

# Cambridge International AS & A Level

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**MATHEMATICS****9709/11**

Paper 1 Pure Mathematics 1

**October/November 2024****MARK SCHEME**Maximum Mark: 75

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **15** printed pages.

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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- marks are awarded when candidates clearly demonstrate what they know and can do
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- marks are not deducted for omissions
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- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
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**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer                                                                                 | Marks     | Guidance                                                           |
|----------|----------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------|
| 1        | Identify correct term and obtain $6(kx)^2 \cdot \left(\frac{2}{x}\right)^2$            | <b>M1</b> | Needs numerical coefficient or $\frac{4!}{2!2!}$ , not ${}^4C_2$ . |
|          | Equate to 150 and obtain $k = \frac{5}{2}$                                             | <b>A1</b> | Ignore $-\frac{5}{2}$                                              |
|          | Identify correct term $4(kx)^3 \cdot \left(\frac{2}{x}\right)$ with their value of $k$ | <b>M1</b> | Needs numerical coefficient or $\frac{4!}{3!1!}$ .                 |
|          | Obtain coefficient 125                                                                 | <b>A1</b> | Accept $125x^2$ as final answer.                                   |
|          |                                                                                        | <b>4</b>  |                                                                    |

| Question | Answer                                                                        | Marks     | Guidance                               |
|----------|-------------------------------------------------------------------------------|-----------|----------------------------------------|
| 2        | Differentiate to obtain $2x + ax^{-2}$ or equivalent                          | <b>B1</b> |                                        |
|          | Equate first derivative to zero, substitute $x = -3$ and attempt value of $a$ | <b>M1</b> | Must be an attempt at differentiation. |
|          | Obtain $a = 54$                                                               | <b>A1</b> |                                        |
|          | Obtain $b = 27$                                                               | <b>A1</b> |                                        |
|          |                                                                               | <b>4</b>  |                                        |

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| Question | Answer                                                                                                                                    | Marks     | Guidance                                                                                              |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------------------------|
| 3        | Use correct sector area formula                                                                                                           | <b>M1</b> |                                                                                                       |
|          | Obtain $\frac{1}{2} \times 15^2 \times \frac{2}{5} \pi - \frac{1}{2} \times x^2 \times \frac{2}{5} \pi = \frac{209}{5} \pi$ or equivalent | <b>A1</b> |                                                                                                       |
|          | Obtain $[x] = 4$                                                                                                                          | <b>A1</b> | AWRT 4.00.                                                                                            |
|          | Use correct arc length formula twice                                                                                                      | <b>M1</b> |                                                                                                       |
|          | Obtain $22 + \frac{38}{5} \pi$                                                                                                            | <b>A1</b> | OE. Must be in terms of $\pi$ .<br>Like terms must be collected.<br>Not from a rounded value of $x$ . |
|          |                                                                                                                                           | <b>5</b>  |                                                                                                       |

| Question | Answer                                                                 | Marks        | Guidance                                                 |
|----------|------------------------------------------------------------------------|--------------|----------------------------------------------------------|
| 4        | Substitute for $y$ (or $x$ ) in first equation and simplify            | <b>*M1</b>   | All terms to one side and brackets expanded.             |
|          | Obtain $10x^2 + 3kx - 40 [= 0]$ (or $10y^2 + 11ky + k^2 - 360 [= 0]$ ) | <b>A1</b>    |                                                          |
|          | Attempt $b^2 - 4ac$ for 3-term quadratic involving $k$                 | <b>DM1</b>   | Not in quadratic formula unless $b^2 - 4ac$ is isolated. |
|          | Obtain $9k^2 + 1600$ (or $81k^2 + 14400$ )                             | <b>A1</b>    |                                                          |
|          | $9k^2 + 1600 > 0$                                                      | <b>A1 FT</b> | FT for $ak^2 + b > 0$ with $a, b > 0$ .                  |
|          |                                                                        | <b>5</b>     |                                                          |

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| Question | Answer                                                                                                         | Marks     | Guidance                                                                                                                         |
|----------|----------------------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------|
| 5(a)     | Attempt correct process for solving 3-term quadratic equation in $\sqrt{x}$                                    | <b>M1</b> | Accept $8y^2 - 6y - 9 \rightarrow (2y - 3)(4y + 3)$ , if $y = \sqrt{x}$ specified.                                               |
|          | Obtain at least $2\sqrt{x} - 3 = 0$ or equivalent                                                              | <b>A1</b> | Ignore $4\sqrt{x} + 3 = 0$ .<br><b>SC B1</b> for $\sqrt{x} = \frac{3}{2}$ with no method shown for solving the 3-term quadratic. |
|          | Conclude $x = \frac{9}{4}$ ignore $\frac{9}{16}$                                                               | <b>A1</b> | <b>SC B1</b> if no method shown for solving the 3-term quadratic.                                                                |
|          | <b>Alternative Method for Q5(a)</b>                                                                            |           |                                                                                                                                  |
|          | $3\sqrt{x} = 4x - \frac{9}{2} \rightarrow 16x^2 - 45x + \frac{81}{4}$ o.e and attempt correct process to solve | <b>M1</b> |                                                                                                                                  |
|          | Obtain $x = \frac{9}{4}$ or $\frac{9}{16}$                                                                     | <b>A1</b> | <b>SC B1</b> if no method shown for solving the 3-term quadratic.                                                                |
|          | $x = \frac{9}{4}$ ignore $\frac{9}{16}$                                                                        | <b>A1</b> | <b>SC B1</b> if no method shown for solving the 3-term quadratic.                                                                |
|          |                                                                                                                | <b>3</b>  |                                                                                                                                  |

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| Question | Answer                                                                                 | Marks     | Guidance                                                                               |
|----------|----------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------|
| 5(b)     | Integrate to obtain form $k_1x^2 + k_2x^{\frac{3}{2}} + k_3x$ where $k_1k_2k_3 \neq 0$ | <b>M1</b> |                                                                                        |
|          | Obtain correct $2x^2 - 2x^{\frac{3}{2}} + x$ or equivalent                             | <b>A1</b> | Allow unsimplified.                                                                    |
|          | Substitute $x=4$ and $y=11$ to attempt value of $c$                                    | <b>M1</b> | Dependent on at least 2 correct terms involving $x$ .                                  |
|          | Obtain $y = 2x^2 - 2x^{\frac{3}{2}} + x - 9$                                           | <b>A1</b> | Must be simplified.<br>Allow 'f(x) ='.<br>Allow $y$ missing if $y$ appears previously. |
|          |                                                                                        | <b>4</b>  |                                                                                        |

| Question | Answer                                                       | Marks     | Guidance |
|----------|--------------------------------------------------------------|-----------|----------|
| 6(a)     | State or imply centre of $C_1$ is $(-3, 5)$                  | <b>B1</b> |          |
|          | State or imply centre of $C_2$ is $(9, -4)$                  | <b>B1</b> |          |
|          | Attempt correct process for finding distance between centres | <b>M1</b> |          |
|          | Obtain 15                                                    | <b>A1</b> |          |
|          |                                                              | <b>4</b>  |          |

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| Question | Answer                            | Marks        | Guidance                                    |
|----------|-----------------------------------|--------------|---------------------------------------------|
| 6(b)     | $R = 4$ and $R = 8$               | <b>B1</b>    |                                             |
|          | Obtain least or greatest distance | <b>B1 FT</b> | ‘15’ – $R_1 - R_2$ or ‘15’ + $R_1 + R_2$ .  |
|          | Obtain 3 and 27                   | <b>B1 FT</b> | ‘15’ – $R_1 - R_2$ and ‘15’ + $R_1 + R_2$ . |
|          |                                   | <b>3</b>     |                                             |

| Question | Answer                                                                               | Marks     | Guidance                                             |
|----------|--------------------------------------------------------------------------------------|-----------|------------------------------------------------------|
| 7(a)     | Differentiate to obtain form $k_1(2x+1)^{-\frac{4}{3}}$                              | <b>M1</b> |                                                      |
|          | Obtain correct $-8(2x+1)^{-\frac{4}{3}}$ or unsimplified equivalent                  | <b>A1</b> |                                                      |
|          | Attempt equation of tangent at $\left(\frac{7}{2}, 6\right)$ with numerical gradient | <b>M1</b> | Gradient must come from a differentiated expression. |
|          | Obtain $y = -\frac{1}{2}x + \frac{31}{4}$ or equivalent of requested form            | <b>A1</b> |                                                      |
|          |                                                                                      | <b>4</b>  |                                                      |

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| Question | Answer                                                            | Marks     | Guidance                                                                                     |
|----------|-------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------|
| 7(b)     | Integrate to obtain form $k_2(2x+1)^{\frac{2}{3}}$                | <b>M1</b> |                                                                                              |
|          | Obtain correct $9(2x+1)^{\frac{2}{3}}$ or unsimplified equivalent | <b>A1</b> |                                                                                              |
|          | Use correct limits correctly to find area                         | <b>M1</b> | Substitute correct limits into an integrated expression.<br>36 – 9 minimum working required. |
|          | Obtain 27                                                         | <b>A1</b> | <b>SC B1</b> if M1 A1 M0 scored.                                                             |
|          |                                                                   | <b>4</b>  |                                                                                              |

| Question | Answer                                                 | Marks     | Guidance                                                                                                                               |
|----------|--------------------------------------------------------|-----------|----------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | Use $\tan^2 \beta = \frac{\sin^2 \beta}{\cos^2 \beta}$ | <b>B1</b> | E.g. $\tan^2 \beta = \frac{\sin^2 \beta}{\cos^2 \beta}$ and then replaces $\sin^2 \beta$ with $a^2$ or $\cos^2 \beta$ with $1 - a^2$ . |
|          | $\cos \beta = -\sqrt{1 - a^2}$                         | <b>B1</b> |                                                                                                                                        |
|          | Obtain $\frac{a^2}{1 - a^2} + 3a\sqrt{1 - a^2}$        | <b>B1</b> |                                                                                                                                        |
|          |                                                        | <b>3</b>  |                                                                                                                                        |

