$\frac{d^4y}{dx^4} = -2\frac{dy}{dx}\frac{d^2y}{dx^2} - 2y\frac{d^3y}{dx^3} - 4\frac{dy}{dx}\frac{d^2y}{dx^2} = -6\frac{dy}{dx}\frac{d^2y}{dx^2} - 2y\frac{d^3y}{dx^3}$

$\frac{d^5y}{dx^5} = -6\left(\frac{d^2y}{dx^2}\right)^2 - 6\frac{dy}{dx}\frac{d^3y}{dx^3} - 2\frac{dy}{dx}\frac{d^3y}{dx^3} - 2y\frac{d^4y}{dx^4}$

$= -2y\frac{d^4y}{dx^4} - 8\frac{dy}{dx}\frac{d^3y}{dx^3} - 6\left(\frac{d^2y}{dx^2}\right)^2$

$\therefore \; a = -2, \quad b = -8, \quad c = -6$

(b)

At $x = 0$, $y = 1$:

$y'(0) = 0 - 1^2 = -1$

$y''(0) = 1 - 2(1)(-1) = 3$

$y'''(0) = -2(1)(3) - 2(-1)^2 = -8$

$y^{(4)}(0) = -6(-1)(3) - 2(1)(-8) = 34$

$y^{(5)}(0) = -2(1)(34) - 8(-1)(-8) - 6(3)^2 = -186$

$y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{(4)}(0) + \frac{x^5}{5!}y^{(5)}(0) + \ldots$

$y = 1 - x + \frac{3}{2}x^2 - \frac{8}{6}x^3 + \frac{34}{24}x^4 - \frac{186}{120}x^5 + \ldots$

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**Question 3 continued**

$y = 1 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3 + \frac{17}{12}x^4 - \frac{31}{20}x^5 + \ldots$

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**Question 3 continued**

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**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4.** The parabola $C$ has equation

$$y^2 = 16x$$

The distinct points $P(p^2, 4p)$ and $Q(q^2, 4q)$ lie on $C$, where $p \neq 0$, $q \neq 0$

The tangent to $C$ at $P$ and the tangent to $C$ at $Q$ meet at the point $R(-28, 6)$.

Show that the area of triangle $PQR$ is 1331

**(8)**

$y^2 = 16x \Rightarrow 2y\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y}$

At $P(p^2, 4p)$: $\frac{dy}{dx} = \frac{8}{4p} = \frac{2}{p}$

Tangent at P: $y - 4p = \frac{2}{p}(x - p^2)$

$\Rightarrow py - 4p^2 = 2x - 2p^2 \Rightarrow py = 2x + 2p^2$

Similarly, tangent at Q: $qy = 2x + 2q^2$

$R(-28, 6)$ lies on both tangents:

$6p = -56 + 2p^2 \Rightarrow p^2 - 3p - 28 = 0$

$(p - 7)(p + 4) = 0 \Rightarrow p = 7$ or $p = -4$

$q$ satisfies the same equation and P, Q are distinct, so $p = 7$, $q = -4$

$P(49, 28)$, $Q(16, -16)$

Area of triangle with vertices $R(-28, 6)$, $P(49, 28)$, $Q(16, -16)$:

Area $= \frac{1}{2}|-28(28 - (-16)) + 49(-16 - 6) + 16(6 - 28)|$

$= \frac{1}{2}|-28(44) + 49(-22) + 16(-22)|$

$= \frac{1}{2}|-1232 - 1078 - 352|$

$= \frac{1}{2} \times 2662$

$= 1331$ as required

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**Question 4 continued**

**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 8 marks)**





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**5.**

$$I = \int \frac{1}{4\cos x - 3\sin x}\,\mathrm{d}x \qquad 0 < x < \frac{\pi}{4}$$

Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that

$$I = \frac{1}{5}\ln\left(\frac{2 + \tan\left(\frac{x}{2}\right)}{1 - 2\tan\left(\frac{x}{2}\right)}\right) + k$$

where $k$ is an arbitrary constant.

