

Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- 1 (a) Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2 . [2]

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- (b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

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- 2** A line has equation $y = 2cx + 3$ and a curve has equation $y = cx^2 + 3x - c$, where c is a constant.

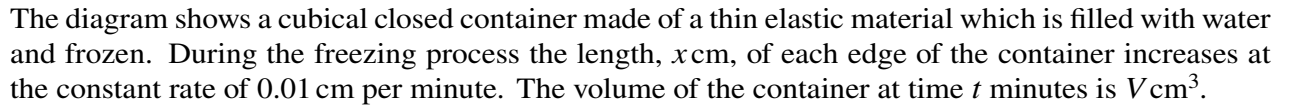
Showing all necessary working, determine which of the following statements is correct.

- A** The line and curve intersect only for a particular set of values of c .

- B** The line and curve intersect for all values of c .

- C** The line and curve do not intersect for any values of c . [4]

[illegible]



[3]

[illegible]

- 4** The transformation R denotes a reflection in the x -axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

- (a) Find the equation, $y = g(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

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- (b) Find the equation, $y = h(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

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- (c) State fully the transformation that maps the curve $y = g(x)$ onto the curve $y = h(x)$. [2]

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- 5 (a) Show that the equation

$$4 \sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$$

may be expressed in the form $a \cos^2 x + b \cos x + c = 0$, where a , b and c are integers to be found. [3]

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- (b) Hence solve the equation $4 \sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]

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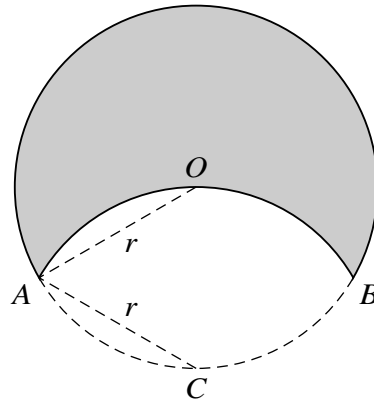
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The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O , and the minor arc AOB of a circle, also with radius r but with centre C . The point C lies on the circle with centre O .

- (a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k . [1]

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- (b) State the perimeter of the shaded motif in terms of π and r . [1]

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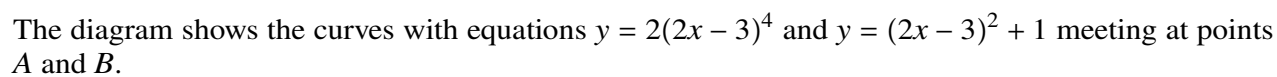
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- 7 The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression.

[7]

[illegible]



- [illegible]

(b) Find the exact area of the shaded region.

[5]

[illegible]

- 9 (a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

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The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

- (b) Given that it is possible to form the composite function gf , find the least possible value of p and the greatest possible value of q . [3]

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(c) Find an expression for $gf(x)$.

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The function h is defined by $h(x) = 4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for $h^{-1}(x)$.

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10 A curve has a stationary point at $(2, -10)$ and is such that $\frac{d^2y}{dx^2} = 6x$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) Find the equation of the curve. [3]

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- (c) Find the coordinates of the other stationary point and determine its nature. [3]

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- (d) Find the equation of the tangent to the curve at the point where the curve crosses the y -axis. [2]

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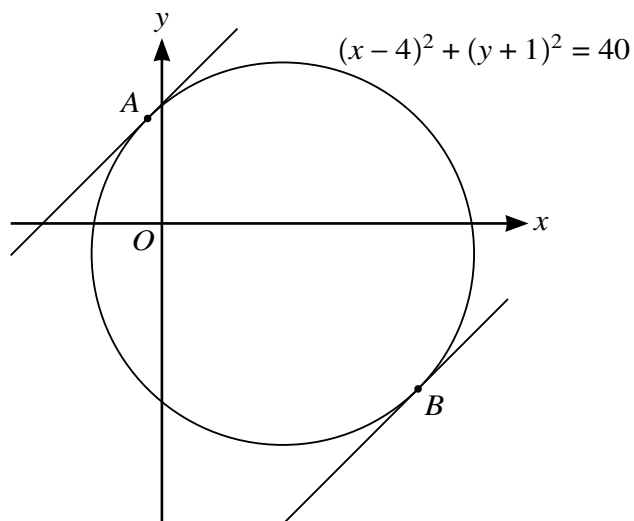
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The diagram shows the circle with equation $(x - 4)^2 + (y + 1)^2 = 40$. Parallel tangents, each with gradient 1, touch the circle at points A and B .

