Please check the examination details below before entering your candidate information

Candidate surname StemUp Model Solution | Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 3 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/01**

Further Mathematics

Advanced

Paper 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where a, b, c and d are real constants.

Given that $-1 + 2i$ and $3 - i$ are two roots of the equation $f(z) = 0$

(a) show all the roots of $f(z) = 0$ on a single Argand diagram,

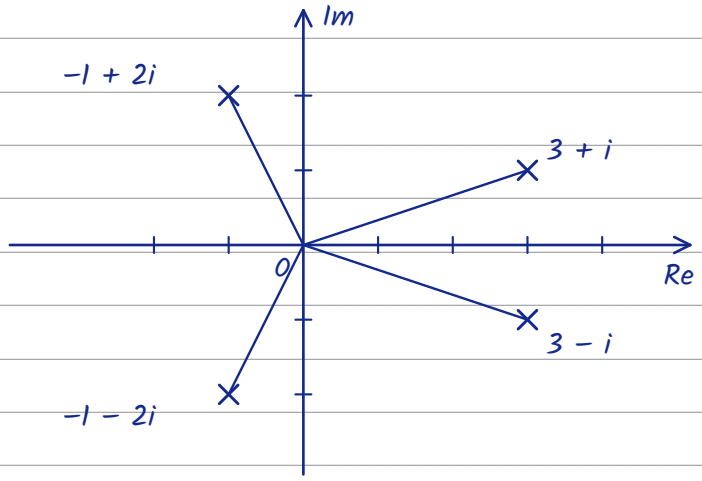
(4)

(b) find the values of a, b, c and d.

(5)

(a) f has real coefficients, so complex roots occur in conjugate pairs.

Roots: $-1 + 2i$, $-1 - 2i$, $3 + i$, $3 - i$



(b) $(z - (-1 + 2i))(z - (-1 - 2i)) = (z + 1)^2 + 4 = z^2 + 2z + 5$

$(z - (3 + i))(z - (3 - i)) = (z - 3)^2 + 1 = z^2 - 6z + 10$

$f(z) = (z^2 + 2z + 5)(z^2 - 6z + 10)$

$= z^4 - 6z^3 + 10z^2 + 2z^3 - 12z^2 + 20z + 5z^2 - 30z + 50$

$= z^4 - 4z^3 + 3z^2 - 10z + 50$

$a = -4, \quad b = 3, \quad c = -10, \quad d = 50$

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

2. Show that

$$\int_0^{\infty} \frac{8x-12}{(2x^2+3)(x+1)}\,dx = \ln k$$

where k is a rational number to be found.

(7)

$\frac{8x-12}{(2x^2+3)(x+1)} \equiv \frac{Ax+B}{2x^2+3} + \frac{C}{x+1}$

$8x - 12 \equiv (Ax + B)(x + 1) + C(2x^2 + 3)$

$x = -1$: $-20 = 5C \Rightarrow C = -4$

$x = 0$: $-12 = B + 3C = B - 12 \Rightarrow B = 0$

coefficient of x^2: $0 = A + 2C \Rightarrow A = 8$

$\int \frac{8x-12}{(2x^2+3)(x+1)}\,dx = \int \left(\frac{8x}{2x^2+3} - \frac{4}{x+1}\right) dx$

$= 2\ln(2x^2 + 3) - 4\ln(x + 1)$

$= 2\ln\left(\frac{2x^2+3}{(x+1)^2}\right) = \ln\left(\frac{(2x^2+3)^2}{(x+1)^4}\right)$

As $x \to \infty$, $\frac{2x^2+3}{(x+1)^2} = \frac{2 + 3/x^2}{(1 + 1/x)^2} \to 2$, so $2\ln\left(\frac{2x^2+3}{(x+1)^2}\right) \to 2\ln 2 = \ln 4$

At $x = 0$: $2\ln(3/1) = 2\ln 3 = \ln 9$

$\int_0^{\infty} \frac{8x-12}{(2x^2+3)(x+1)}\,dx = \ln 4 - \ln 9 = \ln\frac{4}{9}$

$k = \frac{4}{9}$

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

3.