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| Question | Answer                                                                    | Marks       | Guidance                                                                                                                                               |
|----------|---------------------------------------------------------------------------|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(b)     | Use correct identity to obtain 3-term quadratic equation in $\sin \theta$ | <b>*M1</b>  |                                                                                                                                                        |
|          | Obtain $\sin^2 \theta + 4\sin \theta + 1 [= 0]$                           | <b>A1</b>   |                                                                                                                                                        |
|          | Attempt to solve quadratic                                                | <b>DM1</b>  | At least as far as $\frac{-4 \pm \sqrt{12}}{2}$ .<br>–15.5° implies attempt at solving quadratic.                                                      |
|          | Obtain 195.5                                                              | <b>A1</b>   |                                                                                                                                                        |
|          | Obtain 344.5                                                              | <b>A1FT</b> | Following first answer; and no others for $0^\circ < \theta < 360^\circ$ but must be in 4 <sup>th</sup> quadrant.<br><b>SC B1</b> for 3.41° and 6.01°. |
|          |                                                                           | <b>5</b>    |                                                                                                                                                        |

| Question | Answer                                                                          | Marks       | Guidance                                             |
|----------|---------------------------------------------------------------------------------|-------------|------------------------------------------------------|
| 9(a)     | Differentiate to obtain $5 + 12x - 9x^2$                                        | <b>B1</b>   |                                                      |
|          | Attempt to find two critical values by solving quadratic equation or inequality | <b>M1</b>   |                                                      |
|          | Obtain values $-\frac{1}{3}$ and $\frac{5}{3}$                                  | <b>A1</b>   | <b>SC B1</b> if no method for solving the quadratic. |
|          | Conclude $x < -\frac{1}{3}$ , $x > \frac{5}{3}$                                 | <b>A1FT</b> | <b>SC B1</b> if no method for solving the quadratic. |
|          |                                                                                 | <b>4</b>    |                                                      |

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| Question | Answer                                                          | Marks      | Guidance                                                                                                         |
|----------|-----------------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------|
| 9(b)     | Equate first derivative to 9 and simplify to 3 term quadratic   | <b>*M1</b> |                                                                                                                  |
|          | Obtain $x = \frac{2}{3}$                                        | <b>A1</b>  | <b>SC B1</b> for solving $5 + 12x - 9x^2 = 9$ without simplifying to a 3-term quadratic.                         |
|          | Use x-value and corresponding y-value to determine value of $k$ | <b>DM1</b> |                                                                                                                  |
|          | Obtain $k = \frac{28}{9}$                                       | <b>A1</b>  | <b>SC B1</b> for $k = \frac{28}{9}$ from solving $5 + 12x - 9x^2 = 9$ without simplifying to a 3-term quadratic. |
|          |                                                                 | <b>4</b>   |                                                                                                                  |

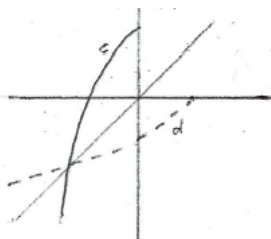
| Question | Answer                                                                                                           | Marks     | Guidance                                                                                               |
|----------|------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------|
| 10(a)    | State or imply that first 3 terms of GP are $5 + d$ , $5 + 4d$ , $5 + 10d$                                       | <b>B1</b> |                                                                                                        |
|          | Form equation $(5 + 4d)^2 = (5 + d)(5 + 10d)$ or equivalent                                                      | <b>M1</b> |                                                                                                        |
|          | Obtain $d = 2.5$                                                                                                 | <b>A1</b> | Ignore 0 as a solution.<br><b>SC B1</b> Obtain $d = 2.5$ and 7.5, 15, 30 by trial and improvement www. |
|          | <b>Alternative Method for Question 10(a):</b>                                                                    |           |                                                                                                        |
|          | State or imply that first 3 terms of GP are $5 + d$ , $5 + 4d$ , $5 + 10d$                                       | <b>B1</b> |                                                                                                        |
|          | $(5 + d)R = 5 + 4d \rightarrow d = \frac{5 - 5R}{R - 4}$ , $(5 + d)R^2 = 5 + 10d \rightarrow R^2 - 3R + 2 [= 0]$ | <b>M1</b> | OE<br>Eliminates $d$ .                                                                                 |
|          | Obtain $d = 2.5$                                                                                                 | <b>A1</b> |                                                                                                        |
|          |                                                                                                                  | <b>3</b>  |                                                                                                        |

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| Question | Answer                                                    | Marks     | Guidance |
|----------|-----------------------------------------------------------|-----------|----------|
| 10(b)    | Use correct formula for sum of AP with their value of $d$ | <b>M1</b> |          |
|          | Obtain or imply 7700                                      | <b>A1</b> |          |
|          | State or imply GP is 7.5, 15, 30, ...                     | <b>B1</b> |          |
|          | Use correct formula for sum of GP with their common ratio | <b>M1</b> |          |
|          | Obtain $S_{77} - G_{10} = 27.5$                           | <b>A1</b> |          |
|          |                                                           | <b>5</b>  |          |

| Question | Answer                                                                                                           | Marks        | Guidance                                                                                          |
|----------|------------------------------------------------------------------------------------------------------------------|--------------|---------------------------------------------------------------------------------------------------|
| 11(a)    | Obtain $b = 2$ and $c = \frac{3}{2}$                                                                             | <b>B1</b>    |                                                                                                   |
|          | Obtain $\frac{15}{2} - 2\left(x - \frac{3}{2}\right)^2$                                                          | <b>B1</b>    |                                                                                                   |
|          | State range is $y \leq \frac{15}{2}$ or $f(x) \leq \frac{15}{2}$ with $\leq$ given or clearly implied (not $<$ ) | <b>B1 FT</b> | Following <i>their</i> value of $a$ .                                                             |
|          |                                                                                                                  | <b>3</b>     |                                                                                                   |
| 11(b)    | State that reflection is in x-axis                                                                               | <b>B1</b>    | Accept transformations in any order.                                                              |
|          | State or imply that translation is by $\begin{pmatrix} -\frac{3}{2} \\ \frac{15}{2} \end{pmatrix}$ or equivalent | <b>B1 FT</b> | Following <i>their</i> values of $a$ and $c$ in part (a).<br>Accept transformations in any order. |
|          |                                                                                                                  | <b>2</b>     |                                                                                                   |

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| Question | Answer                                                                         | Marks     | Guidance                                                                            |
|----------|--------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------------------------|
| 11(c)    | Sketch the correct graph appearing in second and third quadrants only          | <b>B1</b> |  |
|          | State that each y-value is associated with a single x-value or equivalent      | <b>B1</b> | Accept passes the horizontal line test.<br>Ignore passes the vertical line test.    |
|          |                                                                                | <b>2</b>  |                                                                                     |
| 11(d)    | Sketch the correct graph with suitable labelling to distinguish the two curves | <b>B1</b> | Appearing in third and fourth quadrants only.                                       |
|          | Draw the line $y = x$                                                          | <b>B1</b> | See above; no need to label the line.                                               |
|          | Attempt correct process for finding the inverse function                       | <b>M1</b> | Allowing use of $\pm$ and $y$ so far.                                               |
|          | Obtain $\frac{3}{2} - \sqrt{\frac{15}{4} - \frac{1}{2}x}$ or equivalent        | <b>A1</b> | Must involve $x$ at the conclusion.                                                 |
|          |                                                                                | <b>4</b>  |                                                                                     |

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**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer  | Marks     | Guidance                                                             |
|----------|---------|-----------|----------------------------------------------------------------------|
| 1(a)     | $a = 4$ | <b>B1</b> | Allow $4\sin(2x) + 3$ if values of $a$ , $b$ and $c$ are not stated. |
|          | $b = 2$ | <b>B1</b> |                                                                      |
|          | $c = 3$ | <b>B1</b> |                                                                      |
|          |         | <b>3</b>  |                                                                      |
| 1(b)(i)  | 5       | <b>B1</b> | Ignore attempts at finding solutions.                                |
|          |         | <b>1</b>  |                                                                      |
| 1(b)(ii) | 1       | <b>B1</b> | Ignore attempts at finding solutions.                                |
|          |         | <b>1</b>  |                                                                      |

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| Question | Answer                                                                           | Marks      | Guidance                                                                                                                                                                                          |
|----------|----------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2(a)     | $\frac{20}{2}(2 \times -20 + (20 - 1) \times 5)$ or $\frac{20}{2}(-20 + 75)$     | <b>M1</b>  | Correct use of either $S_{20}$ formula with $a = -20$ and $d = 5$ .                                                                                                                               |
|          | 550                                                                              | <b>A1</b>  |                                                                                                                                                                                                   |
|          |                                                                                  | <b>2</b>   |                                                                                                                                                                                                   |
| 2(b)     | $\frac{2k}{2}(-40 + (2k - 1) \times 5)$ or $\frac{k}{2}(-40 + (k - 1) \times 5)$ | <b>M1*</b> | Correct use of $S_n$ formula with $a = -20$ , $d = 5$ and either $k$ or $2k$ .<br><br>This mark can be awarded for clear use of $\frac{n}{2}(a + l)$ when correct values of $a$ and $d$ are used. |
|          | $[-40k + 10k^2 - 5k = -200k + 25k^2 - 25k \Rightarrow] 15k^2 - 180k = 0$         | <b>DM1</b> | Equating their $S_{2k}$ to $10 \times$ <i>their</i> $S_k$ and reaching a 2-term quadratic or 2 term linear equation if $k$ has been cancelled. Condone errors in simplification.                  |
|          | $k = 12$                                                                         | <b>A1</b>  | Condone extra solution $k = 0$ .                                                                                                                                                                  |
|          |                                                                                  | <b>3</b>   |                                                                                                                                                                                                   |

