



Cambridge International AS & A Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2022

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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Mathematics Specific Marking Principles	
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4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
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 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
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Abbreviations

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CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
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Question	Answer	Marks	Guidance
1	$(3x+2)(x-1)=2 \Rightarrow 3x^2-x-4 [=0]$	M1	OE Multiply by denominator and obtain a quadratic.
	$(3x-4)(x+1)[=0]$	M1	Solve by factorising, formula or completing the square.
	$[x =] -1, \frac{4}{3}$	A1	Allow 1.33 If M1 M0, SC B1 possible for two correct answers.
		3	

Question	Answer	Marks	Guidance
2(a)	$12\left(\frac{1}{2} \times 6 - 1\right)^{-4} \left[= 12(2)^{-4} = \frac{3}{4} \right]$	M1	Substitute $x=6$ into $\frac{dy}{dx}$ SOI by gradient $\frac{3}{4}$ used.
	$y-4 = \frac{3}{4}(x-6)$ OR evaluates $c = -\frac{1}{2}$	A1	OE e.g. $y = \frac{3}{4}x - \frac{1}{2}$ or evaluates c in $y = \frac{3}{4}x + c$ using (6, 4) and gradient $\frac{3}{4}$. ISW
		2	

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Question	Answer	Marks	Guidance
2(b)	$[y =] \left(\frac{12 \left(\frac{1}{2}x - 1 \right)^{-3}}{-3} \right) \div \frac{1}{2} \left[= -8 \left(\frac{1}{2}x - 1 \right)^{-3} \right]$	B2, 1, 0	
	$4 = \frac{12 \times \left(\frac{1}{2} \times 6 - 1 \right)^{-3}}{\frac{1}{2} \times -3} + c \left[\Rightarrow 4 = -8 \times 2^{-3} + c \right] \Rightarrow c = [5]$	M1	Must have +c . Substitute $y = 4$, $x = 6$ and solve for c in an integrated expression. May be unsimplified.
	$[y =] -8 \left(\frac{1}{2}x - 1 \right)^{-3} + 5$	A1	OE Must see ‘ $y =$ ’ or ‘ $f(x) =$ ’ in the working.
		4	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = \frac{1}{2}ax^{\frac{1}{2}} - 2$	B2, 1, 0	
	$0 = \frac{1}{2}a(9)^{-\frac{1}{2}} - 2 \Rightarrow \frac{a}{6} - 2 = 0 \Rightarrow a = [12]$	M1	Substitute $x = 9$ and $\frac{dy}{dx} = 0$ into <i>their</i> derivative and solve a linear equation for a .
	$[a =] 12$	A1	
	$\left[y = \text{their } a \times (9)^{\frac{1}{2}} - 18 = \right] 18$	A1 FT	FT on <i>their</i> a .
		5	

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Question	Answer	Marks	Guidance
.4	Coefficient of x^2 in $\left(1 + \frac{2}{p}x\right)^5$ is $10\left(\frac{2}{p}\right)^2 = \frac{10 \times 2^2}{p^2} \left[= \frac{40}{p^2} \right]$	B1	Accept with x^2 present. Must evaluate ${}^5C_2$
	Coefficient of x^2 in $(1 + px)^6$ is $15(p)^2 \left[= 15p^2 \right]$	B1	Accept with x^2 present. Must evaluate ${}^6C_2$
	$\frac{40}{p^2} + 15p^2 = 70$	*M1	Forming an equation in p with <i>their</i> coefficients, the given 70, no x terms and no extra terms.
	$15p^4 - 70p^2 + 40 \left[= 0 \right]$ or $3p^4 - 14p^2 + 8 \left[= 0 \right]$	DM1	Forming a 3-term equation in p (or another variable) with all terms on one side and <i>their</i> coefficients.
	$[5](p^2 - 4)(3p^2 - 2) \left[= 0 \right]$ or $\frac{70 \pm \sqrt{70^2 - 4(15)(40)}}{30}$ or $\frac{14 \pm \sqrt{14^2 - 4(3)(8)}}{6}$	DM1	Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square.
	$p = \pm 2, \pm \sqrt{\frac{2}{3}}$	A1	OE e.g. $\pm \frac{\sqrt{6}}{3}$ or AWRT ± 0.816 If *M1 DM1 DM0, allow SC B1 for 4 correct values.
		6	

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Question	Answer	Marks	Guidance
5(a)	$[2r + 8 = 20 \Rightarrow] r = 6$	B1	
	Angle $AOB = \frac{8}{\text{their } 6}$	*M1	Expect $\frac{4}{3}$ OE (76.4°). M0 Assume triangle is equilateral.
	$AB = 2 \times 6 \sin \text{their } \frac{2}{3}$ or $\sqrt{6^2 + 6^2 - 2 \times 6^2 \cos \text{their } \frac{4}{3}}$ or $AB = \frac{6}{\sin\left(\frac{\pi}{2} - \text{their } \frac{2}{3}\right)} \times \sin \text{their } \frac{4}{3}$	DM1	For 6 read <i>their</i> 6.
	Perimeter = $[7.42 + 8 =]$ 15.4	A1	AWRT
		4	
5(b)	Area = $\frac{1}{2} \times 6^2 \times \text{their } \frac{4}{3} - \frac{1}{2} \times 6^2 \times \sin \text{their } \frac{4}{3}$ or Area = $\frac{1}{2} \times 6^2 \times \text{their } \frac{4}{3} - 2 \times \frac{1}{2} \left(6 \sin \text{their } \frac{2}{3} \right) \left(6 \cos \text{their } \frac{2}{3} \right)$	M1	Sector area – whole triangle area. For 6 read <i>their</i> 6. Sector area – 2(half triangle area).
	= $[24 - 17.49 =]$ 6.51	A1	AWRT
		2	