**(8)**

$t = \tan\left(\frac{x}{2}\right) \Rightarrow \frac{dt}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) = \frac{1 + t^2}{2} \Rightarrow dx = \frac{2}{1 + t^2}\,dt$

$\cos x = \frac{1 - t^2}{1 + t^2}, \qquad \sin x = \frac{2t}{1 + t^2}$

$4\cos x - 3\sin x = \frac{4(1 - t^2) - 3(2t)}{1 + t^2} = \frac{4 - 6t - 4t^2}{1 + t^2}$

$I = \int \frac{1 + t^2}{4 - 6t - 4t^2} \times \frac{2}{1 + t^2}\,dt$

$= \int \frac{2}{4 - 6t - 4t^2}\,dt = \int \frac{1}{2 - 3t - 2t^2}\,dt = \int \frac{1}{(2 + t)(1 - 2t)}\,dt$

Partial fractions: $\frac{1}{(2 + t)(1 - 2t)} \equiv \frac{A}{2 + t} + \frac{B}{1 - 2t}$

$1 \equiv A(1 - 2t) + B(2 + t)$

$t = -2$: $1 = 5A \Rightarrow A = \frac{1}{5}$; $\qquad t = \frac{1}{2}$: $1 = \frac{5}{2}B \Rightarrow B = \frac{2}{5}$

$I = \int \left(\frac{1}{5(2 + t)} + \frac{2}{5(1 - 2t)}\right) dt$

$= \frac{1}{5}\ln|2 + t| - \frac{1}{5}\ln|1 - 2t| + k$

For $0 < x < \frac{\pi}{4}$: $0 < t < \tan\left(\frac{\pi}{8}\right) < \frac{1}{2}$, so $2 + t > 0$ and $1 - 2t > 0$

$I = \frac{1}{5}\ln\left(\frac{2 + t}{1 - 2t}\right) + k$

$\therefore I = \frac{1}{5}\ln\left(\frac{2 + \tan(x/2)}{1 - 2\tan(x/2)}\right) + k$ as required

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**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 8 marks)**

**6.** The concentration of a drug in the bloodstream of a patient, $t$ hours after the drug has been administered, where $t \leqslant 6$, is modelled by the differential equation

$$t^2\frac{\mathrm{d}^2C}{\mathrm{d}t^2} - 5t\frac{\mathrm{d}C}{\mathrm{d}t} + 8C = t^3 \qquad \text{(I)}$$

where $C$ is measured in micrograms per litre.

(a) Show that the transformation $t = \mathrm{e}^x$ transforms equation (I) into the equation

$$\frac{\mathrm{d}^2C}{\mathrm{d}x^2} - 6\frac{\mathrm{d}C}{\mathrm{d}x} + 8C = \mathrm{e}^{3x} \qquad \text{(II)}$$

**(5)**

(b) Hence find the general solution for the concentration $C$ at time $t$ hours.

**(7)**

Given that when $t = 6$, $C = 0$ and $\dfrac{\mathrm{d}C}{\mathrm{d}t} = -36$

(c) find the maximum concentration of the drug in the bloodstream of the patient.

**(5)**

(a)

$t = e^x \Rightarrow \dfrac{dt}{dx} = e^x = t$

$\dfrac{dC}{dx} = \dfrac{dC}{dt}\dfrac{dt}{dx} = t\dfrac{dC}{dt}$

$\dfrac{d^2C}{dx^2} = \dfrac{d}{dx}\left(t\dfrac{dC}{dt}\right) = \dfrac{dt}{dx}\dfrac{dC}{dt} + t\dfrac{d^2C}{dt^2}\dfrac{dt}{dx} = t\dfrac{dC}{dt} + t^2\dfrac{d^2C}{dt^2}$

$\Rightarrow t^2\dfrac{d^2C}{dt^2} = \dfrac{d^2C}{dx^2} - \dfrac{dC}{dx}$ and $t\dfrac{dC}{dt} = \dfrac{dC}{dx}$

Substituting into (I), with $t^3 = e^{3x}$:

$\dfrac{d^2C}{dx^2} - \dfrac{dC}{dx} - 5\dfrac{dC}{dx} + 8C = e^{3x}$

$\dfrac{d^2C}{dx^2} - 6\dfrac{dC}{dx} + 8C = e^{3x}$ (II) as required

(b)