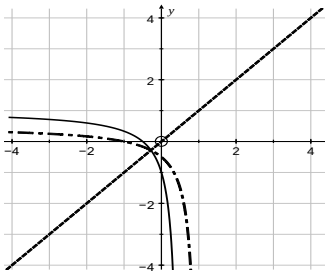
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| Question | Answer                                                                      | Marks       | Guidance                                                                                                                                                                 |
|----------|-----------------------------------------------------------------------------|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3(a)     | $[f(2+h) = ] 2(2+h)^2 - 3$                                                  | <b>B1</b>   | SOI                                                                                                                                                                      |
|          | $\frac{(2(2+h)^2 - 3) - 5}{(2+h) - 2} \left[ = \frac{2h^2 + 8h}{h} \right]$ | <b>M1</b>   | $\frac{\{their(2(2+h)^2 - 3)\} - their 5}{(2+h) - 2}$ can be implied by the simplified expression or the correct answer.<br><i>Their 5</i> must come from $2(2)^2 - 3$ . |
|          | $2h + 8$ or $2(h + 4)$                                                      | <b>A1</b>   |                                                                                                                                                                          |
|          |                                                                             | <b>3</b>    |                                                                                                                                                                          |
| 3(b)     | $h \rightarrow 0$ , or chord [AB] $\rightarrow$ tangent [at A]              | <b>B1</b>   | Either of these statements or any sight of $h = 0$ .                                                                                                                     |
|          | 8                                                                           | <b>B1FT</b> | Could come from anywhere except wrong working.<br>Either correct or FT their linear expression from <b>(a)</b> .                                                         |
|          |                                                                             | <b>2</b>    |                                                                                                                                                                          |

| Question | Answer                                                                                                                           | Marks     | Guidance                                                          |
|----------|----------------------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------|
| 4(a)     | $15$ or $\binom{6}{2} \times x^4 \left(\frac{3}{x^2}\right)^2$ or $x^6 \times \frac{6 \times 5}{2} \left(\frac{3}{x^3}\right)^2$ | <b>B1</b> | OE<br>May be in a list.<br>Allow $\binom{6}{4}$ .                 |
|          | 135                                                                                                                              | <b>B1</b> | Correct term must be identified if in a list.<br>Allow $135x^0$ . |
|          |                                                                                                                                  | <b>2</b>  |                                                                   |

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| Question | Answer                                                                                                                                     | Marks     | Guidance                                                                                                |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------------|
| 4(b)     | $20 \text{ or } \binom{6}{3} \times x^3 \left(\frac{3}{x^2}\right)^3 \text{ or } x^6 \times \frac{6.5.4}{3!} \left(\frac{3}{x^3}\right)^3$ | <b>B1</b> | OE<br>May be in a list.                                                                                 |
|          | $= \frac{540}{x^3}$                                                                                                                        | <b>B1</b> | Identifying $\frac{1}{x^3}$ term.<br>This can be implied by sight of 2160 as part of the constant term. |
|          | $4 \times 540 - 5 \times 135$                                                                                                              | <b>M1</b> | $4 \times \text{their } 540 - 5 \times \text{their } 135$                                               |
|          | 1485                                                                                                                                       | <b>A1</b> | Allow $1485x^0$ .                                                                                       |
|          |                                                                                                                                            | <b>4</b>  |                                                                                                         |

| Question | Answer                                                                              | Marks     | Guidance                                                                                                                                                                                                                                                                                 |
|----------|-------------------------------------------------------------------------------------|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(a)(i)  | $[f(-1) =] \frac{1}{3}$                                                             | <b>B1</b> | Condone 0.333.                                                                                                                                                                                                                                                                           |
|          |                                                                                     | <b>1</b>  |                                                                                                                                                                                                                                                                                          |
| 5(a)(ii) |  | <b>B1</b> | For showing the correct mirror line.                                                                                                                                                                                                                                                     |
|          |                                                                                     | <b>B1</b> | For correct shape: the curves should intersect in the first square in the third quadrant. To the left of the point of intersection, the reflection is below the original and crosses the x-axis. To the right of the point of intersection, the reflection is to the right the original. |
|          |                                                                                     | <b>2</b>  |                                                                                                                                                                                                                                                                                          |

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| Question  | Answer                                                    | Marks      | Guidance                                                                                                                                 |
|-----------|-----------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------------------------------|
| 5(a)(iii) | $\frac{2x+1}{2x-1} = y \Rightarrow 2x+1 = y(2x-1)$        | <b>M1*</b> | Equating $y$ to the given function and clearing of fractions. $x$ and $y$ may be interchanged at this stage.                             |
|           | $2xy - 2x = y + 1$                                        | <b>DM1</b> | Condone $\pm$ errors during simplification.                                                                                              |
|           | $\frac{x+1}{2(x-1)}, \frac{-x-1}{2-2x}$                   | <b>A1</b>  | Allow ' $f^{-1}$ ' or ' $y =$ ' but NOT ' $x =$ ', nor fractions within fractions.                                                       |
|           | [Domain of $f^{-1}$ is] $x < 1$                           | <b>B1</b>  | Accept $-\infty < x < 1$ or $(-\infty, 1)$ , condone $[-\infty, 1)$ .                                                                    |
|           | <b>Alternative Method for Question 5(a)(iii)</b>          |            |                                                                                                                                          |
|           | $y = 1 + \frac{2}{2x-1} \Rightarrow y-1 = \frac{2}{2x-1}$ | <b>M1*</b> | Equating $y$ to the given function after division by $2x-1$ . Isolating the term in $x$ . $x$ and $y$ may be interchanged at this stage. |
|           | $2x = \frac{2}{y-1} + 1$                                  | <b>DM1</b> | Condone $\pm$ errors during simplification.                                                                                              |
|           | $\frac{1}{x-1} + \frac{1}{2}$                             | <b>A1</b>  | OE<br>Allow ' $f^{-1}$ ' or ' $y =$ ' but NOT ' $x =$ ', nor fractions within fractions.                                                 |
|           | [Domain of $f^{-1}$ is] $x < 1$                           | <b>B1</b>  | Accept $-\infty < x < 1$ or $(-\infty, 1)$ , condone $[-\infty, 1)$ .                                                                    |
|           |                                                           | <b>4</b>   |                                                                                                                                          |

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| Question | Answer                                        | Marks     | Guidance                                                                    |
|----------|-----------------------------------------------|-----------|-----------------------------------------------------------------------------|
| 5(b)     | $gf\left(\frac{1}{4}\right) = -7$             | <b>B1</b> |                                                                             |
|          | $\frac{2x+1}{2x-1} = -7$                      | <b>M1</b> | Equating $\frac{2x+1}{2x-1}$ to <i>their</i> $gf\left(\frac{1}{4}\right)$ . |
|          | $[x =] \frac{3}{8}$                           | <b>A1</b> | OE                                                                          |
|          | <b>Alternative solution for Question 5(b)</b> |           |                                                                             |
|          | $gf\left(\frac{1}{4}\right) = -7$             | <b>B1</b> |                                                                             |
|          | $x = f^{-1}(-7)$                              | <b>M1</b> | $x = f^{-1}\left(\text{their } gf\left(\frac{1}{4}\right)\right)$           |
|          | $[x =] \frac{3}{8}$                           | <b>A1</b> | OE                                                                          |
|          |                                               | <b>3</b>  |                                                                             |

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| Question | Answer                                                                                                                                                 | Marks      | Guidance                                                                                                                 |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------------------------------------------------------------------------------------------------------------------|
| 6        | [Perimeter =] $r + r\theta + r + 2r \times 2\theta + r + r\theta + r$ $[= 4r + 6r\theta]$                                                              | <b>B1</b>  |                                                                                                                          |
|          | [Area =] $\frac{1}{2}r^2\theta + \frac{1}{2}(2r)^2 \times 2\theta + \frac{1}{2}r^2\theta$ $[= 5r^2\theta]$                                             | <b>B1</b>  |                                                                                                                          |
|          | $4r + 6r\theta = 14$ and $5r^2\theta = 10$                                                                                                             | <b>M1*</b> | $ar + br\theta = 14$ and $cr^2\theta = 10$ where $a$ , $b$ and $c$ are constants $\neq 0$ .<br>Terms may be uncollected. |
|          | <b>EITHER</b>                                                                                                                                          |            |                                                                                                                          |
|          | $5r^2 \frac{14 - 4r}{6r} = 10$ or $4r + 6r\left(\frac{10}{5r^2}\right) = 14$                                                                           | <b>DM1</b> | Eliminate $\theta$ to get an equation in $r$ .                                                                           |
|          | $[\Rightarrow 2r^2 - 7r + 6 = 0 \Rightarrow] (r - 2)(2r - 3) = 0$                                                                                      | <b>DM1</b> | Factorise or other accepted method for solving their 3-term quadratic.                                                   |
|          | <b>OR</b>                                                                                                                                              |            |                                                                                                                          |
|          | $5\left(\frac{14}{4 + 6\theta}\right)^2 \theta = 10$ or $4\left(\sqrt{\frac{10}{5\theta}}\right) + 6\left(\sqrt{\frac{10}{5\theta}}\right)\theta = 14$ | <b>DM1</b> | Eliminate $r$ to get an equation in $\theta$ .                                                                           |
|          | $[\Rightarrow 18\theta^2 - 25\theta + 8 = 0 \Rightarrow] (9\theta - 8)(2\theta - 1) = 0$                                                               | <b>DM1</b> | Factorise or other accepted method for solving their 3-term quadratic.                                                   |
|          | <b>Then</b>                                                                                                                                            |            |                                                                                                                          |
|          | $r = 2$ and $\theta = 0.5$                                                                                                                             | <b>B1</b>  | Condone extra answers $r = \frac{3}{2}$ and $\theta = \frac{8}{9}$ .                                                     |
|          |                                                                                                                                                        | <b>6</b>   |                                                                                                                          |