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Question	Answer	Marks	Guidance
6(a)	$\frac{\sin \theta - \cos \theta + \sin \theta + \cos \theta}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)} \left[= \frac{\sin \theta - \cos \theta + \sin \theta + \cos \theta}{\sin^2 \theta - \cos^2 \theta} \right] = 1$	*M1	Use common denominator and equate to 1.
	$2 \sin \theta [= \sin^2 \theta - \cos^2 \theta] = \sin^2 \theta - (1 - \sin^2 \theta)$	DM1	Multiply by common denominator and replace $\cos^2 \theta$ by $1 - \sin^2 \theta$.
	$2 \sin^2 \theta - 2 \sin \theta - 1 = 0$	A1	OE In the given form.
		3	
6(b)	$[\sin \theta =] \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{4} \left[= \frac{2 \pm \sqrt{4+8}}{4} = \frac{1 \pm \sqrt{3}}{2} \right]$	M1	Use formula or complete the square to solve a quadratic equation of the correct form.
	201.5° or 338.5°	A1 A1 FT	AWRT; A1 for either solution correct. A1 FT for 540 – (first value). If M0, allow SC B1 B1FT similarly.
		3	

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Question	Answer	Marks	Guidance
7(a)	$r = 0.8$	B1	SOI
	$50 \times (\text{their } 0.8)^7 = 10.5$	M1	Evaluate 8 th or 9 th term in GP.
	$50 \times (\text{their } 0.8)^8 = 8.39$. Hence 9th impact required	A1	AG Two terms correct + conclusion (mention of 9 th impact or u_9 somewhere in the solution). Statement that one is <10 (and the other >10) is insufficient unless it mentions 9 th impact or u_9 .
	Alternative method for final two marks: Logarithm method		
	$50 \times (\text{their } 0.8)^n < 10 \Rightarrow (\text{their } 0.8)^n < 0.5$ $n \log(\text{their } 0.8) < \log 0.5$ $n > \frac{\log 0.5}{\log(\text{their } 0.8)} \Rightarrow [n >] 7.2$	M1	
	$n = 8$ hence 9 th impact required	A1	AG Need conclusion that mentions 9 th impact or u_9 .
7(b)		3	
	$\frac{50(1 - (\text{their } 0.8)^{20})}{1 - \text{their } 0.8}$	M1	OE Use of formula with <i>their</i> r SOI.
	$= 247$	A1	Must be to the nearest mm (not 247.1).
7(c)		2	
	$\frac{50}{1 - \text{their } 0.8}$	M1	Use of sum to infinity formula with <i>their</i> r SOI. Substituting a value of n into the sum formula M0.
	$= 250$	A1	
		2	

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Question	Answer	Marks	Guidance
8(a)	$f'(x) = -3(-1)(4)(4x-p)^{-2} \left[= \frac{12}{(4x-p)^2} \right]$	B2, 1, 0	
	> 0 Hence increasing function	B1FT	Correct conclusion from <i>their</i> $f'(x)$.
		3	
8(b)	$y = 2 - \frac{3}{4x-p} \Rightarrow (y-2)(4x-p) = -3$ or $4xy - py = 8x - 2p - 3$	M1	OE Form horizontal equation. Sign errors only, no missing terms. May go directly to $4y = p - \frac{3}{x-2}$ OE M1 M1
	$4xy - 8x = py - 2p - 3 \Rightarrow 4x(y-2) = p(y-2) - 3$ or $4x = -\frac{3}{y-2} + p$	M1	OE Factorise out $[4]x$ or $[4]y$.
	$x = \frac{p(y-2)-3}{4(y-2)} \left[\Rightarrow x = \frac{p}{4} - \frac{3}{4y-8} \right]$ or $-\frac{3}{x-2} + p$	M1	OE Make x (or y) the subject.
	$[f^{-1}(x) =] \frac{p}{4} - \frac{3}{4x-8}$	A1	OE in correct form (must be in terms of x).
		4	
8(c)	$[p=]8$	B1	
		1	

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Question	Answer	Marks	Guidance
9(a)	$(x-2)^2 + 5$	B1	
		1	
9(b)	$2\left(\{(x+1)^2\} + \{5\}\right)$	B2, 1, 0	
		2	
9(c)	$[g(x) =] 2f(x+3) \text{ or } k=2, h=3$	B1	In correct form. B0 if contradiction.
		1	
9(d)	{Translation} $\left\{\begin{pmatrix} -3 \\ 0 \end{pmatrix}\right\}$	B2, 1, 0 FT	FT on <i>their</i> $x+3$ or $h=3$.
	{Stretch} {y direction, factor 2}	B2, 1, 0 FT	FT on <i>their</i> 2 or $k=2$.
		4	