Auxiliary equation: $m^2 - 6m + 8 = 0 \Rightarrow (m-2)(m-4) = 0$

$m = 2, 4 \Rightarrow$ CF: $C = Ae^{4x} + Be^{2x}$

PI: try $C = ke^{3x}$

$\dfrac{dC}{dx} = 3ke^{3x}$, $\dfrac{d^2C}{dx^2} = 9ke^{3x}$

$9ke^{3x} - 18ke^{3x} + 8ke^{3x} = e^{3x} \Rightarrow -k = 1 \Rightarrow k = -1$

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**Question 6 continued**

$C = Ae^{4x} + Be^{2x} - e^{3x}$

Since $t = e^x$: $C = At^4 + Bt^2 - t^3$

(c)

$t = 6, C = 0$: $1296A + 36B - 216 = 0 \Rightarrow 36A + B = 6$

$\frac{dC}{dt} = 4At^3 + 2Bt - 3t^2$

$t = 6, \frac{dC}{dt} = -36$: $864A + 12B - 108 = -36 \Rightarrow 72A + B = 6$

Subtracting: $36A = 0 \Rightarrow A = 0, \quad B = 6$

$C = 6t^2 - t^3$

$\frac{dC}{dt} = 12t - 3t^2 = 3t(4 - t) = 0 \Rightarrow t = 4$ ($t = 0$ gives $C = 0$)

$\frac{d^2C}{dt^2} = 12 - 6t = -12 < 0$ at $t = 4$, so this is a maximum

$C = 6(4)^2 - 4^3 = 96 - 64 = 32$

Maximum concentration = 32 μg/L (micrograms per litre)





**Question 6 continued**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 17 marks)**

**7.** With respect to a fixed origin $O$, the points $A$, $B$ and $C$ have coordinates (3, 4, 5), (10, –1, 5) and (4, 7, –9) respectively.

The plane $\Pi$ has equation $4x - 8y + z = 2$

The line segment $AB$ meets the plane $\Pi$ at the point $P$ and the line segment $BC$ meets the plane $\Pi$ at the point $Q$.

(a) Show that, to 3 significant figures, the area of quadrilateral $APQC$ is 38.5

**(6)**

The point $D$ has coordinates $(k, 4, -1)$, where $k$ is a constant.

Given that the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ form three edges of a parallelepiped of volume 226

(b) find the possible values of the constant $k$.

**(4)**

(a)

Area APQC = Area ABC − Area PBQ

Line AB: $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -5 \\ 0 \end{pmatrix}$

On Π: $4(3 + 7\lambda) - 8(4 - 5\lambda) + 5 = 2$

$68\lambda - 15 = 2 \Rightarrow \lambda = \frac{1}{4} \Rightarrow P(4.75, 2.75, 5)$

Line BC: $\mathbf{r} = \begin{pmatrix} 10 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ 8 \\ -14 \end{pmatrix}$

On Π: $4(10 - 6\mu) - 8(-1 + 8\mu) + (5 - 14\mu) = 2$

$53 - 102\mu = 2 \Rightarrow \mu = \frac{1}{2} \Rightarrow Q(7, 3, -2)$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 7 \\ -5 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ -14 \end{pmatrix} = \begin{pmatrix} 70 \\ 98 \\ 26 \end{pmatrix}$

Area ABC $= \frac{1}{2}\sqrt{(70^2 + 98^2 + 26^2)} = \frac{1}{2}\sqrt{15180} = 61.6035...$

$\overrightarrow{BP} = \begin{pmatrix} -5.25 \\ 3.75 \\ 0 \end{pmatrix}$, $\overrightarrow{BQ} = \begin{pmatrix} -3 \\ 4 \\ -7 \end{pmatrix}$, $\overrightarrow{BP} \times \overrightarrow{BQ} = \begin{pmatrix} -26.25 \\ -36.75 \\ -9.75 \end{pmatrix}$

Area PBQ $= \frac{1}{2}\sqrt{(26.25^2 + 36.75^2 + 9.75^2)} = 23.1013...$

Area APQC = 61.6035... − 23.1013... = 38.5022...

