

Cambridge International AS & A Level

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MATHEMATICS

9709/51

Paper 5 Probability & Statistics 1

October/November 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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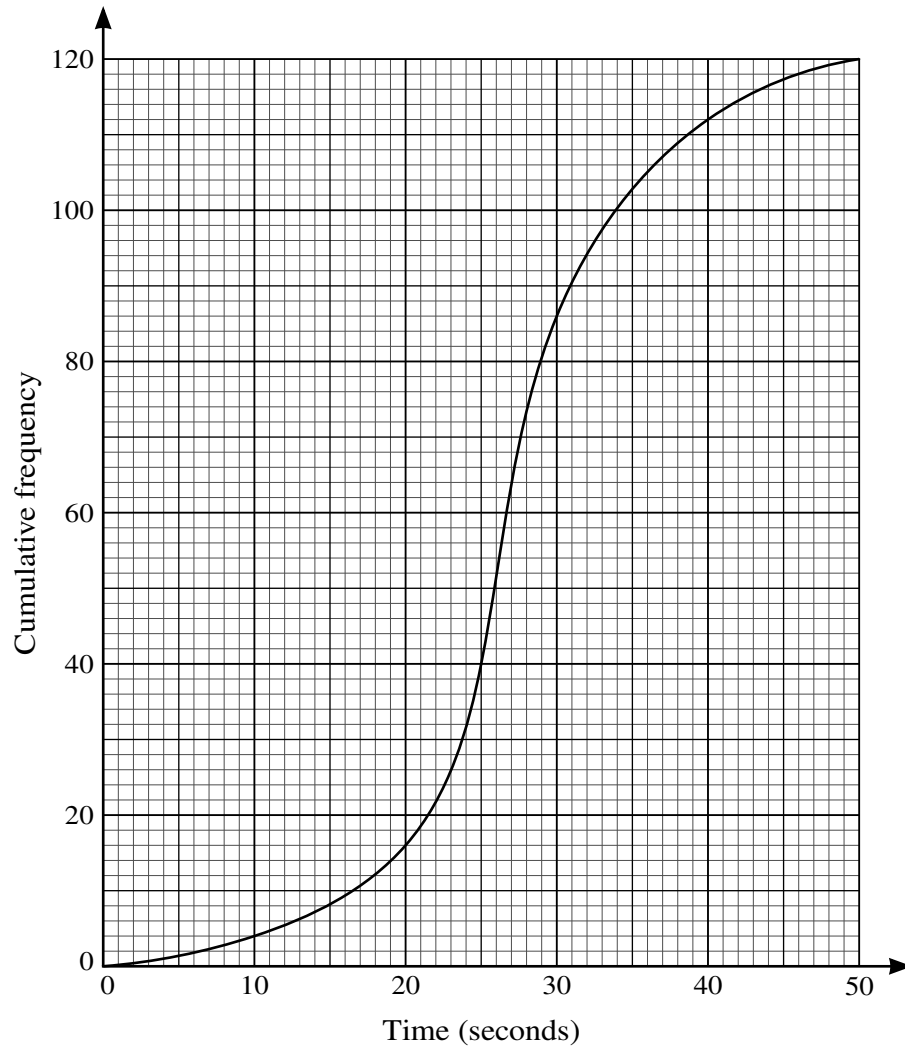
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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1



The times taken by 120 children to complete a particular puzzle are represented in the cumulative frequency graph.

- (a) Use the graph to estimate the interquartile range of the data. [2]

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35% of the children took longer than T seconds to complete the puzzle.

- (b) Use the graph to estimate the value of T . [2]

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- 2** Hazeem repeatedly throws two ordinary fair 6-sided dice at the same time. On each occasion, the score is the sum of the two numbers that she obtains.

- (a)** Find the probability that it takes exactly 5 throws of the two dice for Hazeem to obtain a score of 8 or more. [2]

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- (b)** Find the probability that it takes no more than 4 throws of the two dice for Hazeem to obtain a score of 8 or more. [2]

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- (c) For 8 randomly chosen throws of the two dice, find the probability that Hazeem obtains a score of 8 or more on fewer than 3 occasions. [3]

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- 3** A farmer sells eggs. The weights, in grams, of the eggs can be modelled by a normal distribution with mean 80.5 and standard deviation 6.6. Eggs are classified as small, medium or large according to their weight. A small egg weighs less than 76 grams and 40% of the eggs are classified as medium.