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Question	Answer	Marks	Guidance
10(a)	$\pm \int (2x^{1/2} + 1) - \left(\frac{1}{2}x^2 - x + 1 \right) dx \quad [= \pm \int 2x^{1/2} - \frac{1}{2}x^2 + x dx]$	*M1	
	$\pm \left(\frac{4x^{3/2}}{3} + x - \left(\frac{x^3}{6} - \frac{x^2}{2} + x \right) \right)$ or $\pm \left(\frac{4x^{3/2}}{3} - \frac{x^3}{6} + \frac{x^2}{2} \right)$	B2, 1, 0	OE Coefficients may be unsimplified.
	$\pm \left(\frac{32}{3} - \frac{32}{3} + 8 \right)$ or $\pm \left(\frac{44}{3} - 0 - \frac{20}{3} + 0 \right)$	DM1	$\pm (F(4) - F(0))$ using <i>their</i> integral(s).
	=8	A1	Depends on all previous marks. If *M1 B2 DM0 and limits stated, SC B1 for +8
		5	
10(b)	Upper curve: $\frac{dy}{dx} = x^{-\frac{1}{2}}$. Lower curve: $\frac{dy}{dx} = x - 1$	M1 A1	Attempt at differentiating one function. A1 if both correct.
	At $x = 4$: gradient of upper curve = $\frac{1}{2}$, gradient of lower curve = 3	M1	Evaluate two gradients using $x = 4$.
	$\alpha = \tan^{-1} 3 - \tan^{-1} \frac{1}{2} \quad [= 71.57 - 26.57]$	M1	Use inverse tan to find angles then subtract. OR find equations of both tangents then Pythagoras using a point on each e.g. on axes. OR cosine rule using intercepts or proportion.
	$[\alpha =] 45^\circ$	A1	AWRT
		5	

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Question	Answer	Marks	Guidance
11(a)	$x^2 + (mx + 10)^2 = 20$ or $y^2 + \left(\frac{y-10}{m}\right)^2 = 20$ or $mx + 10 = \sqrt{20 - x^2}$	*M1	Substitute equation of line into equation of circle.
	$x^2(1 + m^2) + 20mx + 80 [= 0]$ or $y^2(m^2 + 1) - 20y + (100 - 20m^2)[= 0]$	A1	Collect terms into a 3 term quadratic.
	$(20m)^2 - 4(1 + m^2) \times 80 [= 0] \Rightarrow 80m^2 - 320 = 0 \Rightarrow [80](m^2 - 4) = 0]$ or $(-20)^2 - 4(m^2 + 1)(100 - 20m^2)[= 0] \Rightarrow [80](m^4 - 4m^2) = 0]$	DM1	Use $b^2 - 4ac [= 0]$.
	$m = \pm 2$	A1	Two values for m .
		4	

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Question	Answer	Marks	Guidance
11(b)	Method 1: Use of quadratic		
	$(1+2^2)x^2 \pm 20(2)x + 80 [= 0 \Rightarrow 5x^2 \pm 40x + 80 = 0]$ or $y^2(2^2 + 1) - 20y + (100 - 20(2^2)) [= 0 \Rightarrow [5](y^2 - 4y + 4) = 0]$	M1	Sub <i>their m</i> into <i>their</i> quadratic in <i>x</i> or <i>y</i> or restart with <i>their</i> tangent equation and equation of circle.
	$[5](x \pm 4)^2 = 0 \Rightarrow x = \pm 4$ or $y = 2$	A1	Correct solutions or one correct pair (<i>x</i> , <i>y</i>).
	$(-4, 2), (4, 2)$	A1	Two correct points with <i>x</i> and <i>y</i> paired correctly.
	Method 2: Using equation of normal		
	$2x + 10 = -\frac{1}{2}x$ or $-2x + 10 = \frac{1}{2}x$	M1	Equate tangent and normal and solve for <i>x</i> .
	$x = \pm 4$	A1	Two correct <i>x</i> -values or one correct pair (<i>x</i> , <i>y</i>).
	$(-4, 2), (4, 2)$	A1	Two correct points with <i>x</i> and <i>y</i> paired correctly.
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Question	Answer	Marks	Guidance
11(c)	Method 1: Using angle at circumference		
	$\cos BOA = \frac{\sqrt{20}}{10}$ or $\sin BOA = \frac{\sqrt{80}}{10}$ or $\tan BOA = \frac{\sqrt{80}}{\sqrt{20}} [= 2]$	*M1	Use a trig function in triangle <i>AOB</i> .
	$BOA = 63.4^\circ \Rightarrow BOC = 126.8^\circ$ or 126.9°	DM1	Strategy involving doubling
	$[BDC =] 63.4^\circ$	A1	AWRT
	Metho 2: Using cosine rule		
	$BC = 8, BD = \sqrt{(\sqrt{20} + 4)^2 + 2^2}, CD = \sqrt{(\sqrt{20} - 4)^2 + 2^2}$	*M1	Calculate two lengths in triangle <i>BCD</i> .
	$64 = 80 - 16\sqrt{5} \cos BDC$	DM1	Use cosine rule with <i>their</i> lengths
	$\cos BDC = \frac{\sqrt{5}}{5} \Rightarrow [BDC =] 63.4^\circ$	A1	AWRT
	Method 3: Subtract angles from 90°		
	Calculate one angle at <i>D</i> [= 13.28]	*M1	<i>ODB</i> or angle between <i>CD</i> and the vertical from <i>D</i>
	Calculate a second angle at <i>D</i> [= 13.28] and subtract both from 90°	DM1	
	$[BDC =] 63.4^\circ$	A1	AWRT
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Question	Answer	Marks	Guidance
1(a)	Mid-point AB is $\left(\frac{10+5}{2}, \frac{2-1}{2}\right) \left[= \left(\frac{15}{2}, \frac{1}{2}\right) \right]$	B1	Accept unsimplified.
	Gradient of $AB = \frac{-1-2}{10-5} = \frac{-3}{5}$ Gradient perpendicular = $\frac{5}{3}$	M1	For use of $\frac{\text{Change in } y}{\text{Change in } x}$, condone inconsistent order of x and y , and $m_1 m_2 = -1$.
	$\frac{y - \frac{1}{2}}{x - \frac{15}{2}} = \frac{5}{3} \left[y - \frac{1}{2} = \frac{5}{3} \left(x - \frac{15}{2} \right) \right]$	A1	OE ISW Any correct version e.g. $y = \frac{5}{3}x - 12$ or $5x - 3y = 36$.
		3	
1(b)	[Radius =] $\sqrt{34}$ or 5.8 AWR or [(radius) ² =] 34	B1	Sight of $\sqrt{34}$ or 34. Condone confusion of r and r^2 .
	$(x-5)^2 + (y-2)^2$	B1	Sight of $(x-5)^2 + (y-2)^2$
	$(x-5)^2 + (y-2)^2 = 34$	B1	CAO ISW
	Alternative method for Question 1(b)		
	$x^2 + y^2 - 10x - 4y$	B1	
	$[c =] 5$ or $[c =] -5$	B1	Substitution of $(10, -1)$ into $x^2 + y^2 - 10x - 4y + c = 0$.
	$x^2 + y^2 - 10x - 4y - 5 = 0$	B1	
		3	

