



Cambridge International AS & A Level

MATHEMATICS

9709/51

Paper 5 Probability & Statistics 1

October/November 2023

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	[IQR =] 31 – 23.7	M1	$30.5 < UQ < 31.25 - 23.25 < LQ \leq 24$ Evidence of graph use must be seen at least once.
	7.3	A1	$7.0 \leq IQR \leq 7.5$ If M0 scored, SC B1 for $7.0 \leq IQR \leq 7.5$ www.
		2	
1(b)	[65% of 120 =]78	B1	Seen or implied by use on graph.
	28.5	B1	$28 < \text{ans} < 29$
		2	

Question	Answer	Marks	Guidance
2(a)	$\left(\frac{21}{36}\right)^4 \left(\frac{15}{36}\right)$	M1	$(1-p)^4 \times p, 0 < p < 1$
	$= \frac{12005}{248832}, 0.0482$	A1	0.0482454... to at least 3SF.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
2(b)	Method 1		
	$[P(X \leq 4) =] 1 - \left(\frac{21}{36}\right)^4$	M1	$1 - b^r$, $b = \text{their } (1 - p)$ in 2(a) or correct; $r = 4, 5$.
	$= \frac{18335}{20736}, 0.884$	A1	0.884211... to at least 3SF.
		2	
	Method 2		
	$[P(X \leq 4) =] \frac{15}{36} + \frac{15}{36} \times \frac{21}{36} + \frac{15}{36} \times \left(\frac{21}{36}\right)^2 + \frac{15}{36} \times \left(\frac{21}{36}\right)^3$	M1	$p + p(1 - p) + p(1 - p)^2 + p(1 - p)^3$ [$+ p(1 - p)^4$] FT from 2(a) or correct.
	$= \frac{18335}{20736}, 0.884$	A1	0.884211... to at least 3SF.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
2(c)	Method 1		
	$[P(0,1,2) =] {}^8C_0 \left(\frac{5}{12}\right)^0 \left(\frac{7}{12}\right)^8 + {}^8C_1 \left(\frac{5}{12}\right)^1 \left(\frac{7}{12}\right)^7 + {}^8C_2 \left(\frac{5}{12}\right)^2 \left(\frac{7}{12}\right)^6$	M1	One term ${}^8C_x (q)^x (1-q)^{8-x}$, $0 < q < 1$, $x \neq 0, 8$.
	0.01341 + 0.07661 + 0.1915	A1 FT	Correct expression, accept unsimplified, no terms omitted leading to final answer. FT only with unsimplified expression.
	= 0.282	B1	$0.2815 \leq q \leq 0.282$
	Method 2		
	$[1 - P(3,4,5,6,7,8) =] 1 - ({}^8C_3 \left(\frac{5}{12}\right)^3 \left(\frac{7}{12}\right)^5 + {}^8C_4 \left(\frac{5}{12}\right)^4 \left(\frac{7}{12}\right)^4 + \dots + {}^8C_7$ $\left(\frac{5}{12}\right)^7 \left(\frac{7}{12}\right)^1 + {}^8C_8 \left(\frac{5}{12}\right)^8 \left(\frac{7}{12}\right)^0)$	M1	One term ${}^8C_x (q)^x (1-q)^{8-x}$, $0 < q < 1$, $x \neq 0, 8$.
	= 1 – (0.2736 + 0.2443 + ... + 0.01017 + 9.084×10 ⁻⁴)	A1 FT	Correct expression, accept unsimplified, no terms omitted leading to final answer. FT only with unsimplified expression.
	= 0.282	B1	$0.2815 \leq q \leq 0.282$
		3	

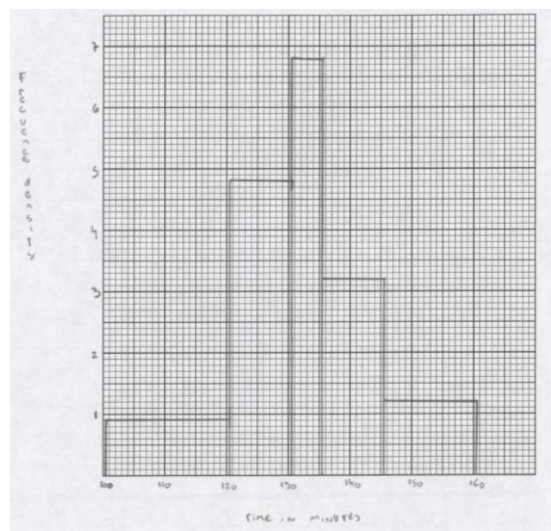
PUBLISHED

Question	Answer	Marks	Guidance
3(a)	$[P(X < 76) =] P(Z < \frac{76 - 80.5}{6.6})$	M1	Use of $\pm$ standardisation formula with 76, 80.5 and 6.6, condone 6.6^2 or $\sqrt{6.6}$, no continuity correction.
	$[= \Phi(-0.6818) = 1 - \Phi(0.6818) =]$ $1 - 0.7524 = 0.2476$	M1	Calculating the appropriate probability area (leading to their final answer).
	24.8%	A1	24.75% < ans $\leq$ 24.8% (percentage value required). If A0 scored, SC B1 for 24.75% < ans $\leq$ 24.8% www.
		3	
3(b)	[% of large eggs = 100 – 40 – 24.76 = 35.24]	B1	0.378 $\leq$ z < 0.3791 or $-0.3791 < z \leq -0.378$ seen.
	$[P(Z > \frac{x - 80.5}{6.6}) = 0.40 + 0.2476 = 0.6476]$ $\frac{x - 80.5}{6.6} = 0.378$	M1	Use of $\pm$ standardisation formula with x, 80.5, 6.6 and a z-value (not 0.6476, 0.3524, 0.4, 0.2476) (treat ± 0.38 as a z-value), not 6.6^2 , not $\sqrt{6.6}$, no continuity correction.
	$x = 83[.0]$	A1	awrt 83.0
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3(c)	Mean = $150 \times 0.4 = 60$ Var = $150 \times 0.4 \times 0.6 = 36$	B1	60 and 36 seen, allow unsimplified.
	$P(X > 68) = P\left(Z > \frac{68.5 - 60}{\sqrt{36}}\right)$	M1	Substituting <i>their</i> 60 and <i>their</i> 6 into $\pm$ standardisation formula (any number for 68.5), condone <i>their</i> σ^2 and <i>their</i> $\sqrt{\sigma}$.
		M1	Using continuity correction 67.5 or 68.5 in <i>their</i> standardisation formula.
	$P(Z > 1.417) = 1 - \Phi(1.417)$ [= $1 - 0.9217$]	M1	Appropriate area Φ , from final process, must be a probability.
	0.0783	A1	$0.07825 < p \leq 0.0783$ If A0 scored, SC B1 for $0.07825 < p \leq 0.0783$.
		5	

PUBLISHED

Question	Answer	Marks	Guidance												
4(a)	<table><tr><td>Class width</td><td>20</td><td>10</td><td>5</td><td>10</td><td>15</td></tr><tr><td>Frequency density</td><td>0.9</td><td>4.8</td><td>6.8</td><td>3.2</td><td>1.2</td></tr></table>	Class width	20	10	5	10	15	Frequency density	0.9	4.8	6.8	3.2	1.2	M1	At least 4 frequency densities calculated by $\frac{f}{cw}$ e.g. $\frac{18}{20}$ (condone $\frac{f}{cw \pm 0.5}$ if unsimplified). Accept unsimplified, may be read from graph using <i>their</i> scale, no lower than 1cm = 1 fd.
	Class width	20	10	5	10	15									
	Frequency density	0.9	4.8	6.8	3.2	1.2									
		A1	All bar heights correct on graph (no FT), using their suitable linear scale with at least 3 values indicated, no lower than 1cm = 1 fd.												
B1		Bar ends at 120.5, 130.5, 135.5, 145.5, 160.5. 5 bars drawn with a horizontal linear scale, no lower than 1 cm = 10 min, with at least 3 values indicated. $100 \leq \text{horizontal scale} \leq 160$.													
		B1	Axes labelled frequency density (fd), time (t) and minutes (min, m) oe, or an appropriate title. (Axes may be reversed).												
		4													