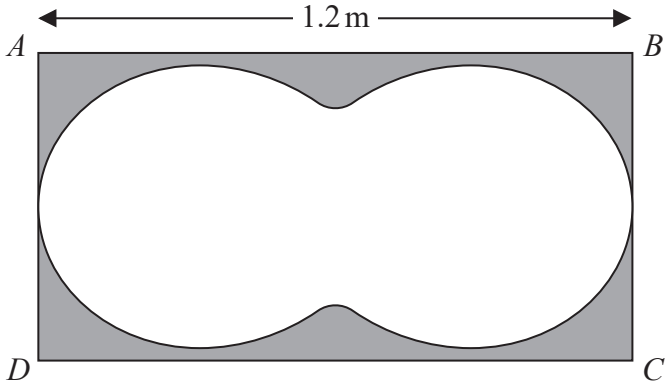


Diagram not to scale

Figure 1

Figure 1 shows the design for a table top in the shape of a rectangle $ABCD$. The length of the table, AB, is 1.2 m. The area inside the closed curve is made of glass and the surrounding area, shown shaded in Figure 1, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a\cos 2\theta \qquad 0 \leqslant \theta < 2\pi$$

where a is a constant.

(a) Show that $a = 0.2$

(2)

Hence, given that $AD = 60$ cm,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures.

(8)

(a) r is greatest when $\cos 2\theta = 1$ ($\theta = 0$), and then $2r = AB = 1.2$

$2(0.4 + a) = 1.2 \Rightarrow 0.4 + a = 0.6 \Rightarrow a = 0.2$

(b) Area of rectangle $= 1.2 \times 0.6 = 0.72\ m^2$

$$\text{Area of glass} = \frac{1}{2}\int_0^{2\pi} (0.4 + 0.2\cos 2\theta)^2\, d\theta$$

$$(0.4 + 0.2\cos 2\theta)^2 = 0.16 + 0.16\cos 2\theta + 0.04\cos^2 2\theta$$

$$= 0.16 + 0.16\cos 2\theta + 0.04\left(\frac{1 + \cos 4\theta}{2}\right)$$

$$= 0.18 + 0.16\cos 2\theta + 0.02\cos 4\theta$$

$$\text{Area of glass} = \frac{1}{2}\left[0.18\theta + 0.08\sin 2\theta + 0.005\sin 4\theta\right]_0^{2\pi}$$

$$= [0.09\theta + 0.04\sin 2\theta + 0.0025\sin 4\theta]_0^{2\pi}$$

$$= (0.18\pi + 0 + 0) - 0$$

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Question 3 continued

$= 0.18\pi = 0.5654...\ m^2$

Area of wood $= 0.72 - 0.18\pi$

$= 0.15451...$

$= 0.155\ m^2$ (3 s.f.)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

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4. Prove that, for $n \in \mathbb{Z}$, $n \geqslant 0$

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where a, b and c are integers to be found.

(5)

$$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{A}{r+1} + \frac{B}{r+2} + \frac{C}{r+3}$$

$$1 \equiv A(r+2)(r+3) + B(r+1)(r+3) + C(r+1)(r+2)$$

$r = -1$: $1 = 2A \Rightarrow A = \frac{1}{2}$; $r = -2$: $1 = -B \Rightarrow B = -1$; $r = -3$: $1 = 2C \Rightarrow C = \frac{1}{2}$

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{1}{2} \sum_{r=0}^{n} \left(\frac{1}{r+1} - \frac{2}{r+2} + \frac{1}{r+3} \right)$$

$r = 0$: $\frac{1}{1} - \frac{2}{2} + \frac{1}{3}$

$r = 1$: $\frac{1}{2} - \frac{2}{3} + \frac{1}{4}$

$r = 2$: $\frac{1}{3} - \frac{2}{4} + \frac{1}{5}$

$\vdots$

$r = n-1$: $\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}$

$r = n$: $\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}$

Terms that do not cancel:

$$\text{Sum} = \frac{1}{2}\left(1 - \frac{2}{2} + \frac{1}{2} + \frac{1}{n+2} - \frac{2}{n+2} + \frac{1}{n+3}\right)$$

$$= \frac{1}{4} - \frac{1}{2(n+2)} + \frac{1}{2(n+3)}$$

$$= \frac{(n+2)(n+3) - 2(n+3) + 2(n+2)}{4(n+2)(n+3)}$$

$$= \frac{n^2 + 5n + 6 - 2n - 6 + 2n + 4}{4(n+2)(n+3)} = \frac{n^2 + 5n + 4}{4(n+2)(n+3)}$$

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Question 4 continued

$= \dfrac{(n+1)(n+4)}{4(n+2)(n+3)}$

so $a = 1,\ b = 4,\ c = 4$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 5 marks)

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5. A tank at a chemical plant has a capacity of 250 litres. The tank initially contains 100 litres of pure water.

Salt water enters the tank at a rate of 3 litres every minute. Each litre of salt water entering the tank contains 1 gram of salt.

It is assumed that the salt water mixes instantly with the contents of the tank upon entry.

At the instant when the salt water begins to enter the tank, a valve is opened at the bottom of the tank and the solution in the tank flows out at a rate of 2 litres per minute.