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| Question | Answer                                           | Marks      | Guidance                         |
|----------|--------------------------------------------------|------------|----------------------------------|
| 7(a)     | $-2((x \pm p)^2 \pm q)$ or $-2(x \pm p)^2 \pm q$ | <b>M1*</b> | $p \neq 0$ .                     |
|          | $-2((x - 2)^2 \pm q)$ or $-2(x - 2)^2 \pm q$     | <b>DM1</b> |                                  |
|          | $-2(x - 2)^2 + 19$ and (2, 19)                   | <b>A1</b>  | Accept $x = 2, y = 19$ or 2, 19. |
|          |                                                  | <b>3</b>   |                                  |

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| Question | Answer                                                                                             | Marks      | Guidance                                                                                                                                                                 |
|----------|----------------------------------------------------------------------------------------------------|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(b)     | <b>Method 1</b>                                                                                    |            |                                                                                                                                                                          |
|          | $[x =] \pm 1$                                                                                      | <b>B1*</b> | Both x co-ordinates for the points of intersection.                                                                                                                      |
|          | Subtract and attempt to integrate                                                                  | <b>M1*</b> |                                                                                                                                                                          |
|          | $\left[ \int (-2x^2 + 2) dx \right] - \frac{2}{3}x^3 + 2x$                                         | <b>B1*</b> | Both terms correct.                                                                                                                                                      |
|          | $\left( -\frac{2}{3} + 2 \right) - \left( \frac{2}{3} - 2 \right)$                                 | <b>M1</b>  | Apply <i>their</i> limits, one positive and one negative, obtained from equating the line and the curve to their integrated expression.                                  |
|          | $= \frac{8}{3}, 2\frac{2}{3}$                                                                      | <b>DB1</b> | AWRT 2.67 WWW.<br>Condone $\frac{-8}{3} \rightarrow \frac{8}{3}$ .<br><b>SC B1</b> for mistaking triangle for trapezium leading to $\frac{11}{3}$ , i.e. a total of 2/5. |
|          | <b>Method 2</b>                                                                                    |            |                                                                                                                                                                          |
|          | $[x =] \pm 1$                                                                                      | <b>B1*</b> | Both x co-ordinates for the points of intersection.                                                                                                                      |
|          | Attempt to integrate and subtract                                                                  | <b>M1*</b> | The second integral can be replaced with what is clearly their area of a trapezium.                                                                                      |
|          | $\left\{ \frac{-2x^3}{3} + \frac{8}{2}x^2 + 11x \right\} [-] \left\{ \frac{8}{2}x^2 + 9x \right\}$ | <b>B1*</b> | OE<br>All terms correct.<br>The second integral can be replaced by $\frac{1}{2}(1+17) \times 2$ OE.                                                                      |

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| Question | Answer                                                                                                              | Marks      | Guidance                                                                                                                                                                                                                                    |
|----------|---------------------------------------------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(b)     | $\left\{ \left( \frac{-2}{3} + 4 + 11 \right) - \left( \frac{2}{3} + 4 - 11 \right) \right\} [-] \{(4+9) - (4-9)\}$ | <b>M1</b>  | Apply <i>their</i> limits, one positive and one negative, obtained from equating the line and the curve, to <i>their</i> integrated expressions.<br>If the trapezium has been used, the second integral can be replaced by <i>their</i> 18. |
|          | $= \frac{8}{3}, 2\frac{2}{3}$                                                                                       | <b>DB1</b> | AWRT 2.67 WWW.<br>Condone $\frac{-8}{3} \rightarrow \frac{8}{3}$ .<br><b>SC B1</b> for mistaking triangle for trapezium leading to $\frac{11}{3}$ , i.e. a total of 2/5.                                                                    |
|          | <b>Method 3</b>                                                                                                     |            |                                                                                                                                                                                                                                             |
|          | $[x =] \pm 1$                                                                                                       | <b>B1*</b> | Both $x$ co-ordinates for the points of intersection.                                                                                                                                                                                       |
|          | Subtract and attempt to integrate                                                                                   | <b>M1*</b> |                                                                                                                                                                                                                                             |
|          | $-\frac{2}{3}(x-2)^3 - \frac{8}{2}x^2 + 10x$                                                                        | <b>B1*</b> | All terms correct.                                                                                                                                                                                                                          |
|          | $\left( \frac{2}{3} - 4 + 10 \right) - (18 - 4 - 10)$                                                               | <b>M1</b>  | Apply <i>their</i> limits, one positive and one negative, obtained from equating the line and the curve, to <i>their</i> integrated expression.                                                                                             |
|          | $= \frac{8}{3}, 2\frac{2}{3}$                                                                                       | <b>DB1</b> | AWRT 2.67 WWW.                                                                                                                                                                                                                              |

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| Question | Answer                                                                            | Marks      | Guidance                                                                                                                                                                                                                                       |
|----------|-----------------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7(b)     | <b>Method 4</b>                                                                   |            |                                                                                                                                                                                                                                                |
|          | $[x =] \pm 1$                                                                     | <b>B1*</b> | Both x co-ordinates for the points of intersection.                                                                                                                                                                                            |
|          | Attempt to integrate and subtract                                                 | <b>M1*</b> | The second integral can be replaced with what is clearly <i>their</i> area of a trapezium.                                                                                                                                                     |
|          | $\left\{-\frac{2}{3}(x-2)^3 + 19x\right\} [-] \left\{\frac{8}{2}x^2 + 9x\right\}$ | <b>B1*</b> | All terms correct.<br>The second integral can be replaced with $\frac{1}{2}(1+17) \times 2$ OE.                                                                                                                                                |
|          | $\left\{\left(\frac{2}{3} + 19\right) - (18 - 19)\right\} [-] \{(4+9) - (4-9)\}$  | <b>M1</b>  | Apply <i>their</i> limits, one positive and one negative, obtained from equating the line and the curve, to <i>their</i> integrated expression.<br>If the trapezium has been used the second integral can be replaced with <i>their</i> 18 OE. |
|          | $= \frac{8}{3}, 2\frac{2}{3}$                                                     | <b>DB1</b> | AWRT 2.67 WWW.<br>Condone $\frac{-8}{3} \rightarrow \frac{8}{3}$ .<br><b>SC B1</b> for mistaking triangle for trapezium leading to $\frac{11}{3}$ , i.e. a total of 2/5.                                                                       |
|          |                                                                                   | <b>5</b>   |                                                                                                                                                                                                                                                |

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| Question | Answer                                                                                                    | Marks      | Guidance                                                                                 |
|----------|-----------------------------------------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------|
| 8(a)     | $\left(x - \left(-\frac{1}{2}p\right)\right)^2 + (y - (-1))^2$ OE                                         | <b>B1*</b> | Allow $a = -\frac{1}{2}p$ and $b = -1$ , or centre is $\left(-\frac{1}{2}p, -1\right)$ . |
|          | $\left(x - \left(-\frac{1}{2}p\right)\right)^2 + (y - (-1))^2 = -q + 1 + \left(-\frac{1}{2}P\right)^2$ OE | <b>DB1</b> |                                                                                          |
|          |                                                                                                           | <b>2</b>   |                                                                                          |
| 8(b)(i)  | [Gradient of tangent =] $-\frac{1}{2}$                                                                    | <b>B1</b>  | OE<br>SOI                                                                                |
|          | [Gradient of normal =] 2                                                                                  | <b>M1</b>  | Use of $m_1m_2 = -1$ with <i>their</i> numeric tangent gradient.                         |
|          | $\frac{y-3}{x-4} = 2$ [ $y = 2x - 5$ ]                                                                    | <b>A1</b>  | OE<br>ISW<br>Allow $y = 2x + c$ , $3 = 2 \times 4 + c \Rightarrow c = -5$ .              |
|          |                                                                                                           | <b>3</b>   |                                                                                          |

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| Question | Answer                                                                                                | Marks      | Guidance                                                                                                          |
|----------|-------------------------------------------------------------------------------------------------------|------------|-------------------------------------------------------------------------------------------------------------------|
| 8(b)(ii) | <b>Method 1 for the first two marks:</b>                                                              |            |                                                                                                                   |
|          | $-1 - 3 = 2\left(-\frac{1}{2}p - 4\right)$ or $-1 = -p - 5$                                           | <b>M1*</b> | Using <i>their</i> stated centre or $\left(\frac{\pm p}{2}, \pm 1\right)$ in <i>their</i> equation of the normal. |
|          | $p = -4$                                                                                              | <b>A1</b>  |                                                                                                                   |
|          | <b>Method 2 for the first two marks:</b>                                                              |            |                                                                                                                   |
|          | $-1 = 2x - 5 \Rightarrow x = 2 \Rightarrow -\frac{1}{2}p = 2$                                         | <b>M1*</b> | Using their normal equation and <i>their</i> stated centre or $\left(\frac{\pm p}{2}, \pm 1\right)$ .             |
|          | $p = -4$                                                                                              | <b>A1</b>  |                                                                                                                   |
|          | <b>Method 3 for the first two marks:</b>                                                              |            |                                                                                                                   |
|          | $2x + 2y \frac{dy}{dx} + p + 2 \frac{dy}{dx} = 0 \left[ \Rightarrow p = -8 - 8 \frac{dy}{dx} \right]$ | <b>M1*</b> |                                                                                                                   |
|          | $\left[ \frac{dy}{dx} = -\frac{1}{2} \Rightarrow \right] p = -4$                                      | <b>A1</b>  |                                                                                                                   |