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Question	Answer	Marks	Guidance
2	$2a - a = a^2 - 2a$	B1	OE An unsimplified correct equation in a or d only, e.g. $a^2 + a = 4a$. Can be implied by correct values for a or d .
	$a = 3$ or $d = 3$	B1	Condone ‘extra’ solution of $a = 0$ or $d = 0$.
	$a = 3$ and $d = 3$	B1	SOI
	$S_{50} = \frac{50}{2}(2 \times \text{their } a + 49 \times \text{their } d)$	M1	May be done using 50th term (=150). Their a and d must be numerical.
	3825	A1	ISW SC B2 for $1275a$ or $1275d$
		5	

Question	Answer	Marks	Guidance
3(a)	$k^2 - 4 \times 8 \times 2 < 0$	M1	Use of $b^2 - 4ac$ but not just in the quadratic formula.
	$-8 < k < 8$ or $-8 < k, k < 8$ or $ k < 8$ or $(-8, 8)$	A1	Condone ‘ $-8 < k$ or $k < 8$ ’, ‘ $-8 < k$ and $k < 8$ ’ but not $\sqrt{64}$.
		2	

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Question	Answer	Marks	Guidance
3(b)	$2(4\cos\theta - 1)(\cos\theta - 1)$ or $(4\cos\theta - 1)(\cos\theta - 1)$	M1	OE Or use of formula or completing the square. Allow use of replacement variable.
	$\cos\theta = \frac{2}{8}, \cos\theta = 1$	A1	OE For both answers. SC: If M0, SC B1 available for sight of $\cos\theta = \frac{2}{8}$ and 1
	$[\theta =] 0^\circ, 75.5^\circ$	A1	AWRT ISW rejection of 0° . For both answers and no others in the range $0^\circ \leq \theta \leq 180^\circ$, must be in degrees. SC: If M0 B1 scored, SC B1 available for correct answers. SC: If M1 A0 scored, SC B1 available for $\cos\theta = \frac{2}{8}$ and $\theta = 75.5^\circ$ only, WWW.
		3	

Question	Answer	Marks	Guidance
4	$a r^2 = 1764$ and $a r + a r^2 = 3444$ or $a r = 1680$ or $\frac{a(1-r^3)}{1-r} - a = 3444$	B1	Two correct algebraic statements.
	Attempt to solve as far as $r =$ or $a =$	M1	Any valid method, e.g. $1764 \div 1680$ or from $20r^2 - 41r + 21$ OE (condone solving using a calculator).
	$r = \frac{1764}{1680} = \frac{21}{20}$ or 1.05 [$a = 1600$]	A1	Note: $r = \frac{1764}{3444 - 1764}$ www implies B1 and M1.
	17 500	A1	AWRT e.g. 17 474.1.....
		4	

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Question	Answer	Marks	Guidance
5(a)	Three points at the bottom of their transformed graph plotted at $y = 2$	B1	All 5 points of the graph must be connected.
	Bottom three points of $\wedge$ at $x = 0$, $x = 1$ & $x = 2$	B1	Must be this shape.
	All correct	B1	Condone extra cycles outside $0 \leq x \leq 2$.
		3	SC: If B0 B0 scored, B1 available for $\wedge$ in one of correct positions or all 5 points correctly plotted and not connected or correctly sized shape in the wrong position.
5(b)	$[g(x) =] f(2x) + 1$	B1 B1	Award marks for their final answer as follows: $f(2x)$ B1, + 1 B1. Condone $y =$ or $f(x) =$.
		2	

Question	Answer	Marks	Guidance
6(a)	$y = 4\left(x + \frac{5}{2}\right)^2 - 19$		There is no requirement for the candidate to list a , b and c . Look at values in their final expression, condone omission of ^{2} , and award marks as follows:
		B1	$a = 4$
		B1	$b = \frac{5}{2}$ OE
		B1	$c = -19$
		3	