- (a) Find the equation of the line AB , giving the answer in the form $y = mx + c$. [3]

[illegible]

(b) Find the coordinates of A , giving each coordinate in surd form.

[4]

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(c) Find the equation of the tangent at A , giving the answer in the form $y = mx + c$, where c is in surd form.

[2]

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Cambridge International AS & A Level

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

- 1** The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a .

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 Find the exact solution of the equation

$$\frac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}(\frac{1}{2}\sqrt{3}). \quad [2]$$

[illegible]

- 3 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P . [2]

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(b) Find the equation of the curve. [4]

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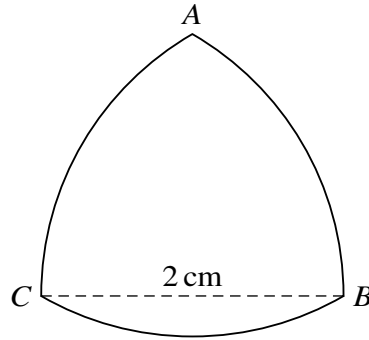
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The diagram shows the shape of a coin. The three arcs AB , BC and CA are parts of circles with centres C , A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

- (a) Find the perimeter of the coin. [2]

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- (b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

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- 5** The first, second and third terms of a geometric progression are $\sin \theta$, $\cos \theta$ and $2 - \sin \theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ .

[3]

[illegible]

- (b) Using this value of θ , find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

[illegible]

6 The equation of a curve is $y = x^2 - 8x + 5$.

- (a)** Find the coordinates of the minimum point of the curve. [2]

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The curve is stretched by a factor of 2 parallel to the y -axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

- (b)** Find the coordinates of the minimum point of the transformed curve. [2]

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- (c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a , b and c are integers to be found. [4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7 (a) Verify the identity $(2x - 1)(4x^2 + 2x - 1) \equiv 8x^3 - 4x + 1$. [1]

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- (b)** Prove the identity $\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} \equiv \frac{1}{1 - 2 \cos^2 \theta}$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

$$\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = 4 \cos \theta,$$

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

8 Functions f and g are defined by

$$f(x) = (x + a)^2 - a \text{ for } x \leq -a,$$

$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $f^{-1}(x)$. [3]

[illegible]

(b) (i) State the domain of the function f^{-1} . [1]

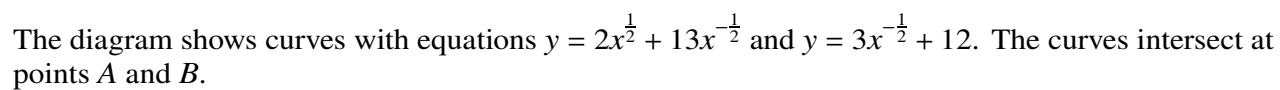
(ii) State the range of the function f^{-1} . [1]

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- (c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$.

[3]

[illegible]



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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 10** The equation of a curve is $y = f(x)$, where $f(x) = (4x - 3)^{\frac{5}{3}} - \frac{20}{3}x$.

- (a) Find the x -coordinates of the stationary points of the curve and determine their nature. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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(b) State the set of values for which the function f is increasing. [1]

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- 11** The coordinates of points A , B and C are $(6, 4)$, $(p, 7)$ and $(14, 18)$ respectively, where p is a constant. The line AB is perpendicular to the line BC .

(a) Given that $p < 10$, find the value of p .

[4]

[illegible]

A circle passes through the points A , B and C .

- (b) Find the equation of the circle. [3]

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- (c) Find the equation of the tangent to the circle at C , giving the answer in the form $dx + ey + f = 0$, where d , e and f are integers. [3]

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[illegible]

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 A curve is such that its gradient at a point (x, y) is given by $\frac{dy}{dx} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve.

[4]

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2 The circle with equation $(x - 3)^2 + (y - 5)^2 = 40$ intersects the y -axis at points A and B .