PUBLISHED

Question	Answer	Marks	Guidance
4(b)	[Midpoints] 110.5 125.5 133 140.5 153	B1	At least 4 correct mid-points seen, may be by data table or used in formula.
	Mean = $\frac{18 \times 110.5 + 48 \times 125.5 + 34 \times 133 + 32 \times 140.5 + 18 \times 153}{150}$ = $\frac{1989 + 6024 + 4522 + 4496 + 2754}{150}$	M1	Correct formula for mean using midpoints ± 0.5 , condone 1 midpoint error within class.
	= 131.9	A1	Accept 132, $131\frac{9}{10}$, or $\frac{1319}{10}$. Must be identified.
	Variance = $\frac{18 \times 110.5^2 + 48 \times 125.5^2 + 34 \times 133^2 + 32 \times 140.5^2 + 18 \times 153^2}{150} - (their\ 131.9)^2$	M1	Appropriate variance formula with <i>their</i> 5 midpoints within class (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency). Condone 1 error. If correct midpoints seen, accept $\left\{ \frac{3200 + 41400 + 194400 + 157300 + 153600}{150} \text{ or } \frac{2630272.5}{150} \right\}$ – {131.9 ² or 17397.61}.
	[= 137.54] [Standard deviation =] 11.7	A1	11.7277448... to at least 3SF. Accept $11.6 \leq \sigma < 11.95$ www. If M0 awarded, SC B1 $11.6 \leq \sigma < 11.95$ www.
		5	

PUBLISHED

Question	Answer	Marks	Guidance																									
5(a)	$0.28 + 6p = 1, \quad p = 0.12$	B1	Using sum of probabilities = 1 to form an equation. Accept $0.28 + p + 2p + 3p = 1, p = 0.12$. Substitution of 0.12 into the expression scores B0.																									
		1																										
5(b)	[For fair spinners (blue and green), probability of any score is 0.25 Scenarios to give total 4 or less:]	B1	Correct probability for 1 identified scenario, accept unsimplified, www.																									
	<table><tr><td>R</td><td>B</td><td>G</td><td></td><td></td></tr><tr><td>1</td><td>1</td><td>2</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>2</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>1</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>2</td><td>1</td><td>1</td><td>$0.12 \times (0.25)^2$</td><td>= 0.0075</td></tr></table>	R	B	G			1	1	2	$0.28 \times (0.25)^2$	= 0.0175	1	2	1	$0.28 \times (0.25)^2$	= 0.0175	1	1	1	$0.28 \times (0.25)^2$	= 0.0175	2	1	1	$0.12 \times (0.25)^2$	= 0.0075	M1	Add values of 4 correct scenarios, may be implied by correct unsimplified expressions. No incorrect/repeated scenarios.
	R	B	G																									
	1	1	2	$0.28 \times (0.25)^2$	= 0.0175																							
	1	2	1	$0.28 \times (0.25)^2$	= 0.0175																							
	1	1	1	$0.28 \times (0.25)^2$	= 0.0175																							
	2	1	1	$0.12 \times (0.25)^2$	= 0.0075																							
	0.06	A1	If A0 scored, SC B1 for 0.06 www.																									
		3																										

PUBLISHED

Question	Answer	Marks	Guidance																																																		
5(c)	[P(X is odd) = 0.28 + 2×0.12 or 0.24]= 0.52[0]	B1	Seen alone or as the denominator of a conditional probability fraction. Accept unsimplified.																																																		
	<table><tr><td>R</td><td>B</td><td>G</td><td></td><td></td></tr><tr><td>1</td><td>1</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>1</td><td>2</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>1</td><td>3</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>1</td><td>4</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>2</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>2</td><td>2</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>3</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>1</td><td>4</td><td>1</td><td>$0.28 \times (0.25)^2$</td><td>= 0.0175</td></tr><tr><td>3</td><td>1</td><td>1</td><td>$0.24 \times (0.25)^2$</td><td>= 0.015</td></tr></table>	R	B	G			1	1	1	$0.28 \times (0.25)^2$	= 0.0175	1	1	2	$0.28 \times (0.25)^2$	= 0.0175	1	1	3	$0.28 \times (0.25)^2$	= 0.0175	1	1	4	$0.28 \times (0.25)^2$	= 0.0175	1	2	1	$0.28 \times (0.25)^2$	= 0.0175	1	2	2	$0.28 \times (0.25)^2$	= 0.0175	1	3	1	$0.28 \times (0.25)^2$	= 0.0175	1	4	1	$0.28 \times (0.25)^2$	= 0.0175	3	1	1	$0.24 \times (0.25)^2$	= 0.015	M1	Values of at least 5 identified correct scenarios added, accept unsimplified, condone incorrect scenarios in calculation.
	R	B	G																																																		
1	1	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	2	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	3	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	1	4	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	2	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	2	2	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	3	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
1	4	1	$0.28 \times (0.25)^2$	= 0.0175																																																	
3	1	1	$0.24 \times (0.25)^2$	= 0.015																																																	
	[P(product of 3 scores ≤ 4 ∩ X is odd) =] $0.28 \times (0.25)^2 \times 8 + 0.24 \times (0.25)^2$	M1	$0.28 \times (0.25)^2 \times x + 0.24 \times (0.25)^2$, or $0.0175 \times x + 0.015$ where $x = 4, 5, 6, 7$, or 8. Seen alone or as numerator/denominator of a conditional probability fraction.																																																		

PUBLISHED

Question	Answer	Marks	Guidance
5(c)	$\left[\frac{P(\text{product of 3 scores} \leq 4 \mid X \text{ is odd})}{P(X \text{ is odd})} = \right]$ $\frac{0.155}{0.52}$	M1	$\frac{0.28 \times (0.25)^2 \times x + 0.24 \times (0.25)^2}{0.28 + 0.24} \quad x = 4, 5, 6, 7, 8$ or $\frac{\text{their identified } P(\text{product of 3 scores is 4 or less and } X \text{ is odd})}{\text{their identified } P(\text{odd})}.$
	$= 0.298, \frac{155}{520}, \frac{31}{104}$	A1	0.2980769... to at least 3SF.
		5	

Question	Answer	Marks	Guidance
6(a)	${}^5C_2 \times 2$	M1	${}^5C_2 \times r$, r = positive integer, 1 implied, no addition.
		M1	$s \times 2$, $s = {}^5C_2$ or ${}^5P_2$ or if ${}^5C_2$ or ${}^5P_2$ not present, s = a single integer > 1 or $t! \times 2$, $2 \leq t \leq 8$, no other terms.
	20	A1	
		3	