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Question	Answer	Marks	Guidance
6(b)	$\left(\text{Their } 4\left(x + \frac{5}{2}\right)^2 - 19 \right) = 45 \Rightarrow \left(x + \frac{5}{2}\right)^2 = 16$	*M1	Equate their quadratic completed square form from 6(a) to 45 or re-start and use completing the square.
	Solve as far as $x =$	DM1	Any valid method leading to two answers.
	$\left[x = \right] \frac{3}{2}, -\frac{13}{2}$	A1	SC: If M0 or M1 DM0 awarded, B1 available for correct final answers.
		3	
6(c)	Quadratic curve that is the right way up (must be seen either side of stationary point)	B1	No axes required, ignore any axes even if incorrect.
	Stationary point stated using any valid method or correctly labelled on their diagram.	B1 FT	FT <i>their</i> values from 6(a) as long as <i>their</i> expression is of the form $p(qx + r)^2 + s$. Expect $\left(-\frac{5}{2}, -19\right)$. Condone if stated correctly but plotted incorrectly.
		B1 FT	
		3	

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Question	Answer	Marks	Guidance
7(a)	$\frac{\sin \theta (\sin \theta - \cos \theta) + \cos \theta (\sin \theta + \cos \theta)}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)} \left[= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} \right]$	*M1	Sight of a correct common denominator, either in one or two fractions, condone missing brackets if recovered. In the numerator condone $\pm$ sign errors only.
	$\frac{\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta}}$	DM1	Divide throughout by $\cos^2 \theta$.
	$\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} \text{ AG}$	A1	
	Alternative method for Question 7(a)		
	$\frac{\frac{\sin^2 \theta}{\cos^2 \theta} + 1}{\frac{\sin^2 \theta}{\cos^2 \theta} - 1} \times \frac{\cos^2 \theta}{\cos^2 \theta} \text{ or the equivalent step } \left[= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} \right]$	*M1	Replace $\tan^2 \theta$ with $\frac{\sin^2 \theta}{\cos^2 \theta}$ and multiply top and bottom by $\cos^2 \theta$. Condone $\pm$ sign errors.
	Sight of convincing use of partial fractions	DM1	
	$\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \text{ AG}$	A1	
		3	Note: M1 DM1 A1 for working on both sides at the same time and finishing at the same correct expression. M1 DM1 for starting separately and finishing at the same correct expression and A1 if there is a final conclusion e.g. QED. Do not allow cross multiplication. Condone use of s, c and t and omission of θ .

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Question	Answer	Marks	Guidance
7(b)	$\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = 2 \Rightarrow \tan^2 \theta + 1 = 2(\tan^2 \theta - 1)$	*M1	Equate expression from (a) to 2 and clear fraction.
	$\tan \theta = [\pm]\sqrt{3}$	DM1	Simplify as far as $\tan \theta =$. May be implied by a correct final answer in degrees or radians.
	Alternative method for first two marks of Question 7(b)		
	$\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} = 2 \Rightarrow 1 = 2 \sin^2 \theta - 2(1 - \sin^2 \theta)$	*M1	Equate expression to 2, clear fraction and use trig identities to form an equation in $\sin \theta$ or $\cos \theta$ only.
	$\sin \theta = [\pm]\sqrt{\frac{3}{4}}$ or $\cos \theta = [\pm]\sqrt{\frac{1}{4}}$	DM1	Simplify as far as $\sin \theta =$, or $\cos \theta =$.
	$\theta = \frac{1}{3}\pi, \frac{2}{3}\pi$	A1	A1 for either correct answer then A1FT For their second value being $\pi -$ (their first) and no others in range $0 \leq \theta \leq \pi$, both values must be exact and in radians. SC: B1 for $\theta = 60^\circ, 120^\circ$ or $0.333\pi, 0.667\pi$ AWRT. or 1.05, 2.09 AWRT.
		A1 FT	
		4	

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Question	Answer	Marks	Guidance
8(a)	$\left[y = \left\{ \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} \right\} + \left\{ -\frac{3x^{\frac{1}{2}}}{\frac{1}{2}} \right\} \right] [+c] \left[= 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} \right]$	B1 B1	Marks can be awarded for correct unsimplified expressions, 1 mark each for contents of { } ISW.
	$5 = 2 \times 3^{\frac{3}{2}} - 6 \times 3^{\frac{1}{2}} + c$	M1	Correct use of (3,5) in an integrated expression (defined by at least one correct power) including + c.
	$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 5$	A1	Condone $c = 5$ as their final line if either $y =$ or $f(x) =$ seen elsewhere in the solution, but coefficients must not contain unresolved double fractions.
		4	
8(b)	$3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} = 0$	M1	Setting given differential to 0.
	$[x =] 1$	A1	CAO WWW Condone extra solution of -1 only if it is rejected.
		2	
8(c)	$x > 1$ or $x > \text{“their 8(b)”}$	B1FT	Allow $\geq$
		1	

Question	Answer	Marks	Guidance
9(a)	$a \left(x + \frac{1}{x} \right) + 1$	B1	ISW
		1	