(a) Find the percentage of eggs that are classified as small. [3]

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(b) Find the least possible weight of an egg classified as large. [3]

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150 of the eggs for sale last week were weighed.

- (c) Use an approximation to find the probability that more than 68 of these eggs were classified as medium. [5]

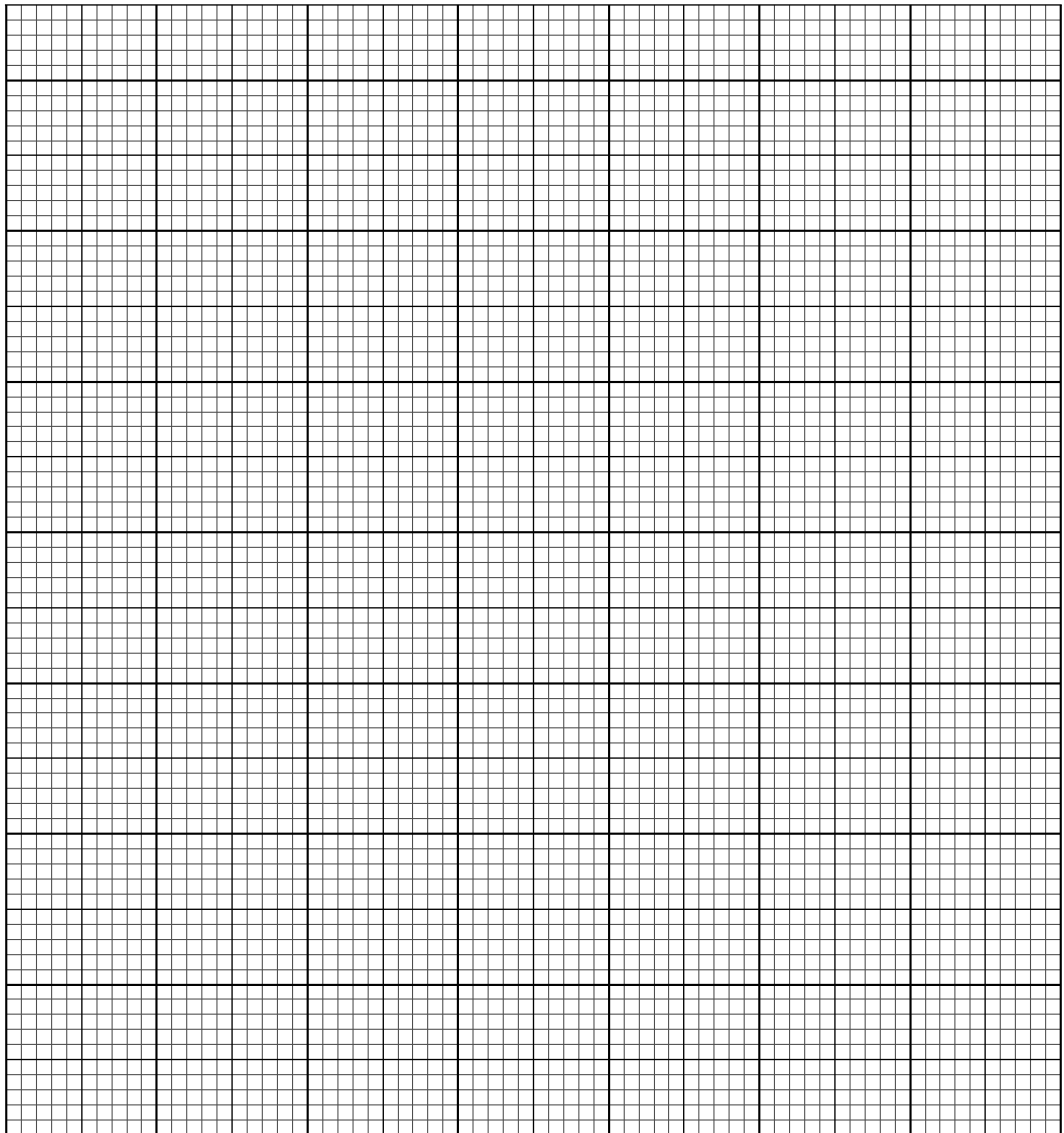
[illegible]

- 4 The times, to the nearest minute, of 150 athletes taking part in a charity run are recorded. The results are summarised in the table.

