

Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

1 Solve the equation $3x + 2 = \frac{2}{x-1}$.

[3]

[illegible]

- 2 The equation of a curve is such that $\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

(a) Find the equation of the tangent to the curve at P . [2]

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(b) Find the equation of the curve. [4]

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- 3** A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P , which has x -coordinate 9.

Find the y -coordinate of P .

[5]

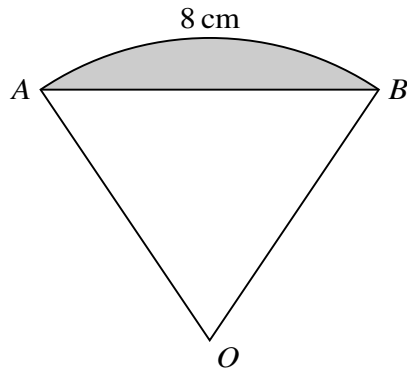
[illegible]

- 4** The coefficient of x^2 in the expansion of $\left(1 + \frac{2}{p}x\right)^5 + (1 + px)^6$ is 70.

Find the possible values of the constant p .

[6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows a sector OAB of a circle with centre O . The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

- (a) Find the perimeter of the shaded segment. [4]

[illegible]

[Turn over

6 (a) Show that the equation

$$\frac{1}{\sin \theta + \cos \theta} + \frac{1}{\sin \theta - \cos \theta} = 1$$

may be expressed in the form $a \sin^2 \theta + b \sin \theta + c = 0$, where a , b and c are constants to be found. [3]

[illegible]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7 A tool for putting fence posts into the ground is called a ‘post-rammer’. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.
- (a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground.

[3]

[illegible]

- (b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

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- (c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

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- 8** The function f is defined by $f(x) = 2 - \frac{3}{4x-p}$ for $x > \frac{p}{4}$, where p is a constant.

- (a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

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- (b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx-d}$, where a, b, c and d are integers. [4]

[illegible]

- (c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

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- 9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$

$$g(x) = 2x^2 + 4x + 12.$$

- (a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]

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- (b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

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- (c) Express $g(x)$ in the form $kf(x + h)$, where k and h are integers. [1]

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- (d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

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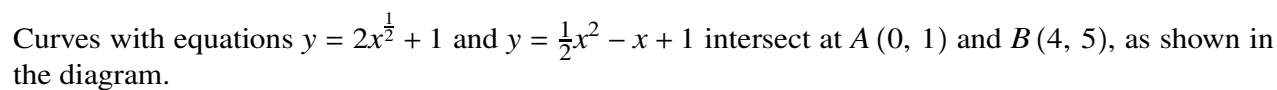
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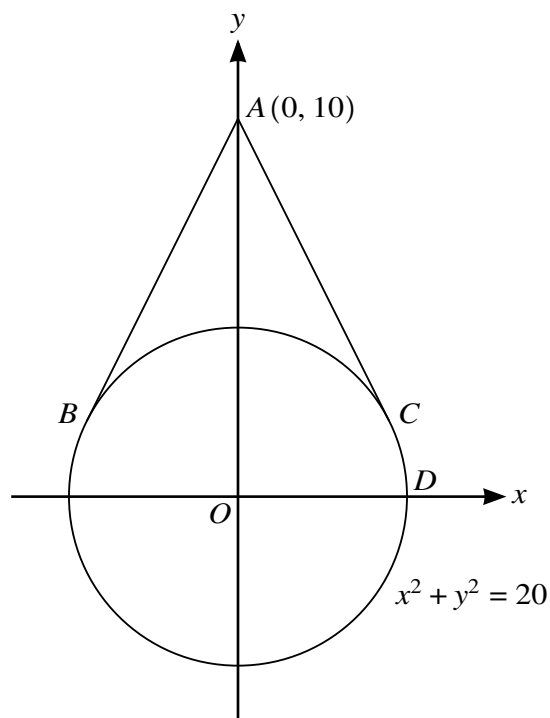
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- This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

The acute angle between the two tangents at B is denoted by α° , and the scales on the axes are the same.

(b) Find α .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point A $(0, 10)$.

- (a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the coordinates of B and C .

[3]

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The point D is where the circle crosses the positive x -axis.

(c) Find angle BDC in degrees.

[3]

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Cambridge International AS & A Level

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Points A and B have coordinates $(5, 2)$ and $(10, -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB . [3]

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(b) Find the equation of the circle with centre A which passes through B . [3]

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- 2** The first, second and third terms of an arithmetic progression are a , $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression.

[5]

[illegible]

- 3 (a) Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

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- (b) Solve the equation $8 \cos^2 \theta - 10 \cos \theta + 2 = 0$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

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- 4 A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term.