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| Question | Answer                                                                                                                                                                   | Marks      | Guidance                                                                                                           |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------------------------------------------------------------------------------------------------------------|
| 8(b)(ii) | <b>Method 1 for the last 3 marks:</b>                                                                                                                                    |            |                                                                                                                    |
|          | $r^2 = (4-2)^2 + (3-(-1))^2 [= 20]$                                                                                                                                      | <b>M1*</b> | Using (4, 3) and <i>their</i> centre or $\left(\frac{\pm \text{their } p}{2}, \pm 1\right)$ to find $r^2$ or $r$ . |
|          | $-q + 1 + \frac{1}{4}p^2 = 20$                                                                                                                                           | <b>DM1</b> | OE<br>Using <i>their</i> expression for $r^2$ (from <b>(a)</b> ) equated to <i>their</i> 20.                       |
|          | $q = -15$                                                                                                                                                                | <b>A1</b>  |                                                                                                                    |
|          | <b>Method 2 for the last 3 marks:</b>                                                                                                                                    |            |                                                                                                                    |
|          | $r = \frac{ 2-2-10 }{\sqrt{5}} \left[ = \frac{10}{\sqrt{5}} \right]$                                                                                                     | <b>M1*</b> | Using (2, -1) and $x + 2y - 10 = 0$ (distance from a point to a line).                                             |
|          | $-q + 1 + \frac{1}{4}p^2 = \left(\frac{10}{\sqrt{5}}\right)^2$                                                                                                           | <b>DM1</b> | OE<br>Using <i>their</i> expression for $r^2$ equated to <i>their</i> $\left(\frac{10}{\sqrt{5}}\right)^2$ .       |
|          | $q = -15$                                                                                                                                                                | <b>A1</b>  |                                                                                                                    |
|          | <b>Method 3 for the last 3 marks:</b>                                                                                                                                    |            |                                                                                                                    |
|          | $4^2 + 3^2 + 4p + 6 + q = 0 \Rightarrow 4p + q + 31 = 0$<br>OR<br>$\left(4 - \left(-\frac{1}{2}p\right)\right)^2 + (3 - (-1))^2 = -q + 1 + \left(-\frac{1}{2}p\right)^2$ | <b>M1*</b> | Substituting (4,3) into their circle equation.                                                                     |
|          | $4(-4) + q + 31 = 0$                                                                                                                                                     | <b>DM1</b> | Substituting <i>their</i> $p = -4$ .                                                                               |
|          | $q = -15$                                                                                                                                                                | <b>A1</b>  |                                                                                                                    |

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| Question | Answer                                                                                                                                                                                                                                                                                                                                                                                                                     | Marks      | Guidance                                                                                                                                                                                                                                                                                                                                                                                    |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(b)(ii) | <b>Alternative Method for Question 8(b)(ii)</b>                                                                                                                                                                                                                                                                                                                                                                            |            |                                                                                                                                                                                                                                                                                                                                                                                             |
|          | $4^2 + 3^2 + 4p + 6 + q = 0$<br>$x^2 + (2x - 5)^2 + px + 2(2x - 5) + q = 0$ with $x = 4$<br>$x^2 + \left(\frac{10 - x}{2}\right)^2 + px + 2\left(\frac{10 - x}{2}\right) + q = 0$ with $x = 4$<br>$\left(\frac{y + 5}{2}\right)^2 + y^2 + p\left(\frac{y + 5}{2}\right) + 2y + q = 0$ with $y = 3$<br>$(10 - 2y)^2 + y^2 + p(10 - 2y) + 2y + q = 0$ with $y = 3$<br>$\{\text{Each of these} \Rightarrow 4p + q + 31 = 0\}$ | <b>M1*</b> | Substituting $(4, 3)$ into <i>their</i> circle equation, or replacing $y$ with $2x - 5$ from the normal equation, or replacing $y$ with $\frac{10 - x}{2}$ from the tangent equation, or replacing $x$ with $\frac{y + 5}{2}$ from the normal equation, or replacing $x$ with $10 - 2y$ from the tangent equation, and using either $x = 4$ or $y = 3$ to form an equation in $p$ and $q$ . |
|          | $\frac{5}{4}x^2 + (p - 6)x + 35 + q = 0 \Rightarrow (p - 6)^2 - 4 \times \frac{5}{4} \times (35 + q) = 0$<br>OR<br>$5y^2 - y(38 + 2p) + 100 + 10p + q = 0 \Rightarrow (38 + 2p)^2 - 4 \times 5 \times (100 + 10p + q) = 0$<br>$\{\text{Each of these} \Rightarrow p^2 - 12p - 139 - 5q = 0\}$                                                                                                                              | <b>M1*</b> | Solving the tangent and circle equations simultaneously to form a quadratic equation in either $x$ or $y$ . Then using $b^2 - 4ac = 0$ on their quadratic to form an equation in $p$ and $q$ .                                                                                                                                                                                              |
|          | Solving the equations simultaneously to find $p$ or $q$                                                                                                                                                                                                                                                                                                                                                                    | <b>DM1</b> |                                                                                                                                                                                                                                                                                                                                                                                             |
|          | $p = -4$                                                                                                                                                                                                                                                                                                                                                                                                                   | <b>A1</b>  |                                                                                                                                                                                                                                                                                                                                                                                             |
|          | $q = -15$                                                                                                                                                                                                                                                                                                                                                                                                                  | <b>A1</b>  |                                                                                                                                                                                                                                                                                                                                                                                             |
|          |                                                                                                                                                                                                                                                                                                                                                                                                                            | <b>5</b>   |                                                                                                                                                                                                                                                                                                                                                                                             |

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| Question | Answer                                                                                                                                                                                                                                                                              | Marks        | Guidance                                                                                                                                                                                                                                                              |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | $\left[ \frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = \frac{1}{2} \right]$ <p>OR</p> $\left[ \frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = k \times \frac{5}{2} + \left( \frac{1}{2} - \frac{5}{2}k \right) \right]$ $25k^2 - 40k + 12 [= 0]$ | <b>M1*</b>   | Using $\left( \frac{5}{2}, \frac{1}{2} \right)$ in the curve equation or equating the line and the curve and then using $x = \frac{5}{2}$ and $p = \frac{1}{2} - \frac{5}{2}k$ .<br>Simplify to get a three-term quadratic in $k$ . Condone errors in simplification. |
|          | $k = \frac{2}{5}$                                                                                                                                                                                                                                                                   | <b>A1</b>    | OE<br>Condone inclusion of $k = \frac{6}{5}$ .                                                                                                                                                                                                                        |
|          | $\frac{1}{2} = \left( \text{their } \frac{2}{5} \right) \left( \frac{5}{2} \right) + p \Rightarrow p =$                                                                                                                                                                             | <b>DM1*</b>  | Using $\left( \frac{5}{2}, \frac{1}{2} \right)$ and <i>their</i> $k$ in an equation in $p$ .<br>Either the line (as shown) or $4p^2 + 12p + 5 = 0$ are the most likely and solving for $p$ .                                                                          |
|          | $p = -\frac{1}{2}$                                                                                                                                                                                                                                                                  | <b>A1</b>    | OE<br>Condone inclusion of $p = -\frac{5}{2}$ .                                                                                                                                                                                                                       |
|          | $\frac{2}{25}x^2 - \frac{6}{5}x + \frac{5}{2} [= 0] \quad [4x^2 - 60x + 125 [= 0]]$                                                                                                                                                                                                 | <b>DM1</b>   | Equating the line and curve using <i>their</i> $k$ and $p$ and simplify to get a three-term quadratic [= 0].                                                                                                                                                          |
|          | $\left( \frac{25}{2}, \frac{9}{2} \right)$                                                                                                                                                                                                                                          | <b>A1 A1</b> | OE<br>Accept $x = \frac{25}{2}, y = \frac{9}{2}$ .                                                                                                                                                                                                                    |

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| Question | Answer                                                                                                                            | Marks        | Guidance                                                                                                                                                                                                                                      |
|----------|-----------------------------------------------------------------------------------------------------------------------------------|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | <b>Alternative Method for Question 9(a)</b>                                                                                       |              |                                                                                                                                                                                                                                               |
|          | $\left[ \frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = k \times \frac{5}{2} + p \right]$ $4p^2 + 12p + 5 [= 0]$ | <b>M1*</b>   | OE<br>Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ in the curve equation or equating the line and the curve and then using $x = \frac{5}{2}$ and $k = \frac{1}{5} - \frac{2}{5}p$ .<br>Simplify to get a three-term quadratic in $p [= 0]$ . |
|          | $p = -\frac{1}{2}$ OE                                                                                                             | <b>A1</b>    | Condone inclusion of $p = -\frac{5}{2}$ .                                                                                                                                                                                                     |
|          | $\frac{1}{2} = \left(\frac{5}{2}k\right) + \left(\text{their} - \frac{1}{2}\right) \Rightarrow k =$                               | <b>DM1*</b>  | Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ and <i>their</i> $p$ in the line equation and solving for $k$ .                                                                                                                                 |
|          | $k = \frac{2}{5}$                                                                                                                 | <b>A1</b>    | OE<br>Condone inclusion of $k = \frac{6}{5}$ .                                                                                                                                                                                                |
|          | $\frac{2}{25}x^2 - \frac{6}{5}x + \frac{5}{2} [= 0] \quad [4x^2 - 60x + 125 [= 0]]$                                               | <b>DM1</b>   | Equating the line and curve using <i>their</i> $k$ and $p$ and simplify to get a three-term quadratic [= 0].                                                                                                                                  |
|          | $\left(\frac{25}{2}, \frac{9}{2}\right)$                                                                                          | <b>A1 A1</b> | OE<br>Accept $x = \frac{25}{2}, y = \frac{9}{2}$ .                                                                                                                                                                                            |
|          |                                                                                                                                   | <b>7</b>     |                                                                                                                                                                                                                                               |

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| Question | Answer                                                                                            | Marks      | Guidance                                                                                                                                                 |
|----------|---------------------------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(b)     | $\left[ \frac{1}{2}k^2x^2 - 2kx + 2 = kx + p \Rightarrow \right] \frac{1}{2}k^2x^2 - 3kx + 2 - p$ | <b>M1*</b> | Equate the original equations of the curve and the line and collect like terms; $k$ and $p$ must still be present.                                       |
|          | $9k^2 - 4 \times \frac{1}{2}k^2(2 - p)$                                                           | <b>DM1</b> | Use of $b^2 - 4ac$ for their quadratic in $x$ to give an expression in $k$ and $p$ . This expression can come from <i>their</i> equation in <b>(a)</b> . |
|          | $p < -\frac{5}{2}$                                                                                | <b>A1</b>  |                                                                                                                                                          |
|          |                                                                                                   | <b>3</b>   |                                                                                                                                                          |

| Question | Answer                             | Marks     | Guidance                                         |
|----------|------------------------------------|-----------|--------------------------------------------------|
| 10(a)    | $-18$                              | <b>B1</b> | SOI                                              |
|          | $\frac{1}{18}$                     | <b>M1</b> | Use of $m_1m_2 = -1$ from $f'(x)$ with $x = 1$ . |
|          | $\frac{y[-0]}{x-1} = \frac{1}{18}$ | <b>A1</b> | OE<br>ISW                                        |
|          |                                    | <b>3</b>  |                                                  |