Time in minutes	101 – 120	121 – 130	131 – 135	136 – 145	146 – 160
Frequency	18	48	34	32	18

- (a) Draw a histogram to represent this information.

[4]



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- 5** A red spinner has four sides labelled 1, 2, 3, 4. When the spinner is spun, the score is the number on the side on which it lands. The random variable X denotes this score. The probability distribution table for X is given below.

x	1	2	3	4
$P(X = x)$	0.28	p	$2p$	$3p$

- (a) Show that $p = 0.12$. [1]

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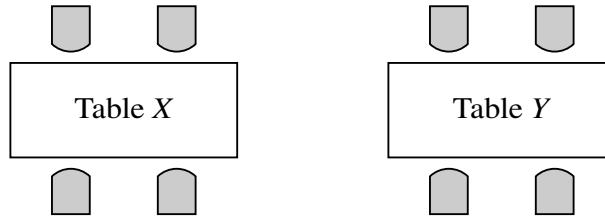
A fair blue spinner and a fair green spinner each have four sides labelled 1, 2, 3, 4. All three spinners (red, blue and green) are spun at the same time.

- (b) Find the probability that the sum of the three scores is 4 or less. [3]

[illegible]

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6



In a restaurant, the tables are rectangular. Each table seats four people: two along each of the longer sides of the table (see diagram). Eight friends have booked two tables, X and Y . Rajid, Sue and Tan are three of these friends.

- (a) The eight friends will be divided into two groups of 4, one group for table X and one group for table Y .

Find the number of ways in which this can be done if Rajid and Sue must sit at the same table as each other and Tan must sit at the other table. [3]

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When the friends arrive at the restaurant, Rajid and Sue now decide to sit at table X on the same side as each other. Tan decides that he does not mind at which table he sits.

- (b) Find the number of different seating arrangements for the 8 friends. [3]

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As they leave the restaurant, the 8 friends stand in a line for a photograph.

- (c) Find the number of different arrangements if Rajid and Sue stand next to each other, but neither is at an end of the line. [4]

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MATHEMATICS

9709/52

Paper 5 Probability & Statistics 1

October/November 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

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INFORMATION

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This document has **12** pages.

- 1 A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below.

x	0	1	2	3
$P(X = x)$	0.4	p	r	0.15

- (a) Given that $E(X) = 1.1$, find the value of p and the value of r . [3]

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- (b) Find the numerical value of $\text{Var}(X)$. [2]

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- 2 George has a fair 5-sided spinner with sides labelled 1, 2, 3, 4, 5. He spins the spinner and notes the number on the side on which the spinner lands.

- (a) Find the probability that it takes fewer than 7 spins for George to obtain a 5. [2]

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George spins the spinner 10 times.

- (b) Find the probability that he obtains a 5 more than 4 times but fewer than 8 times. [3]

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- 3** A factory produces a certain type of electrical component. It is known that 15% of the components produced are faulty. A random sample of 200 components is chosen.

Use an approximation to find the probability that more than 40 of these components are faulty. [5]

[illegible]

- 4 The heights, in cm, of the 11 players in each of two teams, the Aces and the Jets, are shown in the following table.

Aces	180	174	169	182	181	166	173	182	168	171	164
Jets	175	174	188	168	166	174	181	181	170	188	190

- (a) Draw a back-to-back stem-and-leaf diagram to represent this information with the Aces on the left-hand side of the diagram. [4]

- (b) Find the median and the interquartile range of the heights of the players in the Aces. [3]

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- (c) Give one comment comparing the spread of the heights of the Aces with the spread of the heights of the Jets. [1]

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- 5 (a) The heights of the members of a club are normally distributed with mean 166 cm and standard deviation 10 cm.