PUBLISHED

Question	Answer	Marks	Guidance
6(b)	Method 1		
	${}^6C_2 \times 2 \times 2 \times 2 \times 4!$	M1	${}^6C_2 \times 2 \times 2 \times 2 \times t, t = \text{positive integer} \geq 1.$ ${}^6P_2 \times 2 \times 2 \times t, t = \text{positive integer} \geq 1.$
		M1	$u \times 4!, u = \text{positive integer} > 1.$
	2880	A1	If A0 scored, SC B1 for 2880 nfw.
	Method 2		
	$6! \times 2 \times 2$	M1	$6! \times v, v = \text{positive integer} \geq 1.$
		M1	$w \times 2 \times 2, w = \text{positive integer} > 1.$ condone $w \times 4, w = \text{positive integer} > 1.$
	2880	A1	If A0 scored, SC B1 for 2880 nfw.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
6(c)	Method 1: Number of arrangements with Rajid and Sue together – Number of arrangements with Rajid and Sue together and at end of line		
	$7! \times 2 - 6! \times 4$	M1	$7! \times 2 - a$, a = positive integer > 1 .
		M1	$b - 6! \times 4$, b = positive integer > 2880 .
		M1	$7! \times c - 6! \times d$, $c = 1, 2$ and $d = 1, 4$.
	$= 7200$	A1	If A0 scored, SC B1 for 7200 nfw.
	Method 2: Arrangements of 6 people and then place Rajid and Sue		
	$6! \times 2 \times 5$	M1	$6! \times e \times f$, e, f = positive integers ≥ 1 .
		M1	$6! \times 2 \times f$, f = positive integer ≥ 1 . If 5! Used, SC B1 $5! \times 2 \times f$, f = positive integer > 1 .
		M1	$6! \times e \times 5$, e = positive integer ≥ 1 .
	7200	A1	If A0, scored SC B1 for 7200 nfw.
	Method 3: Friends at ends picked first F ^ RS ^ ^ ^ F		
	${}^6P_2 \times 5! \times 2$	M1	${}^6P_2 \times e \times f$, e, f = positive integers ≥ 1 .
		M1	${}^6P_2 \times 5! \times f$, f = positive integer ≥ 1 . Condone ${}^6C_2 \times 5! \times f$, f = positive integer ≥ 1 .
		M1	${}^6P_2 \times e \times 2$, e = positive integer ≥ 1 . Condone ${}^6C_2 \times e \times 2$, e = positive integer ≥ 1 .
	7200	A1	If A0 scored, SC B1 for 7200 nfw.

PUBLISHED

Question	Answer	Marks	Guidance
6(c)	Method 4: RS placed in different possible positions		
	${}^6P_1 \times 2 \times 5! = 1440$	M1	${}^6P_n \times a \times (6-n)!$, a = positive integer, $1 \leq n \leq 5$ seen once.
	${}^6P_2 \times 2 \times 4! = 1440$	M1	${}^6P_n \times 2 \times (6-n)!$, a = positive integer, $1 \leq n \leq 5$ seen at least 3 times in identified scenarios.
	${}^6P_3 \times 2 \times 3! = 1440$		
	${}^6P_4 \times 2 \times 2! = 1440$	M1	Add 5 values of appropriate scenarios only. No additional, incorrect or repeated scenarios. Accept unsimplified.
	${}^6P_5 \times 2 \times 1! = 1440$		
	7200	A1	If A0 scored, SC B1 for 7200 nfw.
		4	



Cambridge International AS & A Level

MATHEMATICS

9709/52

Paper 5 Probability & Statistics 1

October/November 2023

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance
1(a)	$p + r + 0.55 = 1$	M1	Using sum of probabilities = 1 to form an equation. Accept $p + r = 0.45$ oe.
	$p + 2r + 0.45 = 1.1$	M1	Use $E(X) = 1.1$ to form an equation. Accept $p + 2r = 0.65$ oe. NB: These marks can be gained in either order; the second M may have an algebraic substitution.
	$p = 0.25, r = 0.2$	A1	If both Ms not awarded, SC B1 for $p = 0.25, r = 0.2$ stated.
		3	
1(b)	$[\text{Var}(X) =] [0.4 \times 0^2 +] \text{their } 0.25 [\times 1^2] + (\text{their } 0.2) \times 2^2 + 0.15 \times 3^2 - 1.1^2$ [=[0+] 0.25 + 0.8 + 1.35 – 1.21]	M1	Correct formula for variance method using their probability distribution table, $0 < \text{their } P(x) < 1$.
	$= 1.19, 1 \frac{19}{100}$	A1	If M0 awarded, SC B1 for 1.19 www. $\frac{119}{100}$ is A0.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
2(a)	Method 1:		
	$[P(5) = 0.2]$ $[P(X < 7) =] 1 - 0.8^6$	M1	$1 - 0.8^n, n = 6, 7.$
	$= 0.738, \frac{11529}{15625}$	A1	0.737856 to at least 3SF.
	Method 2:		
	$[P(X < 7) =] 0.2 + 0.2 \times 0.8 + 0.2 \times 0.8^2 + 0.2 \times 0.8^3 + 0.2 \times 0.8^4 + 0.2 \times 0.8^5$	M1	$0.2 + 0.2 \times 0.8 + 0.2 \times 0.8^2 + 0.2 \times 0.8^3 + 0.2 \times 0.8^4 + 0.2 \times 0.8^5 (+0.2 \times 0.8^6)$
	$= 0.738, \frac{11529}{15625}$	A1	0.737856 to at least 3SF.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
2(b)	Method 1:		
	$[P(5, 6, 7) =]$ ${}^{10}C_5 (0.2)^5 (0.8)^5 + {}^{10}C_6 (0.2)^6 (0.8)^4 + {}^{10}C_7 (0.2)^7 (0.8)^3$	M1	One term: ${}^{10}C_x (p)^x (1-p)^{10-x}, 0 < p < 1, x \neq 0, 10.$
	$[0.02642 + 5.505 \times 10^{-3} + 7.864 \times 10^{-4}]$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	$= 0.0327$	B1	awrt
	Method 2:		
	$[P(X < 8) - P(X \leq 4) = 1 - P(X \geq 8) - P(X \leq 4) =]$ $1 - \{ {}^{10}C_8 (0.2)^8 (0.8)^2 + {}^{10}C_9 (0.2)^9 0.8 + (0.2)^{10} \}$ $- \{ (0.8)^{10} + {}^{10}C_1 (0.2)(0.8)^9 + {}^{10}C_2 (0.2)^2 (0.8)^8 + {}^{10}C_3 (0.2)^3 (0.8)^7 + {}^{10}C_4 (0.2)^4 (0.8)^6 \}$	M1	One term: ${}^{10}C_x (p)^x (1-p)^{10-x}, 0 < p < 1, x \neq 0, 10.$
	$[1 - \{ 7.373 \times 10^{-5} + 4.096 \times 10^{-6} + 1.024 \times 10^{-7} \} -$ $\{ 0.1074 + 0.2684 + 0.3020 + 0.2013 + 0.08808 \}]$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	$= 0.0327$	B1	awrt
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3	[Mean = $200 \times 0.15 =$] 30 [Var = $200 \times 0.15 \times 0.85 =$] 25.5	B1	30 and 25.5, $25\frac{1}{2}$, $\frac{51}{2}$ seen, allow unsimplified. May be seen in standardisation formula. [$\sigma =$] $5.049 \leq \sigma \leq 5.05[0]$, $\frac{\sqrt{102}}{2}$ implies correct variance. Correct notation is required.
	$[P(X > 40) =] P(Z > \frac{40.5 - 30}{\sqrt{25.5}})$	M1	Substituting <i>their</i> mean and <i>their</i> positive 5.04975 into $\pm$ standardisation formula (any number for 40.5), not <i>their</i> σ^2 or $\sqrt{\text{their } \sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in <i>their</i> standardisation formula.
	$[1 - \Phi(2.079)]$ $1 - 0.9812$	M1	Appropriate area Φ , from final process, must be a probability.
	= 0.0188	A1	$0.01875 < p \leq 0.0188$
		5	