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| Question | Answer                                                                                                                                                                                                                                                                          | Marks       | Guidance                                                                                                                                        |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(b)    | $[f(x) =] \left\{ 8(2x-3)^{\frac{4}{3}} \cdot \frac{1}{2} \cdot \frac{1}{\frac{4}{3}} \right\} \left\{ -10x^{\frac{5}{3}} \cdot \frac{1}{\frac{5}{3}} \right\} [+c]$ $\left[ 3(2x-3)^{\frac{4}{3}} - 6x^{\frac{5}{3}} + c \right]$                                              | <b>B1B1</b> | B1 for each unsimplified {}.<br>Can be implied by equivalent simplified or partly simplified versions.                                          |
|          | $0 = 3(2(1)-3)^{\frac{4}{3}} - 6(1)^{\frac{5}{3}} + c \quad [0 = 3 - 6 + c]$                                                                                                                                                                                                    | <b>M1</b>   | Use of $x=1$ and $y=0$ in <i>their</i> integrated $f'(x)$ , defined as an expression with at least one correct power, which must contain $+c$ . |
|          | $[f(x) \text{ or } y =] 3(2x-3)^{\frac{4}{3}} - 6x^{\frac{5}{3}} + 3$                                                                                                                                                                                                           | <b>A1</b>   | Only condone $c=3$ as their final answer if all coefficients have previously been simplified in a correct statement.                            |
|          |                                                                                                                                                                                                                                                                                 | <b>4</b>    |                                                                                                                                                 |
| 10(c)    | $b^2 - 4ac = 128^2 - 4 \times 125 \times 192$ and stating “ $< 0$ ”<br>OR use of the quadratic formula and stating “No solutions”<br>OR completing the square for the given quadratic and stating positive or $> 0$ .<br>OR sketch of the given quadratic and stating positive. | <b>M1*</b>  | $b^2 - 4ac = -79616$ can be accepted in place of working.                                                                                       |
|          | No turning points [in the original function.]                                                                                                                                                                                                                                   | <b>DM1</b>  |                                                                                                                                                 |
|          | Decreasing because $f'(\text{any positive } x \text{ value}) < 0$                                                                                                                                                                                                               | <b>A1</b>   | WWW<br>e.g. $f'(1) = -18$ .                                                                                                                     |
|          |                                                                                                                                                                                                                                                                                 | <b>3</b>    |                                                                                                                                                 |

# Cambridge International AS & A Level

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**MATHEMATICS****9709/13**

Paper 1 Pure Mathematics 1

**October/November 2024****MARK SCHEME**Maximum Mark: 75

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**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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This document consists of **17** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**PUBLISHED****Mathematics Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer                                                                 | Marks      | Guidance                                                                                                                                               |
|----------|------------------------------------------------------------------------|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1        | $a + 3d = 15$ and $a + 7d = 25$                                        | <b>M1</b>  | Or forming any valid equations which can be used to find $d$ or $a$ .                                                                                  |
|          | Finding $a$ and $d$ $\left[ d = \frac{5}{2}, a = \frac{15}{2} \right]$ | <b>DM1</b> | Or any valid method to find $a$ using <i>their</i> $d$ or $d$ using <i>their</i> $a$ or finding $u_{30}$ directly from either $u_4$ or $u_8$ and $d$ . |
|          | $u_{30} = \frac{15}{2} + 29 \times \frac{5}{2} = 80$                   | <b>A1</b>  |                                                                                                                                                        |
|          |                                                                        | <b>3</b>   |                                                                                                                                                        |

| Question | Answer                                                                                              | Marks     | Guidance                                                                               |
|----------|-----------------------------------------------------------------------------------------------------|-----------|----------------------------------------------------------------------------------------|
| 2        | $\cos\left(\frac{\pi}{6}\right) + \tan 2x + \frac{\sqrt{3}}{2} = 0 \Rightarrow \tan 2x = -\sqrt{3}$ | <b>M1</b> | Making $\tan 2x$ the subject. $\tan 2x = 0$ is M0. Accept decimals and one sign error. |
|          | $\Rightarrow 2x = -\frac{\pi}{3} \Rightarrow x = -\frac{\pi}{6}$                                    | <b>A1</b> | May come from non-exact working. Ignore answers outside the given range.               |
|          |                                                                                                     | <b>2</b>  |                                                                                        |

| Question | Answer                                                                                                                | Marks     | Guidance                                                          |
|----------|-----------------------------------------------------------------------------------------------------------------------|-----------|-------------------------------------------------------------------|
| 3(a)     | $x^3 : \binom{5}{3} 3^2 (-ax)^3 [-10 \times 3^2 \times a^3]$ or $x^4 : \binom{5}{4} 3(-ax)^4 [5 \times 3 \times a^4]$ | <b>M1</b> | Allow for either term, allow sign error and combination notation. |
|          | $x^3 : -90a^3$                                                                                                        | <b>A1</b> | Allow in the full expansion.                                      |
|          | $x^4 : 15a^4$                                                                                                         | <b>A1</b> | Allow in the full expansion.                                      |
|          |                                                                                                                       | <b>3</b>  |                                                                   |

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| Question | Answer                                                                                                         | Marks      | Guidance                                       |
|----------|----------------------------------------------------------------------------------------------------------------|------------|------------------------------------------------|
| 3(b)     | Coefficient of $x^4$ is $a \times \text{their } -90a^3 + 7 \times \text{their } 15a^4$ $\left[ =15a^4 \right]$ | <b>M1</b>  | Must select two appropriate terms only.        |
|          | $15a^4 = 240$                                                                                                  | <b>DM1</b> | Reducing to a simple quartic equation in $a$ . |
|          | $\Rightarrow a^4 = 16 \Rightarrow a = 2$                                                                       | <b>A1</b>  | A0 if $a = -2$ is given as a solution.         |
|          |                                                                                                                | <b>3</b>   |                                                |

| Question | Answer                                                                                       | Marks        | Guidance                                                                                                                                                                                                     |
|----------|----------------------------------------------------------------------------------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4        | Let $x = \sin^2 \theta$<br>$(2x+7)(2x-1) = 0$ or $(2\sin^2 \theta + 7)(2\sin^2 \theta - 1)$  | <b>M1</b>    | Or equivalent method.                                                                                                                                                                                        |
|          | $\Rightarrow \sin^2 \theta = \frac{1}{2} \Rightarrow \sin \theta = [\pm] \frac{1}{\sqrt{2}}$ | <b>M1</b>    | Finding $\sin^2 \theta$ and then $\sin \theta$ (may be implied).                                                                                                                                             |
|          | $\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$                                         | <b>A1 A1</b> | A1 for any two correct values.<br>A1 for all correct and no others within the range.<br>For answers in radians, A1 only for all 4 angles.<br>If no (correct) working, then <b>SC B1</b> for all 4 solutions. |
|          |                                                                                              | <b>4</b>     |                                                                                                                                                                                                              |

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| Question | Answer                                                                                                                                                                     | Marks         | Guidance                                                                                                                                                                                                                                                                                                                                                                                       |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(a)     | Reflection [in] y-axis                                                                                                                                                     | <b>B1 B1</b>  | B1 for reflection B1 mention of y-axis, OE.<br><b>SC B2</b> for stretch, SF $-1$ , parallel to x-axis.                                                                                                                                                                                                                                                                                         |
|          | Translation or shift $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$                                                                                                               | <b>B1*</b>    | B1 for ‘translation’ and a correct vector/description.<br>Do not accept ‘left’/‘right’.<br>If two translations then B0 and B0 for the order.                                                                                                                                                                                                                                                   |
|          | Stretch, factor 2, parallel to y-axis                                                                                                                                      | <b>B2,1,0</b> | B2 all correct OE.<br>B1 any 2 parts correct.<br>This can be at any point in the sequence.                                                                                                                                                                                                                                                                                                     |
|          | Correct order and three correctly named transformations only                                                                                                               | <b>DB1</b>    | If a fourth transformation is given this mark is not awarded and no marks are given for the two transformations of the same type, except where the reflection is described as a stretch.<br>If any transformation is incorrectly named this cannot be given.<br><br>If translation is not $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ then DB0 is given. |
|          | <b>Alternative Solution for first 3 marks</b>                                                                                                                              |               |                                                                                                                                                                                                                                                                                                                                                                                                |
|          | Translation or shift $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$                                                                                                                | <b>B1*</b>    | B1 for ‘translation’ and correct vector/description.                                                                                                                                                                                                                                                                                                                                           |
|          | Reflection [in] y-axis                                                                                                                                                     | <b>B1 B1</b>  | B1 for ‘reflection’, B1 for ‘in y-axis’.                                                                                                                                                                                                                                                                                                                                                       |
|          | <b>Alternative solutions</b>                                                                                                                                               |               |                                                                                                                                                                                                                                                                                                                                                                                                |
|          | There are alternative solutions which can be marked in the same way<br>e.g. the given stretch, translation $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ , reflect in $x = -2.5$ |               |                                                                                                                                                                                                                                                                                                                                                                                                |
|          |                                                                                                                                                                            | <b>6</b>      |                                                                                                                                                                                                                                                                                                                                                                                                |