(i) Find the probability that a randomly chosen member of the club has height less than 170 cm. [2]

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(ii) Given that 40% of the members have heights greater than h cm, find the value of h correct to 2 decimal places. [3]

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- (b) The random variable X is normally distributed with mean μ and standard deviation σ .

Given that $\sigma = \frac{2}{3}\mu$, find the probability that a randomly chosen value of X is positive. [3]

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- 6** Freddie has two bags of marbles.

Bag *X* contains 7 red marbles and 3 blue marbles.

Bag *Y* contains 4 red marbles and 1 blue marble.

Freddie chooses one of the bags at random. A marble is removed at random from that bag and not replaced. A new red marble is now added to each bag. A second marble is then removed at random from the same bag that the first marble had been removed from.

- (a) Draw a tree diagram to represent this information, showing the probability on each of the branches. [3]

- (b) Find the probability that both of the marbles removed from the bag are the same colour. [4]

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- (c) Find the probability that bag Y is chosen given that the marbles removed are **not** both the same colour. [2]

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- 7 (a) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which no consonant is next to another consonant. (The letters D, M, N and R are consonants and the letters A, E and O are **not** consonants.) [3]

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- (b) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which there is an A at each end and the Ds are **not** together. [3]

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- 1** Becky sometimes works in an office and sometimes works at home. The random variable X denotes the number of days that she works at home in any given week. It is given that

$$P(X = x) = kx(x + 1),$$

where k is a constant and $x = 1, 2, 3$ or 4 only.

- (a)** Draw up the probability distribution table for X , giving the probabilities as numerical fractions. [3]

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- (b)** Find $E(X)$ and $\text{Var}(X)$. [3]

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- 2 The weights of large bags of pasta produced by a company are normally distributed with mean 1.5 kg and standard deviation 0.05 kg.

- (a) Find the probability that a randomly chosen large bag of pasta weighs between 1.42 kg and 1.52 kg. [3]

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The weights of small bags of pasta produced by the company are normally distributed with mean 0.75 kg and standard deviation σ kg. It is found that 68% of these small bags have weight less than 0.9 kg.

- (b) Find the value of σ . [3]

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- 3 Tim has two bags of marbles, A and B .

Bag A contains 8 white, 4 red and 3 yellow marbles.

Bag B contains 6 white, 7 red and 2 yellow marbles.

Tim also has an ordinary fair 6-sided dice. He rolls the dice. If he obtains a 1 or 2, he chooses two marbles at random from bag A , without replacement. If he obtains a 3, 4, 5 or 6, he chooses two marbles at random from bag B , without replacement.

- (a) Find the probability that both marbles are white. [3]

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- (b) Find the probability that the two marbles come from bag B given that one is white and one is red. [4]