PUBLISHED

Question	Answer	Marks	Guidance																																													
4(a)	<table><tr><td colspan="4">Aces</td><td></td><td colspan="4">Jets</td></tr><tr><td>9</td><td>8</td><td>6</td><td>4</td><td>16</td><td>6</td><td colspan="3">8</td></tr><tr><td></td><td>4</td><td>3</td><td>1</td><td>17</td><td>0</td><td>4</td><td>4</td><td>5</td></tr><tr><td>2</td><td>2</td><td>1</td><td>0</td><td>18</td><td>1</td><td>1</td><td>8</td><td>8</td></tr><tr><td></td><td></td><td></td><td></td><td>19</td><td>0</td><td colspan="3"></td></tr></table>	Aces					Jets				9	8	6	4	16	6	8				4	3	1	17	0	4	4	5	2	2	1	0	18	1	1	8	8					19	0				B1	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e., stem values are repeated), the remaining B marks are available.
	Aces					Jets																																										
	9	8	6	4	16	6	8																																									
		4	3	1	17	0	4	4	5																																							
	2	2	1	0	18	1	1	8	8																																							
				19	0																																											
		B1	Correct Aces labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.																																													
Key: 1 17 0 means 171 cm for the Aces and 170 cm for the Jets		B1	Correct Jets labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation.																																													
		B1	Correct key for their diagram, need both teams labelled and ‘cm’ stated at least once here, or in leaf headings or title.																																													
		4																																														

PUBLISHED

Question	Answer	Marks	Guidance
4(b)	Median = 173 [cm]	B1	Accept Q_2 ; must be identified.
	[IQR =] 181 – 168	M1	$180 \leq UQ \leq 182 - 166 \leq LQ \leq 169$ Implied if both quartile values are stated and an appropriate IQR calculated accurately.
	13 [cm]	A1	www If M0 scored SC B1 for 13 www.
		3	
4(c)	Jets have a greater variety of heights. Jets have a wider range of height. Jets have a greater/larger/bigger/wider/'more' spread of heights. Aces have a smaller variety of height etc...	B1	[Jets IQR = 18 cm, Range = 24 cm Aces IQR = <i>their</i> 4(b) , Range = 18 cm] Comment about spread in context, must include height. Comparison of values does not score until a comment in context is made. If values for range or IQR are stated, they must be correct or FT from 4(b) . If more than one comment about spread, mark the final comment. Additional comments about central tendency score B0.
		1	

PUBLISHED

Question	Answer	Marks	Guidance
5(a)(i)	$[P(X < 170) =] P(Z < \frac{170-166}{10})$	M1	Use of $\pm$ standardisation formula with 170, 166 and 10 substituted appropriately, condone 10^2 , $\sqrt{10}$, condone continuity correction ± 0.5 .
	$[= P(Z < 0.4) =] 0.655$	A1	$0.655 \leq p < 0.6555$ If M0 awarded, SC B1 for correct answer www.
		2	
5(a)(ii)	$\left[P\left(Z > \frac{h-166}{10} \right) = 0.4 \right]$	B1	$0.253 \leq z \leq 0.2535$ or $-0.2535 \leq z \leq -0.253$ seen.
	$\frac{h-166}{10} = 0.253$	M1	Use of the $\pm$ standardisation formula with h , 166, 10 and a z-value (not $1 - z$ -value), not 10^2 , $\sqrt{10}$, no continuity correction.
	$h = 168.53$	A1	If M0 scored, SC B1 for $168.53 \leq h \leq 168.535$, 168.5. SC B1 for 168.54 from $z = 0.254$.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
5(b)	$\left[P(X > 0) = P\left(Z > \frac{0 - \mu}{\sigma}\right) = \right] P\left(Z > \frac{[0] - \mu}{\frac{2}{3}\mu}\right)$ Or $P\left(Z > \frac{[0] - \frac{3}{2}\sigma}{\sigma}\right)$	M1	Use of the $\pm$ standardisation formula with 0, μ and $\frac{2}{3}\mu$ substituted for σ . Or use of the $\pm$ standardisation formula with 0, σ and $\frac{3}{2}\sigma$ substituted for μ .
	= P(Z > -1.5)	A1	-1.5 seen, no additional terms (e.g. $x - 1.5$ A0). Condone $Z < 1.5$. If M0 scored, SC B1 $Z > -1.5$ or $Z < 1.5$ seen www.
	= 0.933 final answer	A1	$0.933 \leq p < 0.9333$. If M0 scored, SC B1 $0.933 \leq p < 0.9333$ seen www.
		3	

Question	Answer	Marks	Guidance
6(a)		B1	1st column, 2 branches identified X, Y with probabilities $\frac{1}{2}$, $\frac{1}{2}$ indicated.
		B1	2nd column (1st marble pick) of 4 branches identified R B R B (oe) and probabilities $\frac{7}{10}$, $\frac{3}{10}$, $\frac{8}{10}$, $\frac{2}{10}$ indicated appropriately.
		B1	3rd column (2nd marble pick) of 8 branches identified R B R B R B R [B] (oe) and probabilities $\frac{7}{10}$, $\frac{3}{10}$, $\frac{8}{10}$, $\frac{2}{10}$, $\frac{4}{5}$, $\frac{1}{5}$, 1, [0]. Condone omission of YBB branch if YBR branch is fully correct. Ignore any additional columns of branches. If separate tree diagrams for bags X and Y, B0B1B1 max if bags clearly identified.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
6(b)	$[P(\text{both same colour}) = P(BB) + P(RR) = P(XBB) + P(XRR) + P(YRR) =]$ $\frac{1}{2} \times \frac{3}{10} \times \frac{2}{10} + \frac{1}{2} \times \frac{7}{10} \times \frac{7}{10} + \frac{1}{2} \times \frac{4}{5} \times \frac{4}{5}$ $\left[= \frac{6}{200} + \frac{49}{200} + \frac{16}{50}, 0.03 + 0.245 + 0.32 \right]$	B1 FT	$\left[P(BB) = \right] \left[\frac{1}{2} \times \frac{3}{10} \times \frac{2}{10} \left[+ \frac{1}{2} \times \frac{1}{5} \times 0 \right] \right] = \frac{6}{200}$ seen. Accept unsimplified. FT from 6(a) unsimplified only with 3 term probabilities.
		B1 FT	Either $[P(XRR) =] \frac{1}{2} \times \frac{7}{10} \times \frac{7}{10}$ or $[P(YRR) =] \frac{1}{2} \times \frac{4}{5} \times \frac{4}{5}$ seen. FT from 6(a) unsimplified only with 3 term probabilities.
		M1	$[P(BB) + P(XRR) + P(YRR) =]$ <i>their</i> $\frac{6}{200} + \text{their } \frac{49}{200} + \text{their } \frac{16}{50}$ Accept unsimplified, consistent with tree diagram if not clearly identified by notation.
	$= \frac{119}{200}, 0.595$	A1	
		4	Special case: if $\frac{1}{2}$ omitted consistently in the tree diagram and the calculation (i.e., no probability for picking the bags), no FT. SC B1 $[P(BB) =] \frac{3}{10} \times \frac{2}{10} \left[+ \frac{1}{5} \times 0 \right]$ SC B1 $[P(RR) =] \frac{7}{10} \times \frac{7}{10} + \frac{4}{5} \times \frac{4}{5}$ SC B1 $\frac{3}{10} \times \frac{2}{10} \left[+ \frac{1}{5} \times 0 \right] + \frac{7}{10} \times \frac{7}{10} + \frac{4}{5} \times \frac{4}{5}$