Given that there are S grams of salt in the tank after t minutes,

(a) show that the situation can be modelled by the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = 3 - \frac{2S}{100 + t}$$

(4)

(b) Hence find the number of grams of salt in the tank after 10 minutes.

(5)

When the concentration of salt in the tank reaches 0.9 grams per litre, the valve at the bottom of the tank must be closed.

(c) Find, to the nearest minute, when the valve would need to be closed.

(3)

(d) Evaluate the model.

(1)

(a) 3 litres enter and 2 litres leave each minute, so the volume increases by 1 litre per minute: volume = $100 + 3t - 2t = 100 + t$ litres

Concentration of salt in the tank = $\frac{S}{100 + t}$ grams per litre

2 litres leave per minute, so salt out = $2 \times \frac{S}{100 + t} = \frac{2S}{100 + t}$ g per minute

Salt in = 3 litres × 1 g per litre = 3 g per minute

$\therefore \frac{dS}{dt}$ = salt in − salt out = $3 - \frac{2S}{100 + t}$

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Question 5 continued

(b) $\frac{dS}{dt} + \frac{2}{100 + t} S = 3$

Integrating factor: $e^{\int 2/(100 + t)\, dt} = e^{2 \ln(100 + t)} = (100 + t)^2$

$\frac{d}{dt}\left[S(100 + t)^2\right] = 3(100 + t)^2$

$S(100 + t)^2 = \int 3(100 + t)^2\, dt = (100 + t)^3 + c$

$t = 0, S = 0$: $0 = 100^3 + c \Rightarrow c = -10^6$

$S = 100 + t - \frac{10^6}{(100 + t)^2}$

$t = 10$: $S = 110 - \frac{10^6}{110^2} = \frac{3310}{121} = 27.355...$

Salt after 10 minutes = 27.4 g (3 s.f.)

(c) Concentration $= \frac{S}{100 + t} = 1 - \frac{10^6}{(100 + t)^3} = 0.9$

$\frac{10^6}{(100 + t)^3} = 0.1 \Rightarrow (100 + t)^3 = 10^7$

$100 + t = 215.443... \Rightarrow t = 115.443...$

The valve must be closed after 115 minutes

(volume then 215 litres < 250 litres, so the tank is not yet full)

(d) The model assumes the salt water mixes instantly with the contents of the tank, which is unlikely in reality, so the model is only approximate.

Question 5 continued

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Question 5 continued

(Total for Question 5 is 13 marks)

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6. Prove by induction that for all positive integers n

$$f(n) = 3^{2n+4} - 2^{2n}$$

is divisible by 5

(6)

When $n = 1$: $f(1) = 3^6 - 2^2 = 729 - 4 = 725 = 5 \times 145$

so $f(1)$ is divisible by 5 and the statement is true for $n = 1$.

Assume true for $n = k$, i.e. $f(k) = 3^{2k+4} - 2^{2k}$ is divisible by 5.

$$\begin{aligned} f(k+1) - f(k) &= 3^{2k+6} - 2^{2k+2} - 3^{2k+4} + 2^{2k} \\ &= 9 \times 3^{2k+4} - 4 \times 2^{2k} - 3^{2k+4} + 2^{2k} \\ &= 8 \times 3^{2k+4} - 3 \times 2^{2k} \\ &= 8(3^{2k+4} - 2^{2k}) + 5 \times 2^{2k} \\ &= 8f(k) + 5 \times 2^{2k} \end{aligned}$$

$\therefore f(k+1) = 9f(k) + 5 \times 2^{2k}$

$9f(k)$ is divisible by 5 (by the assumption) and 5×2^{2k} is divisible by 5, so $f(k+1)$ is divisible by 5.

If the statement is true for $n = k$ then it is true for $n = k + 1$, and as it is true for $n = 1$, the statement is true for all positive integers n.

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Question 6 continued

(Total for Question 6 is 6 marks)

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7. The line l_1 has equation

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

The line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

where t is a scalar parameter.

(a) Show that l_1 and l_2 lie in the same plane.

(3)

(b) Write down a vector equation for the plane containing l_1 and l_2

(1)

(c) Find, to the nearest degree, the acute angle between l_1 and l_2

(3)

(a) l_1: $r = i - j + 4k + \lambda(2i - j + 3k)$

Equating components with l_2:

x: $1 + 2\lambda = 1 + t$

y: $-1 - \lambda = -t$

z: $4 + 3\lambda = 3 + 2t$

From x: $t = 2\lambda$; from y: $t = 1 + \lambda \Rightarrow \lambda = 1,\ t = 2$

Check z: $4 + 3(1) = 7$ and $3 + 2(2) = 7$, so the equations are consistent

The lines intersect at $(3, -2, 7)$.