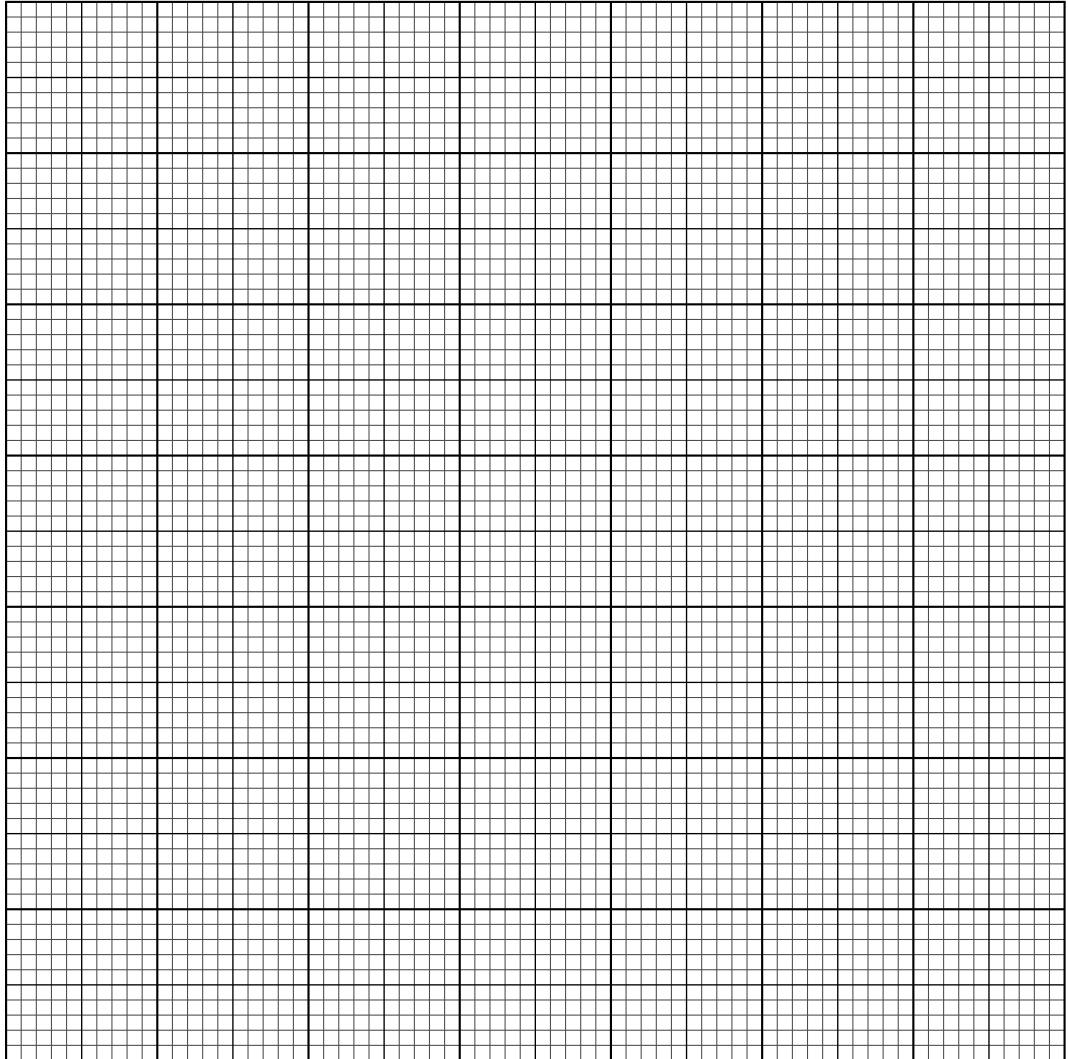
[illegible]

- 4 The weights, x kg, of 120 students in a sports college are recorded. The results are summarised in the following table.

Weight (x kg)	$x \leq 40$	$x \leq 60$	$x \leq 65$	$x \leq 70$	$x \leq 85$	$x \leq 100$
Cumulative frequency	0	14	38	60	106	120

- (a) Draw a cumulative frequency graph to represent this information.

[2]



- (b) It is found that 35% of the students weigh more than W kg.

Use your graph to estimate the value of W .

[2]

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- (c) Calculate estimates for the mean and standard deviation of the weights of the 120 students. [6]

[illegible]

5 The probability that a driver passes an advanced driving test is 0.3 on any given attempt.

(a) Dipak keeps taking the test until he passes. The random variable X denotes the number of attempts required for Dipak to pass the test.

(i) Find $P(2 \leq X \leq 6)$. [2]

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(ii) Find $E(X)$. [1]

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Five friends will each take their advanced driving test tomorrow.

(b) Find the probability that at least three of them will pass tomorrow. [3]

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75 people will take their advanced driving test next week.

- (c) Use an approximation to find the probability that more than 20 of them will pass next week. [5]

[illegible]

6 Jai and his wife Kaz are having a party. Jai has invited five friends and each friend will bring his wife.

(a) At the beginning of the party, the 12 people will stand in a line for a photograph.

- (i)** How many different arrangements are there of the 12 people if Jai stands next to Kaz and each friend stands next to his own wife? [3]

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- (ii)** How many different arrangements are there of the 12 people if Jai and Kaz occupy the two middle positions in the line, with Jai's five friends on one side and the five wives of the friends on the other side? [2]

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- (b) For a competition during the party, the 12 people are divided at random into a group of 5, a group of 4 and a group of 3.

Find the probability that Jai and Kaz are in the same group as each other.

[5]

[illegible]

[illegible]

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