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**Question 7 continued**

$= 38.5$ (3 s.f.) as required

(b)

$\overrightarrow{AD} = \begin{pmatrix} k-3 \\ 0 \\ -6 \end{pmatrix}$ Volume $= |(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}|$

$(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD} = \begin{pmatrix} 70 \\ 98 \\ 26 \end{pmatrix} \cdot \begin{pmatrix} k-3 \\ 0 \\ -6 \end{pmatrix} = 70(k-3) + 0 - 156 = 70k - 366$

$|70k - 366| = 226$

$70k - 366 = 226 \Rightarrow 70k = 592 \Rightarrow k = \frac{296}{35}$

$70k - 366 = -226 \Rightarrow 70k = 140 \Rightarrow k = 2$

$k = 2$ or $k = \frac{296}{35}$

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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 10 marks)**

**8.** The hyperbola $H$ has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

The line $l_1$ is the tangent to $H$ at the point $P(4\cosh\theta, 3\sinh\theta)$.

The line $l_1$ meets the $x$-axis at the point $A$.

The line $l_2$ is the tangent to $H$ at the point $(4, 0)$.

The lines $l_1$ and $l_2$ meet at the point $B$ and the midpoint of $AB$ is the point $M$.

(a) Show that, as $\theta$ varies, a Cartesian equation for the locus of $M$ is

$$y^2 = \frac{9(4-x)}{4x} \qquad p < x < q$$

where $p$ and $q$ are values to be determined.

**(11)**

Let $S$ be the focus of $H$ that lies on the positive $x$-axis.

(b) Show that the distance from $M$ to $S$ is greater than 1

**(3)**

(a)

$\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow \frac{x}{8} - \frac{2y}{9}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{9x}{16y}$

At P: $\frac{dy}{dx} = \frac{36\cosh\theta}{48\sinh\theta} = \frac{3\cosh\theta}{4\sinh\theta}$

$l_1$: $y - 3\sinh\theta = \frac{3\cosh\theta}{4\sinh\theta}(x - 4\cosh\theta)$

$4y\sinh\theta - 12\sinh^2\theta = 3x\cosh\theta - 12\cosh^2\theta$

$\Rightarrow \quad 3x\cosh\theta - 4y\sinh\theta = 12(\cosh^2\theta - \sinh^2\theta) = 12$

A: $y = 0 \Rightarrow x = \frac{4}{\cosh\theta}, \quad A\left(\frac{4}{\cosh\theta}, 0\right)$

$l_2$ is the tangent at the vertex $(4, 0)$: $x = 4$

B: $x = 4 \Rightarrow 12\cosh\theta - 4y\sinh\theta = 12 \Rightarrow y = \frac{3\cosh\theta - 3}{\sinh\theta}$

M: $x = \frac{1}{2}\left(4 + \frac{4}{\cosh\theta}\right) = 2 + \frac{2}{\cosh\theta}, \quad y = \frac{3\cosh\theta - 3}{2\sinh\theta}$

From x: $\cosh\theta = \frac{2}{x-2}$

$y^2 = \frac{9(\cosh\theta - 1)^2}{4\sinh^2\theta} = \frac{9(\cosh\theta - 1)^2}{4(\cosh\theta - 1)(\cosh\theta + 1)} = \frac{9(\cosh\theta - 1)}{4(\cosh\theta + 1)}$

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**Question 8 continued**

$\cosh\theta - 1 = \frac{2}{x-2} - 1 = \frac{4-x}{x-2}$, $\quad \cosh\theta + 1 = \frac{2}{x-2} + 1 = \frac{x}{x-2}$

$\Rightarrow \quad y^2 = \frac{9(4-x)}{4(x-2)} \times \frac{x-2}{x} = \frac{9(4-x)}{4x}$ as required

$\theta \neq 0$ (otherwise $l_1$ and $l_2$ coincide), so $\cosh\theta > 1$

$\Rightarrow \quad 0 < \frac{2}{\cosh\theta} < 2 \quad \Rightarrow \quad 2 < x < 4$

$p = 2, \quad q = 4$

(b)

$b^2 = a^2(e^2 - 1) \quad \Rightarrow \quad 9 = 16(e^2 - 1) \quad \Rightarrow \quad e^2 = \frac{25}{16} \quad \Rightarrow \quad e = \frac{5}{4}$

$S = (ae, 0) = \left(4 \times \frac{5}{4}, 0\right) = (5, 0)$

$MS = \sqrt{((5-x)^2 + y^2)} \geqslant 5 - x$

Since $x < 4$ for all points on the locus, $\quad 5 - x > 1$

$\therefore \quad MS > 1$

**Question 8 continued**

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**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 14 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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