PUBLISHED

Question	Answer	Marks	Guidance
6(c)	$\left[P(\text{bag } Y \mid \text{different colours}) = \left(\frac{P(\text{bag } Y \cap \text{different colours})}{P(\text{different colours})} \right) \right]$ $\frac{\frac{1}{2} \times \frac{4}{5} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} [\times 1]}{1 - \text{their} \left(\frac{119}{200} \right)} \text{ or } \frac{\frac{1}{2} \times \frac{4}{5} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} [\times 1]}{\frac{1}{2} \times \frac{7}{10} \times \frac{3}{10} + \frac{1}{2} \times \frac{3}{10} \times \frac{8}{10} + \frac{1}{2} \times \frac{4}{5} \times \frac{1}{5} + \frac{1}{2} \times \frac{1}{5} [\times 1]}$	M1	FT from <i>their 6(a)</i> and <i>their 6(b)</i> with 3 term probabilities unsimplified only or correct. Accept $\frac{\frac{4}{5} + \frac{1}{5}}{\frac{50}{81} + \frac{10}{200}}, \frac{\frac{2}{25} + \frac{1}{10}}{\frac{81}{200}}, \frac{0.08 + 0.1}{0.405}$.
	$= \left[\frac{\frac{9}{50}}{\frac{81}{200}} \right] = \frac{4}{9}, 0.444$	A1	Accept $\frac{36}{81}, 0.4$.
		2	Special case: if $\frac{1}{2}$ omitted consistently in the tree diagram and the calculation (ie no probability for picking the bags), no FT. SC B1 $\frac{\frac{4}{5} \times \frac{1}{5} + \frac{1}{5} [\times 1]}{1 - \text{their } \mathbf{6(b)}}$, or $\frac{\frac{4}{5} \times \frac{1}{5} + \frac{1}{5} [\times 1]}{\frac{7}{10} \times \frac{3}{10} + \frac{3}{10} \times \frac{8}{10} + \frac{4}{5} \times \frac{1}{5} + \frac{1}{5} [\times 1]}$.

PUBLISHED

Question	Answer	Marks	Guidance
7(a)	$\frac{5! \times 4!}{2! \times 2!}$	M1	$\frac{5! \times 4!}{e}$, e a positive integer, 1 can be implied. No other terms on numerator. No addition etc.
		M1	$\frac{f}{2! \times g!}$, f a positive integer, $g = 1, 2$. No other terms on denominator.
	720	A1	
		3	

PUBLISHED

Question	Answer	Marks	Guidance																		
7(b)	Method 1 Number of arrangements with A at each end – Number of arrangements with A at each end and 2 Ds together.																				
	$\frac{7!}{2!} - 6!$	B1	$\frac{7!}{2!} - e, {}^7P_5 - e, e$ a positive integer or 0.																		
		M1	$d - \frac{6!}{r!}, d > 720, r = 1, 2.$																		
	= 1800	A1																			
	Method 2 A ^ ^ ^ ^ A and Ds inserted separately																				
	$5! \times \frac{{}^6P_2}{2!}$ or $5! \times \frac{6 \times 5}{2}$ or $5! \times {}^6C_2$	B1	$5! \times s, s$ a positive integer, 1 may be implied.																		
		M1	$t \times \frac{6 \times 5}{u}, t$ a positive integer $> 1, u = 1, 2.$																		
	= 1800	A1																			
	Method 3 Number of arrangements with As at each end and Ds placed in different scenarios.																				
	<table><tr><td>Scenario position of first D</td><td></td><td></td></tr><tr><td>A D ^ ^ ^ ^ A</td><td>$5! \times 5$</td><td>600</td></tr><tr><td>A ^ D ^ ^ ^ A</td><td>$5! \times 4$</td><td>480</td></tr><tr><td>A ^ ^ D ^ ^ A</td><td>$5! \times 3$</td><td>360</td></tr><tr><td>A ^ ^ ^ D ^ A</td><td>$5! \times 2$</td><td>240</td></tr><tr><td>A ^ ^ ^ ^ D ^ A</td><td>$5! \times 1$</td><td>120</td></tr></table>		Scenario position of first D			A D ^ ^ ^ ^ A	$5! \times 5$	600	A ^ D ^ ^ ^ A	$5! \times 4$	480	A ^ ^ D ^ ^ A	$5! \times 3$	360	A ^ ^ ^ D ^ A	$5! \times 2$	240	A ^ ^ ^ ^ D ^ A	$5! \times 1$	120	B1
Scenario position of first D																					
A D ^ ^ ^ ^ A	$5! \times 5$	600																			
A ^ D ^ ^ ^ A	$5! \times 4$	480																			
A ^ ^ D ^ ^ A	$5! \times 3$	360																			
A ^ ^ ^ D ^ A	$5! \times 2$	240																			
A ^ ^ ^ ^ D ^ A	$5! \times 1$	120																			
		M1	Add values of 5 correct scenarios, no incorrect/repeated scenarios.																		
[Total =] 1800	A1																				
	3																				

PUBLISHED

Question	Answer	Marks	Guidance									
7(c)	Method 1:											
	<table><tr><td>Scenarios</td><td></td><td></td></tr><tr><td>A D ^ ^</td><td>${}^2C_1 \times {}^2C_1 \times {}^5C_2$</td><td>= 40</td></tr><tr><td>A D D ^</td><td>${}^2C_1 \times [{}^2C_2 \times] {}^5C_1$</td><td>= 10</td></tr></table>	Scenarios			A D ^ ^	${}^2C_1 \times {}^2C_1 \times {}^5C_2$	= 40	A D D ^	${}^2C_1 \times [{}^2C_2 \times] {}^5C_1$	= 10	M1	At least one correct unsimplified expression for an identified scenario.
	Scenarios											
	A D ^ ^	${}^2C_1 \times {}^2C_1 \times {}^5C_2$	= 40									
	A D D ^	${}^2C_1 \times [{}^2C_2 \times] {}^5C_1$	= 10									
	[Total =] 40 + 10 or 50 soi											
[Total number of selections =] ${}^9C_4$ [= 126]		B1	Accept evaluated, accept as denominator of probability expression. Do not condone ${}^9C_5$ unless there is a clear explanation for selecting the letters not in the group.									
[Probability =] $\frac{50}{126}, \frac{25}{63}$		B1 FT	0.396825... to at least 3SF. FT $\frac{\text{their attempted } 40 + 10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios and must be evaluated.									

Question	Answer	Marks	Guidance									
7(c)	Method 2:											
	<table><tr><td>Scenarios</td><td></td><td></td></tr><tr><td>A D ^ ^</td><td>$\frac{2}{9} \times \frac{2}{8} \times \frac{5}{7} \times \frac{4}{6} \times {}^4P_2$</td><td>$= \frac{960}{3024}, \frac{20}{63}$</td></tr><tr><td>A D D ^</td><td>$\frac{2}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6} \times \frac{{}^4P_3}{2!}$</td><td>$= \frac{240}{3024}, \frac{5}{63}$</td></tr></table>	Scenarios			A D ^ ^	$\frac{2}{9} \times \frac{2}{8} \times \frac{5}{7} \times \frac{4}{6} \times {}^4P_2$	$= \frac{960}{3024}, \frac{20}{63}$	A D D ^	$\frac{2}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6} \times \frac{{}^4P_3}{2!}$	$= \frac{240}{3024}, \frac{5}{63}$	M1	Numerator for at least one correct unsimplified expression for an identified scenario. <i>either</i> $\frac{2 \times 2 \times 5 \times 4 \times 12}{a \times b \times c \times d}$ or $\frac{2 \times 2 \times 1 \times 5 \times 12}{a \times b \times c \times d}$ seen, $6 \leq a, b, c, d \leq 9$.
	Scenarios											
	A D ^ ^	$\frac{2}{9} \times \frac{2}{8} \times \frac{5}{7} \times \frac{4}{6} \times {}^4P_2$	$= \frac{960}{3024}, \frac{20}{63}$									
	A D D ^	$\frac{2}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{5}{6} \times \frac{{}^4P_3}{2!}$	$= \frac{240}{3024}, \frac{5}{63}$									
[Total Probability =] $\frac{20}{63} + \frac{5}{63}$	A1	$\frac{2 \times 2 \times 5 \times 4 \times 12}{a \times b \times c \times d} + \frac{2 \times 2 \times 1 \times 5 \times 12}{a \times b \times c \times d}$, $6 \leq a, b, c, d \leq 9$. If M0 scored, SC B1 $\frac{1200}{g}$, $g > 1200$, or $\frac{25}{63}$ seen.										
	B1	$\frac{p}{9} \times \frac{q}{8} \times \frac{r}{7} \times \frac{s}{6}$ present in all scenarios attempted, accept $\frac{t}{3024}$, $t < 3024$.										
$\frac{1200}{3024}, \frac{25}{63}$ oe	B1 FT	0.396825... to at least 3SF. FT $\frac{\text{their attempted } 960 + 240}{3024}$. Numerator must be from an attempt to find the 2 appropriate scenarios.										