[4]

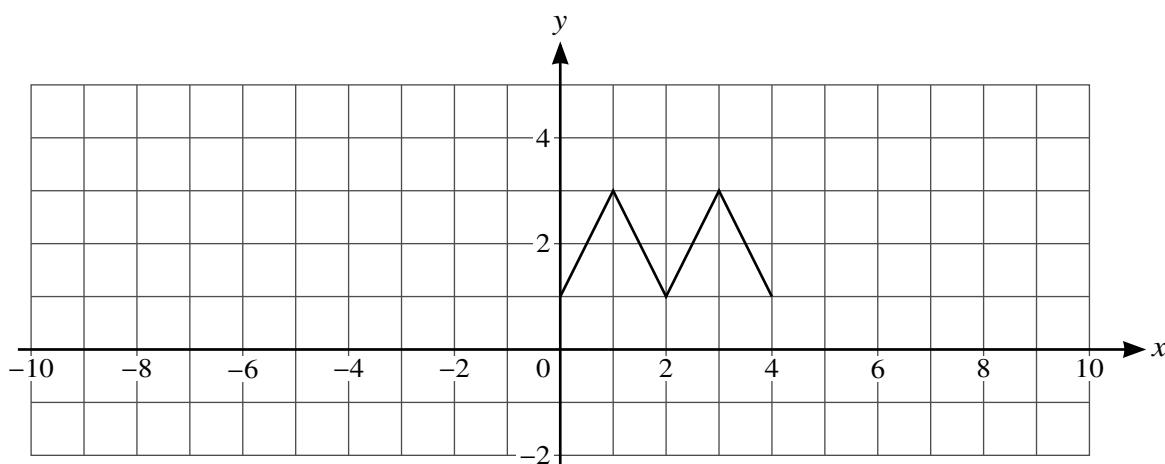
[illegible]

- 5** The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x -direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

- (a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$.

[3]



- (b)** Find an expression for $g(x)$ in terms of $f(x)$.

[2]

[illegible]

6 The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a , b and c are constants. [3]

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(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

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- (c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x - and y -axes. [3]

7 (a) Prove the identity $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \equiv \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$. [3]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 8** The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(a) Find the equation of the curve.

[4]

[illegible]

(b) Find the x -coordinate of the stationary point.

[2]

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(c) State the set of values of x for which y increases as x increases.

[1]

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- 9 Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$

$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

- (a) Find an expression for $gf(x)$. [1]

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- (b) Given that $gf(2) = 11$, find the value of a . [2]

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- (c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

- (d) Find and simplify an expression for $g^{-1}f(x)$. [3]

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- (e) Explain why the composite function fg cannot be formed. [1]

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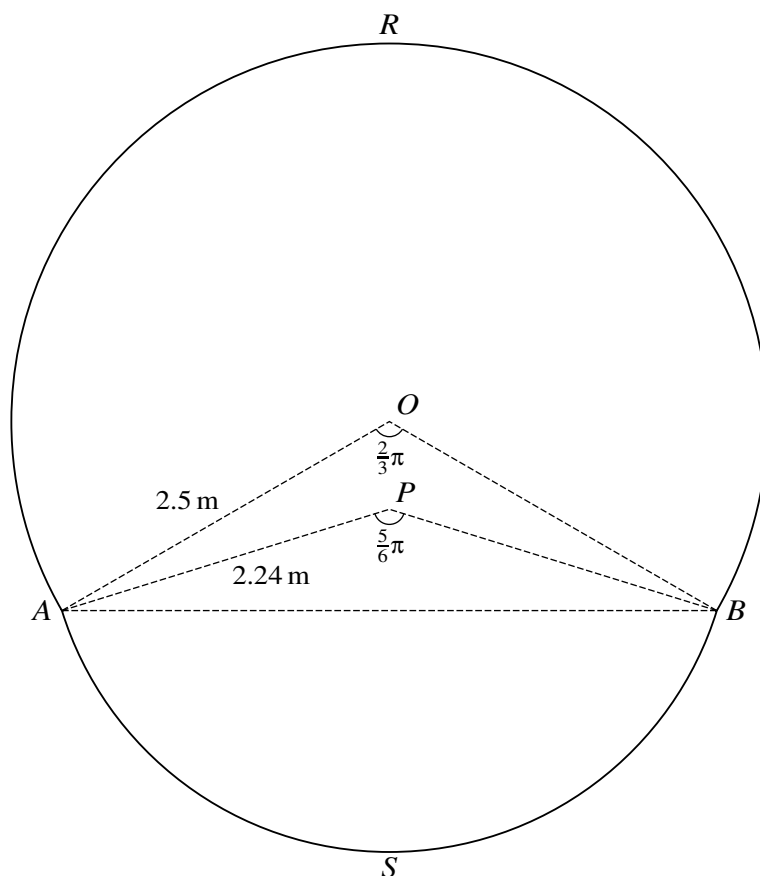
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The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O , a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

- (a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places.