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Question	Answer	Marks	Guidance
9(b)	$a\left(2 + \frac{1}{2}\right) + 1 = 11$	M1	Substitute $x = 2$ into <i>their</i> expression from (a) and equate to 11. This may be done in 2 stages: $f(2) = 2.5, g(2.5) = 11$.
	$[a =] 4$	A1	
		2	
9(c)	No,[because it is] not one-one	B1	Or other suitable explanation that may include one to many or many to one.
		1	
9(d)	$[g^{-1}(x)] = \frac{x-1}{5}$ WWW	B1	Condone use of a instead of 5.
	$[g^{-1}f(x)] = \frac{x + \frac{1}{5} - 1}{5}$ OE	M1	Correct combination of their $g^{-1}(x)$ with given $f(x)$ Condone use of a instead of 5.
	$\frac{x^2 - x + 1}{5x}$ or $\frac{1}{5}\left(x + \frac{1}{x} - 1\right)$ or $\frac{1}{5}(x + x^{-1} - 1)$ OE ISW	A1	Must not contain unresolved fractions e.g. $\frac{x + x^{-1} - 1}{5}$.
		3	
9(e)	The domain of f does not include the whole of the range of g . Or The range of g does not lie in the domain of f .	B1	Accept an answer that includes an example outside the domain of f , e.g. $g(-1) = -4$ but for $f, x > 0$.
		1	

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Question	Answer	Marks	Guidance
10(a)	$2.5 \times \frac{4\pi}{3} + 2.24 \times \frac{5\pi}{6} [= 10.47[2] + 5.86[4] \text{ or } \frac{10\pi}{3} + \frac{28\pi}{15}]$	B1	For either arc correct. Arc ARB could be AR+RB.
		M1	For adding two (or three) arc lengths using different radii and angles and nothing else. SOI
	16.34 or $\frac{26\pi}{5}$	A1	AWRT Condone 16.33 only.
		3	
10(b)	Area AOB = $\frac{1}{2} \times 2.5^2 \sin \frac{2\pi}{3} [=2.706]$ Area APB = $\frac{1}{2} \times 2.24^2 \sin \frac{5\pi}{6} [=1.254]$	M1	For either $\triangle AOB$ or $\triangle APB$ (AB = 4.33, h= 1.25, 0.58) or any other valid method.
	[Difference =] 1.45	A1	AWRT Condone 1.46 only.
		2	
10(c)	Area AOB = $\frac{1}{2} \times 2.5^2 \times \frac{4\pi}{3} [=13.09]$ Area APB = $\frac{1}{2} \times 2.24^2 \times \frac{5\pi}{6} [=6.57]$	B1	For either sector area correct
	[Area of cross section =] $\frac{1}{2} \times 2.5^2 \times \frac{4\pi}{3} + \frac{1}{2} \times 2.24^2 \times \frac{5\pi}{6} + \text{“their10(b)”}$ [=13.09+6.57+ “their10(b)”]	M1	Adding two sector areas from different sectors and ‘ <i>their10(b)</i> ’ and nothing else. SOI
	21.1	A1	CAO Condone slight inaccuracies in intermediate working if the correct answer is arrived at.
		3	

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Question	Answer	Marks	Guidance
11(a)	$\left[\frac{dy}{dx} = \frac{9}{2}x - 12 [= 0] \text{ or } [y =] \frac{9}{4}\left\{\left(x - \frac{8}{3}\right)^2 + \frac{8}{9}\right\} \text{ or } \frac{9}{4}\left(x - \frac{8}{3}\right)^2 + 2\right.$	B1	OE Either $\frac{dy}{dx}$ or a correct expression in completed square form. Allow unsimplified.
	$x = \frac{24}{9}$	B1	OE Condone 2.67 AWRT.
	$y = 2$	B1	CAO Note: $x = \frac{-b}{2a} = \frac{8}{3}$ B1; substitute $\frac{8}{3}$ for x in $y =$ B1; $y=2$ B1.
		3	

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Question	Answer	Marks	Guidance
For 11(b) look for working to be marked on page 19 or annotate it as BP or SEEN			
11(b)	$[\text{Area} =] \int \left(18 - \frac{3}{8}x^{\frac{5}{2}} - \left(\frac{9}{4}x^2 - 12x + 18 \right) \right) dx$	M1	Intention to integrate and subtract areas (either way around). Can be two separate functions or combined. Using y^2 scores 0/5 but condone inclusion of π except for the final mark.
	Note: Subtraction not required for these marks. Either separately $\left([18x] - \frac{3x^{\frac{7}{2}}}{8 \times \frac{7}{2}} \right)$, $\left(\frac{9x^3}{4 \times 3} - \frac{12x^2}{2} [+18x] \right)$ Or combined $[18x] - \frac{3x^{\frac{7}{2}}}{8 \times \frac{7}{2}} - \frac{9x^3}{4 \times 3} + \frac{12x^2}{2} [-18x]$	B1, B1	One mark for correct integration of each curve, allow unsimplified. $\left([18x] - \frac{3}{28}x^{\frac{7}{2}} \right) \left(\frac{3}{4}x^3 - 6x^2 [+18x] \right)$ or $[18x] - \frac{3}{28}x^{\frac{7}{2}} - \frac{3}{4}x^3 + 6x^2 [-18x]$ BUT condone sign errors that are only due to missing brackets.
	$= \left(-\frac{3}{28} \times 4^{\frac{7}{2}} - \frac{3}{4} \times 4^3 + 6 \times 4^2 \right) [- (0)]$	M1	Clear substitution of 4 into at least one integrated expression (defined by at least one correct power) which can be unsimplified.
	$= \frac{240}{7} \text{ or } 34.3 \text{ AWRT}$	A1	SC: If all marks awarded except the final M1, SCB1 is available for the correct final answer.
		5	