As the lines intersect at a point, l_1 and l_2 lie in the same plane.

(b) $r = i + 3k + p(2i - j + 3k) + q(i - j + 2k)$

(c) $(2i - j + 3k)\cdot(i - j + 2k) = 2 + 1 + 6 = 9$

$$\cos\theta = \frac{9}{\sqrt{(2^2 + (-1)^2 + 3^2)}\sqrt{(1^2 + (-1)^2 + 2^2)}} = \frac{9}{\sqrt{14}\sqrt{6}}$$

$\cos\theta = 0.98198\ldots \Rightarrow \theta = 10.893\ldots°$

Acute angle between l_1 and l_2 = 11° (nearest degree)

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Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 7 marks)

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8. A scientist is studying the effect of introducing a population of white-clawed crayfish into a population of signal crayfish.
At time t years, the number of white-clawed crayfish, w, and the number of signal crayfish, s, are modelled by the differential equations

$$\frac{dw}{dt} = \frac{5}{2}(w - s)$$
$$\frac{ds}{dt} = \frac{2}{5}w - 90e^{-t}$$

(a) Show that

$$2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t}$$

(3)

(b) Find a general solution for the number of white-clawed crayfish at time t years.

(6)

(c) Find a general solution for the number of signal crayfish at time t years.

(2)

The model predicts that, at time T years, the population of white-clawed crayfish will have died out.

Given that $w = 65$ and $s = 85$ when $t = 0$

(d) find the value of T, giving your answer to 3 decimal places.

(6)

(e) Suggest a limitation of the model.

(1)

(a) $\frac{d^2w}{dt^2} = \frac{5}{2}\left(\frac{dw}{dt} - \frac{ds}{dt}\right) \Rightarrow \frac{ds}{dt} = \frac{dw}{dt} - \frac{2}{5}\frac{d^2w}{dt^2}$

Substituting: $\frac{dw}{dt} - \frac{2}{5}\frac{d^2w}{dt^2} = \frac{2}{5}w - 90e^{-t}$

Multiply by -5:

$-5\frac{dw}{dt} + 2\frac{d^2w}{dt^2} = -2w + 450e^{-t}$

$\Rightarrow 2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t}$

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Question 8 continued

(b) Auxiliary equation: $2m^2 - 5m + 2 = 0 \Rightarrow (2m - 1)(m - 2) = 0$

$m = \frac{1}{2}, \quad m = 2$

Complementary function: $w = Ae^{0.5t} + Be^{2t}$

Particular integral: try $w = ke^{-t}$, $dw/dt = -ke^{-t}$, $d^2w/dt^2 = ke^{-t}$

$2ke^{-t} + 5ke^{-t} + 2ke^{-t} = 450e^{-t} \Rightarrow 9k = 450 \Rightarrow k = 50$

General solution: $w = Ae^{0.5t} + Be^{2t} + 50e^{-t}$

(c) From the first equation: $s = w - \frac{2}{5}\frac{dw}{dt}$

$s = Ae^{0.5t} + Be^{2t} + 50e^{-t} - \frac{2}{5}\left(\frac{A}{2}e^{0.5t} + 2Be^{2t} - 50e^{-t}\right)$

$s = \frac{4A}{5}e^{0.5t} + \frac{B}{5}e^{2t} + 70e^{-t}$

(d) $t = 0, w = 65$: $A + B + 50 = 65 \Rightarrow A + B = 15$

$t = 0, s = 85$: $\frac{4A}{5} + \frac{B}{5} + 70 = 85 \Rightarrow 4A + B = 75$

$\Rightarrow A = 20, \quad B = -5$

$w = 0$: $20e^{0.5t} - 5e^{2t} + 50e^{-t} = 0$

Multiply by e^t and divide by -5: $e^{3t} - 4e^{1.5t} - 10 = 0$

$e^{1.5t} = \frac{4 \pm \sqrt{(4^2 - 4(1)(-10))}}{2} = \frac{4 \pm \sqrt{56}}{2} = 2 \pm \sqrt{14}$

$e^{1.5t} > 0$, so reject $2 - \sqrt{14} < 0$: $e^{1.5t} = 2 + \sqrt{14}$

$1.5T = \ln(2 + \sqrt{14}) \Rightarrow T = \frac{2}{3}\ln(2 + \sqrt{14}) = 1.16517...$

$T = 1.165$ (3 d.p.)

(e) The model would give a negative number of white-clawed crayfish for $t > T$, which is not possible, so the model is not valid once they have died out.



Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

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