[3]

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- (b) Find the difference in area of the two triangles AOB and APB , giving your answer correct to 2 decimal places. [2]

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- (c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

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- 11 (a)** Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]

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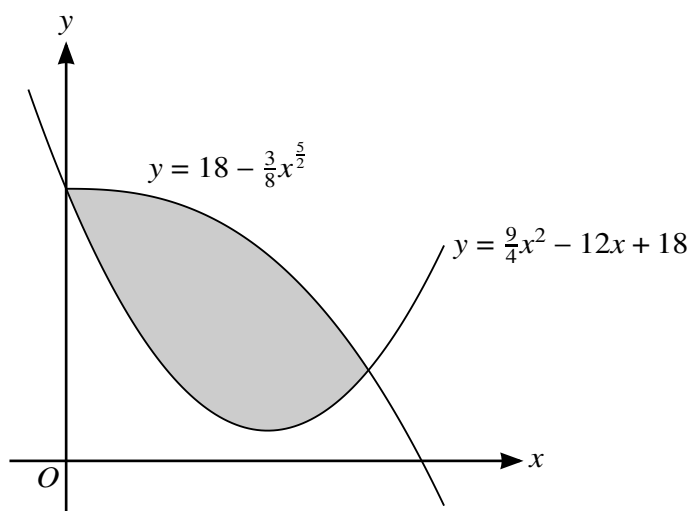
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The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

- (b)** Find the area of the shaded region. [5]

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- (c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x -coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y -coordinate of P is changing when $x = 4$. [3]

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[illegible]

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2022

1 hour 50 minutes

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You will need: List of formulae (MF19)

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INFORMATION

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This document has **20** pages. Any blank pages are indicated.

- 1** Solve the equation $8 \sin^2 \theta + 6 \cos \theta + 1 = 0$ for $0^\circ < \theta < 180^\circ$. [3]

[illegible]

2 The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

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(b) Find the range of f . [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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- 3 (a)** Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$. [2]

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- (b)** Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$. [2]

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- (c)** Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$. [2]

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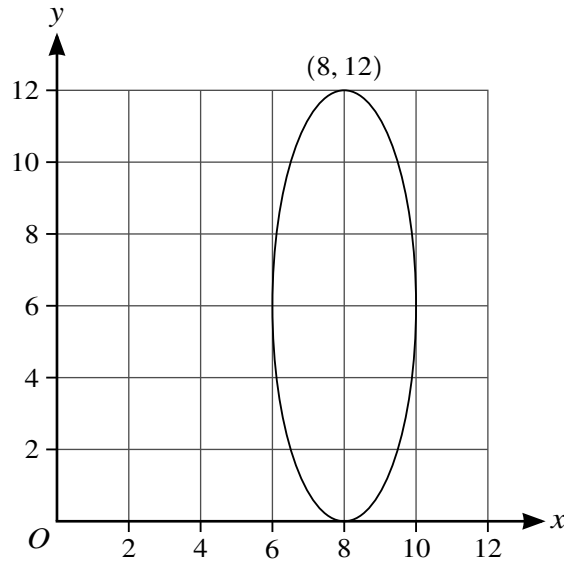
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- 4** A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of 3.6m^3 per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

[illegible]



The diagram shows a curve which has a maximum point at $(8, 12)$ and a minimum point at $(8, 0)$. The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y -direction.

- (a) State the scale factor of the stretch. [1]

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- (b) State the radius of the original circle. [1]

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- (c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

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- (d) State the coordinates of the centre of the original circle. [2]

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- 6** It is given that $\alpha = \cos^{-1}(\frac{8}{17})$.

Find, without using the trigonometric functions on your calculator, the exact value of $\frac{1}{\sin \alpha} + \frac{1}{\tan \alpha}$.

[5]

[illegible]

7 The curve $y = f(x)$ is such that $f'(x) = \frac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

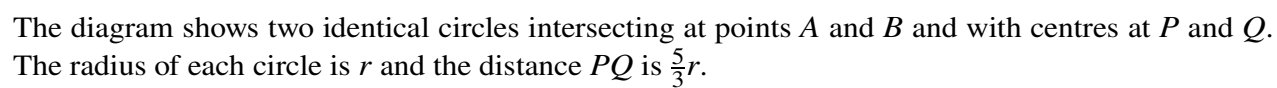
Find the possible values of a .

[4]

[illegible]

[3]

[illegible]



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(b) Find the area of the shaded region in terms of r .

[3]

[illegible]

9 The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression.

[3]

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The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. [6]

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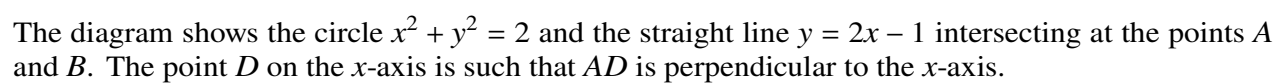
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- (b)** Find the volume of revolution when the shaded region is rotated through 360° about the x -axis.
Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a , b , c and d are integers. [4]

[illegible]

- (c) Find an exact expression for the perimeter of the shaded region. [2]

[illegible]

11 The coordinates of points A , B and C are $A(5, -2)$, $B(10, 3)$ and $C(2p, p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p . [3]

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(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p . [4]

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- (ii) Find the equation of the circle which passes through A , B and C , giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a , b and c are constants. [4]

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