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| Question | Answer                                       | Marks     | Guidance                                                                               |
|----------|----------------------------------------------|-----------|----------------------------------------------------------------------------------------|
| 5(b)     | $g(x) = 2f(-x-1)$ or $a = 2, b = -1, c = -1$ | <b>B1</b> | First B1 for $a = 2$ and no additional terms added to the function.<br>$a = -2$ is B0. |
|          |                                              | <b>B1</b> | Second B1 for $b = -1$ and $c = -1$ .                                                  |
|          |                                              | <b>2</b>  |                                                                                        |

| Question | Answer                                                                                                                                          | Marks      | Guidance                                                                                                                                                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6        | $\frac{10(1-r^8)}{\frac{1-r}{10(1-r^4)}} = \frac{17}{16} \left[ a \frac{(1-r^8)}{(1-r)} = \frac{17}{16} \times a \frac{(1-r^4)}{(1-r)} \right]$ | <b>M1*</b> | OE, i.e. substituting $p$ and $q$ expressions into ratio $\frac{17}{16}$ .<br>$16 = a \frac{(1-r^4)}{(1-r)}, 17 = a \frac{(1-r^8)}{(1-r)}$ gets M0 unless recovered later. |
|          | Simplifying to $16r^8 - 17r^4 + 1 [=0]$ (or equivalent form)                                                                                    | <b>DM1</b> | Or $\frac{(1-r^8)}{(1-r^4)} = (1+r^4) = \frac{17}{16}$ .                                                                                                                   |
|          | $\left[ (16r^4 - 1)(r^4 - 1) = 0 \right] \Rightarrow r = \pm \frac{1}{2}$                                                                       | <b>A1</b>  | Or $r^4 = \frac{1}{16} \Rightarrow r = \pm \frac{1}{2}$ (condone extra $r = \pm 1$ solution).                                                                              |
|          | $S_{\infty} = \frac{10}{1 - \left( \left[ \pm \right] \frac{1}{2} \right)}$                                                                     | <b>DM1</b> | Use of correct sum to infinity formula with either of <i>their</i> $r$ values providing $ r  < 1$ .                                                                        |
|          | $S_{\infty} = 20 \text{ and } \frac{20}{3}$                                                                                                     | <b>A1</b>  | Allow 6.67 or better.<br>A0 if there is only one or more than two $S_{\infty}$ values.                                                                                     |
|          |                                                                                                                                                 | <b>5</b>   |                                                                                                                                                                            |

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| Question | Answer                                                                                                           | Marks      | Guidance                                                                                                                                     |
|----------|------------------------------------------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| 7(a)     | Area of sector $BOF = \frac{1}{2} \times 20^2 \times (2\pi - 2.4)$ [= 776.63...]                                 | <b>M1</b>  | Or combination of large semi-circle and small sector:<br>$\frac{1}{2} \times 20^2 \times \pi + \frac{1}{2} \times 20^2 \times (\pi - 2.4)$ . |
|          | Length $BD = DF = 2 \times 20 \sin 0.6$ or $\sqrt{20^2 + 20^2 - 2 \times 20 \times 20 \cos 1.2}$<br>[= 22.58...] | <b>M1*</b> | Length of radius of small circles is acceptable for M1.                                                                                      |
|          | Area of two semicircles = $\pi \times (20 \sin 0.6)^2$ [= 400.64...]                                             | <b>DM1</b> |                                                                                                                                              |
|          | Area of triangles = $2 \times \frac{1}{2} \times 20 \times 20 \sin 1.2$ [= 372.81...]                            | <b>M1</b>  |                                                                                                                                              |
|          | Total area = 1550 [cm <sup>2</sup> ]                                                                             | <b>A1</b>  | Expect 1550.09 but accept AWRT to 3sf.                                                                                                       |
|          |                                                                                                                  | <b>5</b>   |                                                                                                                                              |
| 7(b)     | $\frac{1}{2} \pi r^2 = 50\pi \Rightarrow r = 10$                                                                 | <b>B1</b>  | May be seen as $20 \sin \frac{\theta}{2}$ , where $\theta = \frac{\pi}{3}$ .                                                                 |
|          | $\Rightarrow \theta = \frac{\pi}{3}$                                                                             | <b>M1*</b> | OE<br>Finding $\theta$ using <i>their</i> $r$ . Allow working in degrees.                                                                    |
|          | Arc length of sector $BOF = 20 \times \left( 2\pi - \text{their } \frac{2\pi}{3} \right)$                        | <b>DM1</b> |                                                                                                                                              |
|          | Total perimeter = $20 \times \left( 2\pi - \text{their } \frac{2\pi}{3} \right) + 2\pi \times \text{their } 10$  | <b>DM1</b> | Dependent on the first dM1.                                                                                                                  |
|          | $\frac{140\pi}{3}$ or $46\frac{2}{3}\pi$                                                                         | <b>A1</b>  | Must be a single exact term.                                                                                                                 |
|          |                                                                                                                  | <b>5</b>   |                                                                                                                                              |

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| Question | Answer                                                       | Marks        | Guidance                                                        |
|----------|--------------------------------------------------------------|--------------|-----------------------------------------------------------------|
| 8(a)     | $3(x-2)^2 + 2$ or $a = -2, b = 2$                            | <b>B1 B1</b> |                                                                 |
|          |                                                              | <b>2</b>     |                                                                 |
| 8(b)     | 2 or $k = 2$ or $k \geq 2$                                   | <b>B1FT</b>  | FT on <i>their a</i> .<br>Do not accept $x = 2$ or $x \geq 2$ . |
|          |                                                              | <b>1</b>     |                                                                 |
| 8(c)     | $3(x-2)^2 + 14 - 12 = y \Rightarrow (x-2)^2 = \frac{y-2}{3}$ | <b>M1</b>    | Using <i>their</i> completed square form.                       |
|          | $x = [\pm] \sqrt{\frac{y-2}{3}} + 2$                         | <b>DM1</b>   |                                                                 |
|          | $f^{-1}(x) = \sqrt{\frac{x-2}{3}} + 2$                       | <b>A1</b>    | OE, e.g. $y = \frac{\sqrt{3x-6}}{3} + 2$ .                      |
|          |                                                              | <b>3</b>     |                                                                 |

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| Question | Answer                                                                     | Marks      | Guidance                                                                                                         |
|----------|----------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------|
| 8(d)     | Finding $f^{-1}(29)$ [= 5]                                                 | <b>M1</b>  | Or solving $f(x) = 29$ [using <i>their</i> completed square form, OE].                                           |
|          | Finding $f^{-1}(\text{their } 5)$                                          | <b>M1</b>  | Or solving $f(x) = \text{their } 5$ .                                                                            |
|          | $x = 3$                                                                    | <b>A1</b>  | If using $f(x)$ method, $x = 1$ must be discarded.                                                               |
|          | <b>Alternative solution for Question 8(d)</b>                              |            |                                                                                                                  |
|          | $3(3(x-2)^2 + 2) - 2)^2 + 2 = 29$ using <i>their</i> completed square form | <b>M1</b>  | Or $3(3x^2 - 12x + 14)^2 - 12(3x^2 - 12x + 14) + 14 = 29$ .<br>Allow if the '= 29' appears later in the working. |
|          | Solving as far as $9(x-2)^4 = 9$ or $x^2 - 4x + 3 = 0$                     | <b>DM1</b> | OE<br>Or $[27](x^4 - 8x^3 + 24x^2 - 32x + 15) = 0$ .                                                             |
|          | $x = 3$ only                                                               | <b>A1</b>  | WWW<br>Only dependent on the first M1.                                                                           |
|          |                                                                            | <b>3</b>   |                                                                                                                  |

| Question | Answer                                                                         | Marks      | Guidance                                                   |
|----------|--------------------------------------------------------------------------------|------------|------------------------------------------------------------|
| 9(a)     | $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3 \Rightarrow x^3 - 4x^2 + 3x [= 0]$ | <b>M1</b>  | Reducing to 3-term cubic or quadratic if $x$ cancelled.    |
|          | $[x](x-1)(x-3)[= 0]$                                                           | <b>DM1</b> | Factorising the cubic or quadratic.                        |
|          | $x = 0, 1$ and $3$ { $x = 0$ may be seen in the working}                       | <b>A1</b>  | <b>SC B1</b> for $x = 1, 3$ only, with no M marks awarded. |
|          |                                                                                | <b>3</b>   |                                                            |

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| Question | Answer                                                                                                                                                                                                                                                                                                                                                               | Marks      | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                  |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9(b)     | Attempt at integration of both functions. Can be before or after subtraction of the functions or integrals                                                                                                                                                                                                                                                           | <b>M1</b>  | Expect integration of $\int((x^3 - 3x + 3) - (2x^3 - 4x^2 + 3))dx$ or $\int(-x^3 + 4x^2 - 3x)dx$ .<br>At this stage, subtraction can be done either way.                                                                                                                                                                                                                                                                  |
|          | $= \pm \left( -\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} \right)$ or $\pm \left\{ \left( \frac{x^4}{4} - \frac{3}{2}x^2 + 3x \right) - \left( \frac{2}{4}x^4 - \frac{4}{3}x^3 + 3x \right) \right\}$                                                                                                                                                           | <b>A1</b>  | OE<br>$\pm$ covers A1 being awarded to those who subtract the ‘other’ way.                                                                                                                                                                                                                                                                                                                                                |
|          | $= \left[ \left( -\frac{81}{4} + \frac{108}{3} - \frac{27}{2} \right) - \left( -\frac{1}{4} + \frac{4}{3} - \frac{3}{2} \right) \right],$<br>or<br>$\left( \frac{81}{4} - \frac{27}{2} + 9 \right) - \left( \frac{1}{4} - \frac{3}{2} + 3 \right) - \left\{ \left( \frac{81}{2} - \frac{108}{3} + 9 \right) - \left( \frac{1}{2} - \frac{4}{3} + 3 \right) \right\}$ | <b>DM1</b> | OE<br>Minimum required is $\left( \frac{63}{4} - \frac{7}{4} \right) - \left( \frac{27}{2} - \frac{13}{6} \right)$ , i.e. four fractions.<br>Correctly apply limits <i>their</i> 1 and 3.<br>Do not allow if $x=0$ used.<br>Need at least one correct substitution in every bracket.<br>If two integrals, need to see substitution into both.<br>Allow one sign error only in each expression, if brackets are not shown. |
|          | $= \frac{8}{3}$                                                                                                                                                                                                                                                                                                                                                      | <b>A1</b>  | Accept if this comes from use of limits $f(1) - f(3)$ or $\int(x^3 - 4x^2 + 3x)dx$ , if $\left  \frac{-8}{3} \right $ used.<br>Only dependent on the first method mark.<br>Accept AWRT 2.67.                                                                                                                                                                                                                              |
|          |                                                                                                                                                                                                                                                                                                                                                                      | <b>4</b>   |                                                                                                                                                                                                                                                                                                                                                                                                                           |