PUBLISHED

Question	Answer	Marks	Guidance
7(c)	Method 3: selecting the A and then selecting 3 any letters and removing selections without Ds.		
	${}^2C_1 \times ({}^7C_3 - {}^5C_3) [= 2 \times (35 - 10)]$	M1	$a \times ({}^7C_3 - {}^5C_3)$, $a = 1, 2$.
	[Total =] 50	A1	www If M0 scored, SC B1 [total =]50 www.
	[Total number of selections =] ${}^9C_4 [= 126]$	B1	Accept evaluated, accept as denominator of probability expression. Do not condone ${}^9C_5$ unless there is a clear explanation for selecting the letters not in the group.
	[Probability =] $\frac{50}{126}, \frac{25}{63}$	B1 FT	0.396825... to at least 3SF. FT $\frac{\text{their attempted } 40 + 10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios.
	Method 4: Listing outcomes.		
	Either 10 correct outcomes for ADD^ listed or 40 correct outcomes for AD^^ listed	M1	
	50 stated	A1	www If M0 scored, SC B1 [total =]50 www.
	126 stated or correct outcomes listed	B1	
	[Probability =] $\frac{50}{126}, \frac{25}{63}$	B1	0.396825... to at least 3SF. FT $\frac{\text{their attempted } 40 + 10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios.
		4	



Cambridge International AS & A Level

MATHEMATICS

9709/53

Paper 5 Probability & Statistics 1

October/November 2023

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for questions 1 and 4. This published mark scheme for these questions was created alongside the question paper, but has not been used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

PUBLISHED

Question	Answer	Marks	Guidance																									
1(a)	$2k + 6k + 12k + 20k = 1, \left[k = \frac{1}{40} \right]$	M1	Using sum of probabilities = 1 to form an equation in k . Accept $1 \times 2 \times k + 2 \times 3 \times k + 3 \times 4 \times k + 4 \times 5 \times k = 1$.																									
	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X)$</td><td>$\frac{2}{40}$</td><td>$\frac{6}{40}$</td><td>$\frac{12}{40}$</td><td>$\frac{20}{40}$</td></tr><tr><td></td><td>0.05</td><td>0.15</td><td>0.3</td><td>0.5</td></tr></table>	X	1	2	3	4	$P(X)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$		0.05	0.15	0.3	0.5	M1	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X)$</td><td>$2k$</td><td>$6k$</td><td>$12k$</td><td>$20k$</td></tr></table> Two correctly linked, accurate probabilities. May be in terms of k . May not be in a table.	X	1	2	3	4	$P(X)$	$2k$	$6k$	$12k$	$20k$
	X	1	2	3	4																							
	$P(X)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$																							
	0.05	0.15	0.3	0.5																								
X	1	2	3	4																								
$P(X)$	$2k$	$6k$	$12k$	$20k$																								
		A1	Table with correct X values and correct probabilities.																									
		3																										
1(b)	$[E(X) =] [E(X) = \frac{1 \times 2 + 2 \times 6 + 3 \times 12 + 4 \times 20}{40}] \frac{2 + 12 + 36 + 80}{40}$	M1	$[E(X) = 1 \times 2k + 2 \times 6k + 3 \times 12k + 4 \times 20k = 130k]$ Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 3 or more probabilities summing to 1 ($0 < p < 1$). If there are outcomes in the table without probabilities, condone and treat as $p = 0$.																									
	$\left[\text{Var}(X) = \frac{1^2 \times 2 + 2^2 \times 6 + 3^2 \times 12 + 4^2 \times 20}{40} - (their\ E(X))^2 = \right]$ $\frac{1 \times 2 + 4 \times 6 + 9 \times 12 + 16 \times 20}{40} - \left(their\ \frac{13}{4} \right)^2$ $\left[\frac{2 + 24 + 108 + 320}{40} - \left(their\ \frac{13}{4} \right)^2 \right]$	M1	$[\text{Var}(X) = 1^2 \times 2k + 2^2 \times 6k + 3^2 \times 12k + 4^2 \times 20k - (130k)^2]$ Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the highlighted calculation (or less simplified) seen, Note: if table is correct, $\frac{454}{40} \left(\text{or } \frac{227}{20} \text{ or any calculation} \right) - (their\ E(X))^2$ implies M1.																									
	$E(X) = \frac{13}{4}, 3\frac{1}{4}, 3.05 \quad \text{Var}(X) = \frac{63}{80}, 0.7875$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in variance. Condone E, V, μ, σ etc. If A0 earned, SC B1 for identified correct final solutions.																									
			3																									

PUBLISHED

Question	Answer	Marks	Guidance
2(a)	$[P(1.42 < X < 1.52) =] P\left(\frac{1.42-1.5}{0.05} < Z < \frac{1.52-1.5}{0.05}\right)$	M1	Use of $\pm$ standardisation formula once with 1.5, 0.05 and either 1.42 or 1.52, allow σ^2 or $\sqrt{\sigma}$, no continuity correction.
	$[= P(-1.6 < Z < 0.4) = \Phi(0.4) + \Phi(1.6) - 1]$ $= 0.6554 + 0.9452 - 1$ or $0.6554 - 0.0548$	M1	Calculating the appropriate probability area (leading to their final answer, expect > 0.5).
	$= 0.601$	A1	$0.6005 < p \leq 0.601$ SC B1 for 0.601 with no standardisation seen.
		3	
2(b)	$\left[P(X < 0.9) = P\left(Z < \frac{0.9-0.75}{\sigma}\right) = 0.68 \right]$	B1	$0.467 < z \leq 0.468$ or $-0.468 \leq z < -0.467$ seen
	$\frac{0.9-0.75}{\sigma} = 0.468$	M1	$\pm$ standardisation formula with 0.9, 0.75, σ equating to a z-value (not 0.32, 0.68, 0.532, 0.7517, 0.2483, 0.6255,). Condone continuity correct ± 0.05 , not $\sigma^2, \sqrt{\sigma}$. Condone $\pm \frac{0.15}{\sigma} = 0.468$.
	$\sigma = 0.321, \frac{25}{78}$	A1	$0.3205 \leq \sigma < 0.3215$ SC B1 if M0 www.
		3	