(a) Find the y -coordinates of A and B , expressing your answers in terms of surds. [2]

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(b) Find the equation of the circle which has AB as its diameter. [2]

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- 3 (a) Show that the equation

$$5 \cos \theta - \sin \theta \tan \theta + 1 = 0$$

may be expressed in the form $a \cos^2 \theta + b \cos \theta + c = 0$, where a , b and c are constants to be found. [3]

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- (b) Hence solve the equation $5 \cos \theta - \sin \theta \tan \theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

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- 4 (a)** Expand the following in ascending powers of x up to and including the term in x^2 .

(i) $(1 + 2x)^5$. [1]

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(ii) $(1 - ax)^6$, where a is a constant. [2]

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In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of x^2 is -5 .

- (b)** Find the possible values of a . [4]

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5 The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p . [3]

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(b) One of the values of p found in (a) is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

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- 6** A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where c is a constant. The line is a tangent to the curve at point P .

Find the possible values of c and the corresponding coordinates of P .

[7]

[illegible]

7 The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f .

[1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$.

[2]

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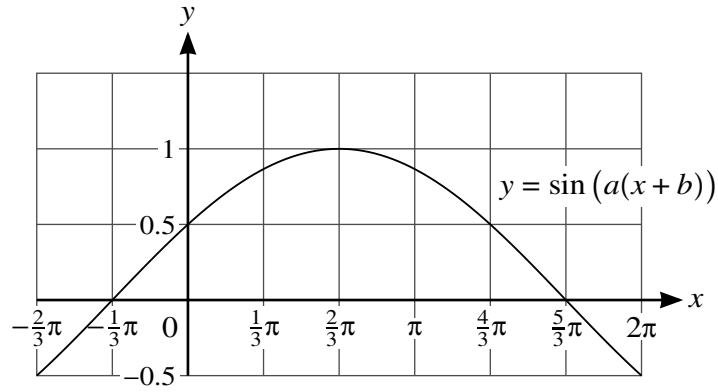
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The diagram shows part of the graph of $y = \sin(a(x+b))$, where a and b are positive constants.

- (a) State the value of a and one possible value of b .

[2]

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Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q) , where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\frac{1}{4}(x+8)\right).$$

- (b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q .

[3]

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9 A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

- (a) Find the equation of the normal to the curve at the point $A(4, 3)$, giving your answer in the form $y = mx + c$. [3]

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A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at A the rate of increase of the x -coordinate is 3 cm s^{-1} .

- (b) Find the rate of increase of the y -coordinate at A . [2]

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At A the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the y -coordinate is constant at 5 cm s^{-1} .

- (c) As the point moves down the normal, find the rate of change of its x -coordinate. [3]

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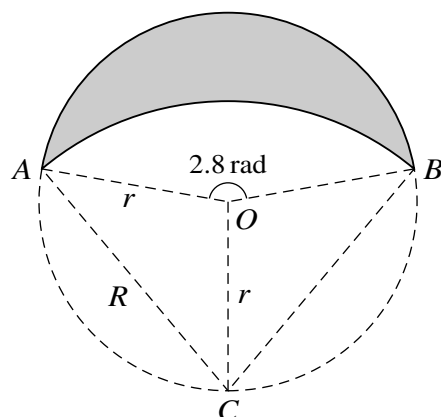
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The diagram shows points A , B and C lying on a circle with centre O and radius r . Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r . The lower arc is part of a circle with centre C and radius R .

- (a) State the size of angle ACO in radians. [1]

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- (b) Find R in terms of r . [1]

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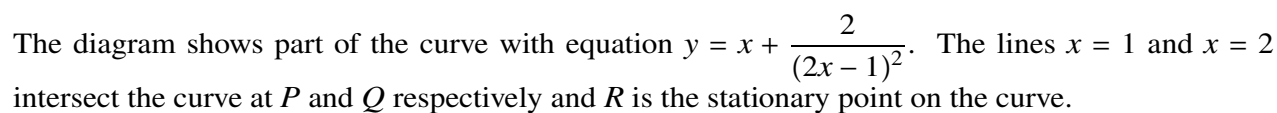
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- (c) Find the area of the shaded region in terms of r .

[7]

[illegible]



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- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

(b) Find the exact value of the area of the shaded region.

[6]

[illegible]

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