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| Question | Answer                                                                                                         | Marks      | Guidance                                                                        |
|----------|----------------------------------------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------------|
| 10(a)    | Gradient of $AB = \frac{-5-3}{8-4} [= -2]$                                                                     | <b>M1*</b> |                                                                                 |
|          | Midpoint $AB = \left(\frac{8+4}{2}, \frac{-5+3}{2}\right) [(6, -1)]$                                           | <b>M1</b>  |                                                                                 |
|          | Gradient of normal $= -\frac{1}{-2} \left[ = \frac{1}{2} \right]$ and an attempt to find the required equation | <b>DM1</b> | Must be used to find equation of perpendicular through <i>their</i> $(6, -1)$ . |
|          | Equation of perpendicular bisector is $y+1 = \frac{1}{2}(x-6)$ , so $y = \frac{1}{2}x - 4$                     | <b>A1</b>  | WWW<br>AG – working involving the perpendicular bisector must be seen.          |
|          | <b>Alternative Method for Question 10(a)</b>                                                                   |            |                                                                                 |
|          | $AC^2 = (a-4)^2 + (b-3)^2$ , $BC^2 = (a-8)^2 + (b+5)^2$ both expanded                                          | <b>M1*</b> |                                                                                 |
|          | Solving $AC = BC$ [= 10]                                                                                       | <b>DM1</b> | Only allow a single sign error.                                                 |
|          | Eliminating $a^2$ and $b^2$                                                                                    | <b>DM1</b> | May be awarded before the previous DM1.                                         |
|          | $a = 2b + 8$ , concluding $y = \frac{x}{2} - 4$                                                                | <b>A1</b>  | WWW                                                                             |
|          |                                                                                                                | <b>4</b>   |                                                                                 |

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| Question | Answer                                                                                                                     | Marks      | Guidance                                                                                                                    |
|----------|----------------------------------------------------------------------------------------------------------------------------|------------|-----------------------------------------------------------------------------------------------------------------------------|
| 10(b)    | Using the centre as $\left(a, \frac{1}{2}a - 4\right)$                                                                     | <b>M1</b>  | May see centre as $(2y + 8, y)$ OE.<br>May be seen in an incorrect equation.                                                |
|          | $(4 - a)^2 + (3 - 0.5a + 4)^2 = 100$                                                                                       | <b>M1</b>  | Sub in $(4, 3)$ or $(8, -5)$ .<br>Could use circle with $(6, -1)$ and $r = \sqrt{80}$ .                                     |
|          | $1.25a^2 - 15a - 35 [= 0] \Rightarrow a^2 - 12a - 28 [= 0] \text{ (or } b^2 + 2b - 15 [= 0])$                              | <b>DM1</b> | Obtain a 3-term quadratic in <i>their</i> $x$ or $y$ .                                                                      |
|          | $[(a - 14)(a + 2) = 0] \Rightarrow a = 14, a = -2$                                                                         | <b>A1</b>  | Or $[(b - 3)(b + 5) = 0] \Rightarrow b = 3, b = -5$ .                                                                       |
|          | $\Rightarrow (x - 14)^2 + (y - 3)^2 = 100 \text{ and } (x + 2)^2 + (y + 5)^2 = 100$                                        | <b>A1</b>  |                                                                                                                             |
|          | <b>Alternative Method 1 for the first 3 marks:</b>                                                                         |            |                                                                                                                             |
|          | Make $a$ or $b$ the subject from a circle centre $(a, b)$ using $A$ or $B$                                                 | <b>M1</b>  | E.g. $b = \sqrt{100 - (y - 3)^2} + 4$ from circle through $A$ .<br>These equations may have been found in part <b>(a)</b> . |
|          | Form an equation in $a$ or $b$ only                                                                                        | <b>M1</b>  | Substitute <i>their</i> $a$ or $b$ into their second circle equation.                                                       |
|          | Simplify to a quadratic in $a$ or $b$                                                                                      | <b>DM1</b> | Expect $a^2 - 12a - 28 = 0$ or $b^2 + 2b - 15 = 0$ , OE.                                                                    |
|          | <b>Alternative Method 2 for the first 3 marks:</b>                                                                         |            |                                                                                                                             |
|          | Obtaining $CM$ ( $C$ , centre; $M$ , mid-point of $AB$ )                                                                   | <b>M1</b>  | Expect $\sqrt{80}$ . Must be clear this is $CM$ , not $AB$ .                                                                |
|          | Using the triangle $CMT$ , where $CT$ is parallel to the $x$ -axis, to find the vertical distance of $C$ from $M$ , $MT$   | <b>DM1</b> | Expect $MT = 4$ .                                                                                                           |
|          | Using the triangle $CMT$ , where $MT$ is parallel to the $y$ -axis, to find the horizontal distance of $C$ from $M$ , $CT$ | <b>DM1</b> | Expect $CT = 8$ .                                                                                                           |
|          |                                                                                                                            | <b>5</b>   |                                                                                                                             |

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| Question | Answer                                                                                                                                                                                                                                                                                 | Marks        | Guidance                                                                                                                                                                                                 |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11(a)    | $\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - 8x$                                                                                                                                                                                                                                    | <b>B1</b>    |                                                                                                                                                                                                          |
|          | $\frac{d^2y}{dx^2} = -\frac{1}{4}kx^{-\frac{3}{2}} - 8$                                                                                                                                                                                                                                | <b>B1</b>    |                                                                                                                                                                                                          |
|          |                                                                                                                                                                                                                                                                                        | <b>2</b>     |                                                                                                                                                                                                          |
| 11(b)    | $x^{-\frac{1}{2}} - 8x = 0 \Rightarrow 1 - 8x^{\frac{3}{2}} = 0$ or $x^{-1} = 64x^2 \left[ \Rightarrow x^3 = \frac{1}{64} \text{ or } 8x^{\frac{3}{2}} = 1 \right]$<br><br>Setting their $\frac{dy}{dx}$ to zero and solving, providing their only error(s) are incorrect coefficients | <b>M1</b>    | OE<br>Award if working leads to $x = \frac{1}{4}$ WWW.<br>Squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ to $x^{-1} - 64x^2 = 0$ gets M0.                                                                        |
|          | $x = \frac{1}{4}$ only                                                                                                                                                                                                                                                                 | <b>A1</b>    | If $x = 0$ included, A0 and max of 3/4.<br><b>SC B1</b> only for $x = \frac{1}{4}$ only from squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ directly to $x^{-1} - 64x^2 = 0$ ( <b>SC B1</b> replacing the M1A1). |
|          | $y = \frac{11}{4}$                                                                                                                                                                                                                                                                     | <b>A1</b>    | <b>SC B1</b> for $y = \frac{11}{4}$ from squaring $x^{-\frac{1}{2}} - 8x^2 = 0$ to $x^{-1} - 64x^2 = 0$ .                                                                                                |
|          | $\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}} - 8$ which is negative, so maximum                                                                                                                                                                                                   | <b>B1 FT</b> | WWW<br>FT <i>their</i> $x$ -value and <i>their</i> $\frac{d^2y}{dx^2}$ .<br>No FT if $x = 0$ is the only solution.                                                                                       |
|          |                                                                                                                                                                                                                                                                                        | <b>4</b>     |                                                                                                                                                                                                          |

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| Question | Answer                                                                                                                                                        | Marks      | Guidance                                                                                                                                                   |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 11(c)    | When $x = 1$ , attempting to find $y = k - 2$ and gradient $= \frac{1}{2}k - 8$                                                                               | <b>M1*</b> | OE<br><b>SC B1</b> if both correct gradients only, or both correct $y$ -coordinates only.                                                                  |
|          | Equation of tangent is $y - k + 2 = \left(\frac{1}{2}k - 8\right)(x - 1)$                                                                                     | <b>A1</b>  | OE, e.g. $y = \left(\frac{k}{2} - 8\right)x + \frac{k}{2} + 6$ or $y = \frac{k}{2}x - 8x + \frac{k}{2} + 6$ .                                              |
|          | When $x = \frac{1}{4}$ , attempting to find $y = \frac{1}{2}k + 1.75$ and gradient $= k - 2$                                                                  | <b>M1*</b> | OE                                                                                                                                                         |
|          | Equation of tangent is $y - \frac{1}{2}k - 1.75 = (k - 2)(x - 0.25)$                                                                                          | <b>A1</b>  | OE, e.g. $y = (k - 2)x + \frac{k}{4} + \frac{9}{4}$ or $y = kx - 2x + \frac{k}{4} + \frac{9}{4}$ .                                                         |
|          | Meet at $\left(\frac{1}{2}k - 8\right)(0.6 - 1) + k - 2 = (k - 2)(0.6 - 0.25) + \frac{1}{2}k + 1.75$<br>Equate two tangent equations and substitute $x = 0.6$ | <b>DM1</b> | OE, e.g. $\left(\frac{k}{2} - 8\right)0.6 + \frac{k}{2} + 6 = (k - 2)0.6 + \frac{k}{4} + \frac{9}{4}$ .<br>M0 if constants in both equations are the same. |
|          | $\Rightarrow [-0.2k + k + 3.2 - 2 = 0.35k - 0.7 + 0.5k + 1.75]$<br>$\Rightarrow 0.05k = 0.15$<br>$k = 3$                                                      | <b>A1</b>  |                                                                                                                                                            |
|          |                                                                                                                                                               | <b>6</b>   |                                                                                                                                                            |