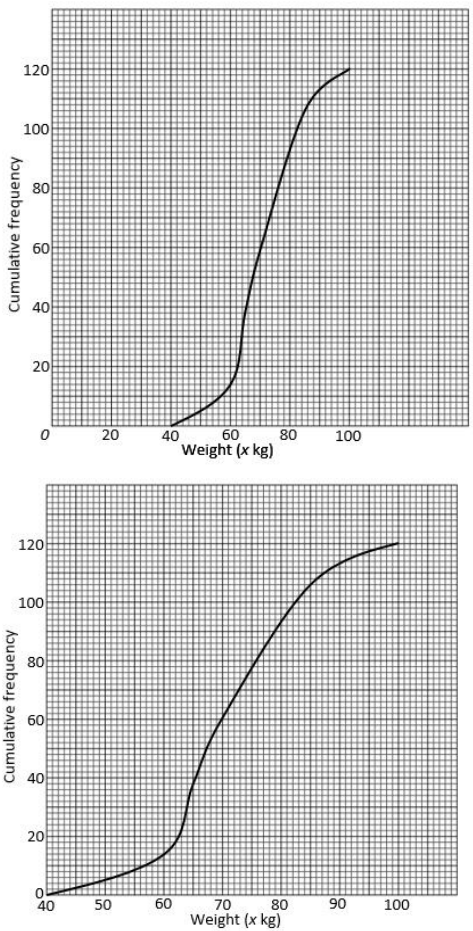
PUBLISHED

Question	Answer	Marks	Guidance
3(a)	$[P(WW) = P(AWW) + P(BWW) =]$ $\frac{2}{6} \times \frac{8}{15} \times \frac{7}{14} + \frac{4}{6} \times \frac{6}{15} \times \frac{5}{14}$	M1	Either $\frac{2}{6} \times \frac{8}{15} \times \frac{7}{14}$ or $\frac{4}{6} \times \frac{6}{15} \times \frac{5}{14}$ seen, accept unsimplified.
		M1	$\frac{q}{6} \times \frac{r}{15} \times \frac{r-1}{14} + \frac{6-q}{6} \times \frac{s}{15} \times \frac{s-1}{14}$ seen, no additional terms, accept unsimplified. Condone $\frac{q}{6} \times \frac{r}{15} \times \frac{r}{15} + \frac{6-q}{6} \times \frac{s}{15} \times \frac{s}{15}$, $1 \leq q \leq 5, 1 < r, s < 9$.
	$\left[= \frac{56}{630} + \frac{60}{630} = \frac{4}{45} + \frac{2}{21} \right] = \frac{58}{315}$ or 0.184	A1	SC B1 for 58/315 if either M mark withheld.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
3(b)	$\left[P(B WR \text{ or } RW) = \frac{P(W \& R \text{ from bag B})}{P(W \text{ and } R)} = \right]$ $\frac{\frac{4}{6} \times \frac{6}{15} \times \frac{7}{14} + \frac{4}{6} \times \frac{7}{15} \times \frac{6}{14}}{\frac{2}{6} \times \frac{8}{15} \times \frac{4}{14} + \frac{2}{6} \times \frac{4}{15} \times \frac{8}{14} + \frac{4}{6} \times \frac{6}{15} \times \frac{7}{14} + \frac{4}{6} \times \frac{7}{15} \times \frac{6}{14}}$ $\text{or } \frac{2 \times \frac{4}{6} \times \frac{6}{15} \times \frac{7}{14}}{2 \times \frac{2}{6} \times \frac{8}{15} \times \frac{4}{14} + 2 \times \frac{4}{6} \times \frac{6}{15} \times \frac{7}{14}}$	B1	P(W & R from bag B) $= \frac{2}{3} \times \frac{6}{15} \times \frac{7}{14} + \frac{2}{3} \times \frac{7}{15} \times \frac{6}{14} \text{ or } 2 \times \frac{2}{3} \times \frac{6}{15} \times \frac{7}{14} [= \frac{4}{15} \text{ or } 0.267]$ Seen alone or as numerator/denominator of conditional probability.
		M1	P(WR or RW) = P(W & R from bag A) + P(W & R from bag B) $= a \times \frac{2}{6} \times \frac{8}{15} \times \frac{4}{14} + a \times \frac{4}{6} \times \frac{6}{15} \times \frac{7}{14} \text{ or}$ $= a \times \frac{2}{6} \times \frac{8}{15} \times \frac{4}{14} + \text{their } P(W \& R \text{ from bag B}).$ $a = 1 \text{ or } 2.$ [expect $\frac{116}{315}$ or 0.368] Seen alone or as numerator/denominator of conditional probability.
	$\frac{\frac{168}{630}}{\frac{232}{630}} = \frac{\frac{4}{15}}{\frac{116}{315}}$	M1	<u>their identified P(W & R from bag B)</u> <u>their identified P(WR or RW)</u> Accept unsimplified.
	$= \frac{168}{232}, \frac{21}{29} \text{ or } 0.724$	A1	0.7241379 to at least 3SF.
		4	

PUBLISHED

Question	Answer	Marks	Guidance
4(a)		M1	At least 3 points plotted accurately at class upper end points (40,0) (60,14) (65,38) (70,60) (85,106) (100,120). Linear cumulative frequency scale $0 \leq cf \leq 120$ and linear weight scale $40 \leq \text{weight(kg)} \leq 100$ with at least 3 values identified on each axis. Condone scale reversed.
		A1	All points plotted correctly, curve drawn (within tolerance) and joined to (40,0). Axes labelled cumulative frequency (cf), weight (w) and kg (kilograms) – or a suitable title.
		2	
4(b)	$[120 \times 0.65 =] 78 \text{ seen}$	M1	May be implied by use on graph.
	76 [kg]	A1	$75 < \text{hours} < 79$. Indication of use of graph required.
		2	

PUBLISHED

Question	Answer	Marks	Guidance
4(c)	Frequencies: [0] 14 24 22 46 14	B1	At least 5 correct frequencies seen (condone omission of 0).
	Midpoints: 20 50 62.5 67.5 77.5 92.5	B1	At least 5 correct midpoints seen (condone omission of 20).
	Mean = $\frac{0 \times 20 + 14 \times 50 + 24 \times 62.5 + 22 \times 67.5 + 46 \times 77.5 + 14 \times 92.5}{120}$ $= \frac{[0] + 700 + 1500 + 1485 + 3565 + 12950}{120} \left[= \frac{8545}{120} \right]$	M1	Correct formula for mean using <i>their</i> midpoints and <i>their</i> frequencies, implied by $\frac{8545}{120}$ if correct midpoints & frequencies seen. May be gained in variance calculation. If midpoints not clearly identified, condone midpoints ± 0.5 .
	= 71.2	A1	Accept $\frac{1709}{24}$, $71\frac{5}{24}$ or 71.208333 to at least 3SF. If M0 scored, SC B1 for $\frac{1709}{24}$, $71\frac{5}{24}$ or 71.208333 to at least 3SF www.
	Variance = $\frac{0 \times 20^2 + 14 \times 50^2 + 24 \times 62.5^2 + 22 \times 67.5^2 + 46 \times 77.5^2 + 14 \times 92.5^2}{120} - 71.2^2$ $\frac{[0] + 35000 + 93750 + 100237.5 + 276287.5 + 119787.5}{120} - \left(\frac{8545}{120} \right)^2$ [= 138.23]	M1	Correct formula for variance using <i>their</i> midpoints, <i>their</i> frequencies and <i>their</i> mean. Implied by $\frac{625062.5}{120} - \left(\frac{8545}{120} \right)^2$ if correct midpoints & frequencies seen.
	Standard deviation = 11.8	A1	11.757016 to at least 3SF.
		6	

PUBLISHED

Question	Answer	Marks	Guidance
5(a)(i)	Method 1		
	$[P(2 \leq X \leq 6) = P(X \leq 6) - P(X \leq 1) =] 1 - (0.7)^6 - (1 - 0.7)$	M1	$1 - 0.7^n$ seen, $n = 5, 6$.
	$= 0.582$	A1	www 0.582351 to at least 3SF.
	Method 2		
	$P(X = 2, 3, 4, 5, 6)$ $= 0.7 \times 0.3 + 0.7^2 \times 0.3 + 0.7^3 \times 0.3 + 0.7^4 \times 0.3 + 0.7^5 \times 0.3$ $= 0.21 + 0.147 + 0.1029 + 0.07203 + 0.050421$	M1	Sum of first 4 or 5 correct terms – no incorrect terms.
	$= 0.582$	A1	www 0.582351 to at least 3SF.
		2	
5(a)(ii)	$3\frac{1}{3}$	B1	Condone 3.33, $3.\dot{3}$ or $\frac{10}{3}$ – NOT $\frac{1}{0.3}$.
		1	