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Question	Answer	Marks	Guidance
11(c)	$\left[\frac{dy}{dx} = \right] \frac{-5 \times 3}{2 \times 8} x^{\frac{3}{2}} \left[= -\frac{15}{16} x^{\frac{3}{2}} \right]$	B1	Allow unsimplified.
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{15}{16} \times 8 \times 2$	M1	Substitute $x=4$ into their $\frac{dy}{dx}$ and multiply by 2.
	-15	A1	Accept decreasing [at/by] 15
		3	Note: If incorrect curve used, this is not a MR and only M1 mark is available. Expect $(\frac{9(4)}{2} - 12) \times 2 [=12]$



Cambridge International AS & A Level

MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2022

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **14** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$8(1 - \cos^2 \theta) + 6 \cos \theta + 1 = 0$	M1	Expect $8 \cos^2 \theta - 6 \cos \theta - 9 = 0$.
	$(4 \cos \theta + 3)(2 \cos \theta - 3) = 0$	A1	Factors or formula or completing square must be shown.
	$[\rightarrow \cos \theta = -0.75 \rightarrow \theta =] 138.6^\circ$ only,	A1	AWRT, ignore solutions outside the given range, answer in radians A0.
		3	

Question	Answer	Marks	Guidance
2(a)	$[f(x)] = \{-2(x+2)^2\} - \{5\}$	B1 B1	
		2	
2(b)	$[f(x)] < -7$	B1	Allow $y < -7, < -7, (-\infty, -7)$ or less than -7 $-\infty < f(x) < -7, -7 < f(x) < -\infty, f < -7$
		1	
2(c)	$y = -2(x+2)^2 - 5 \rightarrow (x+2)^2 = \frac{-(y+5)}{2}$	M1	Operations correct. Allow sign errors. FT <i>their</i> quadratic from (a) .
	$x = [\pm] \sqrt{\frac{-(y+5)}{2}} - 2$	M1	Operations correct. Allow sign errors. FT <i>their</i> quadratic from (a) .
	$[f^{-1}(x)] = -2 - \sqrt{\frac{-(x+5)}{2}}$ or $-2 - \sqrt{\frac{(x+5)}{2}}$	A1	Allow $[f^{-1}(x)] = -2 - \sqrt{\frac{x+5}{-2}}$.
		3	

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Question	Answer	Marks	Guidance
3(a)	$1+10x+40x^2$ May be part of a complete expansion	B2, 1, 0	1^5 must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer. Mis-reads not condoned in this question.
		2	
3(b)	$1-12x+54x^2$ May be part of a complete expansion	B2, 1, 0	1^4 must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer. Mis-reads not condoned in this question.
		2	
3(c)	$54-120+40$	M1	Forming exactly 3 products correctly using their terms.
	-26	A1	Allow $-26x^2$ If in a list with other terms it must be clear this is the required term otherwise A0.
		2	

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Question	Answer	Marks	Guidance
4	$\left[\frac{dv}{dx}\right] = (9-x)^2$	B1	Allow unsimplified forms. Allow any or no notation
	Substitute $x = 4$ into <i>their</i> differentiated V,	*M1	Expect 25.
	$\frac{dx}{dt} = \frac{1}{\text{their derivative}} \times 3.6$ (accept $\frac{dt}{dx} = \frac{\text{their derivative}}{3.6}$)	M1	Correct use of the chain rule, ignore incorrect conversions at this point. Expect 0.144
	$= \frac{1}{\text{their numerical derivative}} \times 3.6 \times \frac{100}{60}$	DM1	Correct use of the conversion factors.
	$= \frac{1}{25} \times 3.6 \times \frac{100}{60} = 0.24$	A1	
		5	

Question	Answer	Marks	Guidance
5(a)	3	B1	Ignore any description.
		1	
5(b)	2	B1	Ignore any description.
		1	
5(c)	(8, 2)	B1 B1	Ignore any description. Allow vector notation and absence of brackets.
		2	

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Question	Answer	Marks	Guidance
5(d)	(1, 5)	B1 FT	FT each coordinate, (<i>their</i> 8 – 7, <i>their</i> 2 + 3) Allow vector notation and absence of brackets.
		B1 FT	
		2	

Question	Answer	Marks	Guidance
6	Use of $\sin^2 \alpha + \cos^2 \alpha = 1$ eg $\sin \alpha = [\pm] \sqrt{1 - \left(\frac{8}{17}\right)^2}$	*M1	Or Pythagoras seen (may quote 8, 15, 17 triple).
	$\sin \alpha = \frac{15}{17}$	A1	
	$\tan \alpha = \frac{15}{8}$	A1	
	$\frac{1}{\sin \alpha} + \frac{1}{\tan \alpha} = \frac{17}{15} + \frac{8}{15}$	DM1	Dealing with reciprocals and addition of fractions correctly.
	$= \frac{5}{3}$ oe	A1	Correct answer with no working shown scores 0. Extra answers from $\sin \alpha = -\frac{15}{17}$ are allowed.
		5	