PUBLISHED

Question	Answer	Marks	Guidance
5(b)	Method 1		
	$[P(3, 4, 5) =] {}^5C_3(0.3)^3(0.7)^2 + {}^5C_4(0.3)^4(0.7)^1 + {}^5C_5(0.3)^5(0.7)^0$	M1	One term seen ${}^5C_x (p)^x (1-p)^{5-x}$, $0 < p < 1$, $x \neq 0, 5$.
	$= 0.1323 + 0.02835 + 0.00243$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	$= 0.163, \frac{4077}{25000}$	B1	0.16308 to at least 3SF.
	Method 2		
	$[1 - P(0, 1, 2) =$ $1 - ({}^5C_0(0.3)^0(0.7)^5 + {}^5C_1(0.3)^1(0.7)^4 + {}^5C_2(0.3)^2(0.7)^3)$	M1	One term ${}^5C_x (p)^x (1-p)^{5-x}$, $0 < p < 1$, $x \neq 0, 5$.
	$= 1 - (0.16807 + 0.36015 + 0.3087)$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	$= 0.163, \frac{4077}{25000}$	B1	0.16308 to at least 3SF.
		3	

PUBLISHED

Question	Answer	Marks	Guidance
5(c)	[Mean = $75 \times 0.3 =$] 22.5 [Var = $75 \times 0.3 \times 0.7 =$] 15.75	B1	22.5, $22\frac{1}{2}$ and 15.75, $15\frac{3}{4}$ seen, allow unsimplified. $(\sigma = \frac{3\sqrt{7}}{2}$ or 3.9686269... to at least 3SF implies correct variance)
	$[P(X > 20) =] P\left(Z > \frac{20.5 - 22.5}{\sqrt{15.75}}\right)$	M1	Substituting their μ and σ into $\pm$ standardisation formula (any number for 20.5), not σ^2 not $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in <i>their</i> standardisation formula.
	$[P(Z > -0.504) = \Phi(0.504)]$ = 0.693	M1	Appropriate area Φ , from final process, must be a probability. Expect final answer > 0.5 . Note: correct final answer implies this M1.
		A1	$0.6925 < p \leq 0.693$
		5	

PUBLISHED

Question	Answer	Marks	Guidance
6(a)(i)	Method 1		
	$6! \times 2^6$	M1	$6! \times a$, a integer > 1 .
		M1	$b \times 2^6$, b integer ≥ 1 .
	$= 46080$	A1	Accurate answer required. SC B1 for 46080 if M0 M0 www.
	Alternative method for question 6(a)(i)		
	$12 \times 10 \times 8 \times 6 \times 4 \times 2$	M1	$c \times d \times e \times f \times g \times h$ $2 \leq c, d, e, f, g, h$ (different integers) ≤ 12
		M1	Correct unsimplified.
	$= 46080$	A1	Accurate answer required. SC B1 for 46080 if M0 M0 www.
		3	
6(a)(ii)	$5! \times 5! \times 2 \times 2$	M1	$5! \times 5! \times k$, k positive integer, 1 may be implied (no adding/subtracting).
	$= 57600$	A1	
		2	

PUBLISHED

Question	Answer	Marks	Guidance									
6(b)	Method 1 probabilities of J & K being placed:											
	<table><tr><td>In the group of 5</td><td>$\frac{5}{12} \times \frac{4}{11}$</td><td>$\left[= \frac{20}{132}, \frac{5}{33} \right]$</td></tr><tr><td>In the group of 4</td><td>$\frac{4}{12} \times \frac{3}{11}$</td><td>$\left[= \frac{12}{132}, \frac{1}{11} \right]$</td></tr><tr><td>In the group of 3</td><td>$\frac{3}{12} \times \frac{2}{11}$</td><td>$\left[= \frac{6}{132}, \frac{1}{22} \right]$</td></tr></table>	In the group of 5	$\frac{5}{12} \times \frac{4}{11}$	$\left[= \frac{20}{132}, \frac{5}{33} \right]$	In the group of 4	$\frac{4}{12} \times \frac{3}{11}$	$\left[= \frac{12}{132}, \frac{1}{11} \right]$	In the group of 3	$\frac{3}{12} \times \frac{2}{11}$	$\left[= \frac{6}{132}, \frac{1}{22} \right]$	B1	Correct probability for one identified scenario.
		In the group of 5	$\frac{5}{12} \times \frac{4}{11}$	$\left[= \frac{20}{132}, \frac{5}{33} \right]$								
		In the group of 4	$\frac{4}{12} \times \frac{3}{11}$	$\left[= \frac{12}{132}, \frac{1}{11} \right]$								
		In the group of 3	$\frac{3}{12} \times \frac{2}{11}$	$\left[= \frac{6}{132}, \frac{1}{22} \right]$								
	M1	Denominator 12×11 for all probabilities, (1, 2 or 3 scenarios).										
	A1	3 correct probabilities, accept unsimplified.										
$\frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11}$		M1	Adding probabilities for 3 correct scenarios.									
$\frac{19}{66}$, 0.288		A1	0.2878787 to at least 3SF.									

PUBLISHED

Question	Answer	Marks	Guidance										
6(b)	Method 2 number of arrangements of J & K being placed:												
	In the group of 5	$^{10}C_3 \times ^7C_4$	[= $120 \times 35 = 4200$]	B1	Correct value of one identified scenario seen, accept unsimplified.								
	In the group of 4	$^{10}C_2 \times ^8C_5$	[= $45 \times 56 = 2520$]	M1									
	In the group of 3	$^{10}C_1 \times ^9C_5$	[= $10 \times 126 = 1260$]										
	[Total number of ways of arranging the 3 groups =] $^{12}C_5 \times ^7C_4 = 792 \times 35 = 27720$ or $^{12}C_3 \times ^9C_4$ or $^{12}C_4 \times ^8C_5$			A1	27720 Seen alone or as denominator of probability –accept unsimplified. SC B1 if M0.								
	4200 + 2520 + 1260 = 7980			M1	Values of 3 correct scenarios added, accept unsimplified – or correct.								
	[Probability =] $\frac{7980}{27720}, \frac{19}{66}, 0.288$			A1	0.2878787 to at least 3SF.								
				5	Note, alternative arrangement calculations possible e.g. <table><tr><td>In the group of 5</td><td>$^{10}C_3 \times ^7C_4$</td><td>[= $120 \times 35 = 4200$]</td></tr><tr><td>In the group of 4</td><td>$^{10}C_5 \times ^5C_2$</td><td>[= $252 \times 10 = 2520$]</td></tr><tr><td>In the group of 3</td><td>$^{10}C_5 \times ^5C_4$</td><td>[= $252 \times 5 = 1260$]</td></tr></table>	In the group of 5	$^{10}C_3 \times ^7C_4$	[= $120 \times 35 = 4200$]	In the group of 4	$^{10}C_5 \times ^5C_2$	[= $252 \times 10 = 2520$]	In the group of 3	$^{10}C_5 \times ^5C_4$
In the group of 5	$^{10}C_3 \times ^7C_4$	[= $120 \times 35 = 4200$]											
In the group of 4	$^{10}C_5 \times ^5C_2$	[= $252 \times 10 = 2520$]											
In the group of 3	$^{10}C_5 \times ^5C_4$	[= $252 \times 5 = 1260$]											