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Question	Answer	Marks	Guidance
7(a)	$\frac{-3}{(a+2)^4} = -\frac{16}{27} \rightarrow \text{e.g. } 16(a+2)^4 = 81$	M1	Equate first derivative and $-\frac{16}{27}$ and move term in a (or x) into the numerator.
	$\rightarrow (a+2)^2 = \frac{9}{4} \rightarrow a+2 = [\pm]\frac{3}{2}$	M1	Solve for $(a+2)$ or $(x+2)$
	$a = -\frac{1}{2}$ or $-\frac{7}{2}$	A1 A1	Allow 'x ='
		4	
7(b)	$[f(x)] = \frac{1}{(x+2)^3} [+c]$	B1	Allow unsimplified form and 'y ='
	$5 = 1 + c$	M1	Sub $x = -1$, $y = 5$ into an integral.
	$[f(x)] = \frac{1}{(x+2)^3} + 4$	A1	Allow 'y ='
		3	

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Question	Answer	Marks	Guidance
8(a)	$APQ = \cos^{-1} \frac{\frac{5}{6}r}{r} \left[= \cos^{-1} \frac{5}{6} \right]$	*M1	May use cosine rule to find APB. Stating APQ or APB as an incorrect multiple of π is M0.
	$= 0.5857$	A1	Accept 0.586 or 33.6° or APB (1.171 or 67.1°).
	Perimeter $= 4 \times r \times \text{their } 0.5857 = 2.34r$ or $0.745\pi r$ or $(293/125)r$	DM1 A1	Must use a numerical value of <i>their</i> angle.
		4	
8(b)	Use of sector formula: Sector APB $= \frac{1}{2}r^2 \times (2 \times \text{their } 0.5857)$ or Sector APC (C is on PQ so PC = r) $= \frac{1}{2}r^2 \times (\text{their } 0.5857)$	M1	Any sector with <i>their</i> appropriate angle. It must be clear the appropriate numerical angle is being used.
	Use of appropriate formula for area of triangle and correct combination with the sector to find the area of a half segment, one segment or both segments	M1	e.g. Area APB $= \frac{1}{2}r^2 \times \sin(2 \times \text{their } 0.5857)$.
	Shaded area $[= 2 \times 0.1250r^2] = 0.250r^2$	A1	or $0.0796\pi r^2$, allow $\frac{1}{4}r^2$ or $0.25r^2$.
		3	

Question	Answer	Marks	Guidance
9(a)	$216r^3 = 64 \rightarrow r = 2/3$	B1	Allow decimal to 3sf (AWRT).
	$S_{\infty} = \frac{216}{1 - \text{their } \frac{2}{3}} = 648$ cao	M1 A1	M1 depends on <i>their</i> $ r < 1$.
		3	

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Question	Answer	Marks	Guidance
9(b)	$216\left(\frac{2}{3}\right) = 144 \rightarrow 144 = a + d$	B1 FT	SOI, may be implied in the use of $96 = 144 + 3d$ and finding a . Mis-reads not condoned in 9(b) .
	$216\left(\frac{2}{3}\right)^2 = 96 \rightarrow 96 = a + 4d$	B1 FT	SOI, may be implied in the use of $96 = 144 + 3d$ and finding a .
	Solve simultaneously	*M1	No working may be seen.
	$d = -16, a = 160$	A1	Both required.
	$S_{21} = \frac{21}{2}\{320 + 20(-16)\} = 0$	DM1 A1	Or use of $\frac{21}{2}(a + u_{21})$.
		6	

Question	Answer	Marks	Guidance
10(a)	$x^2 + (2x - 1)^2 - 2 [= 0] \rightarrow 5x^2 - 4x - 1 [= 0]$	*M1 A1	Or $5y^2 + 2y - 7 [= 0]$.
	$(5x + 1)(x - 1) [= 0]$ or $(5y + 7)(y - 1) [= 0]$	DM1	May see factors or formula or completing square.
	$x = 1, y = 1$ or $(1, 1)$ only	A1	May be implied on the diagram.
		4	

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Question	Answer	Marks	Guidance
10(b)	$(\pi) \int (2 - x^2) dx = (\pi) \left(2x - \frac{x^3}{3} \right)$	*M1 A1	Attempt integration of y^2 , allow $\int (2 - y^2) dy$.
	$(\pi) \left(2\sqrt{2} - \frac{(\sqrt{2})^3}{3} \right) - \left(2 - \frac{1}{3} \right)$	DM1	Apply limits $1 \rightarrow \sqrt{2}$.
	$\frac{\pi}{3} (4\sqrt{2} - 5)$	A1	CAO, allow $\frac{\pi}{3} (2\sqrt{8} - 5)$, must be in given form.
		4	
10(c)	Arc length = $\frac{1}{8} (2\pi\sqrt{2})$ or $\frac{\pi\sqrt{2}}{4}$ oe	B1	Must be exact.
	Perimeter = $\sqrt{2} + \text{their arc length}$	B1 FT	Must be exact, do not allow inverse trig functions.
		2	

Question	Answer	Marks	Guidance
11(a)	$(5 - 2p)^2 + (p + 2)^2 = (10 - 2p)^2 + (3 - p)^2$	M1 A1	Allow one sign error for M mark only.
	$25 - 20p + 4p^2 + p^2 + 4p + 4 = 100 - 40p + 4p^2 + 9 - 6p + p^2$ $30p = 80 \rightarrow p = \frac{8}{3}$ oe	A1	Allow 2.67 AWRT.
		3	

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Question	Answer	Marks	Guidance
11(b)(i)	$m_{AC} = \frac{p+2}{2p-5} \quad m_{BC} = \frac{p-3}{2p-10}$	M1	Allow a sign error.
	$\frac{p+2}{2p-5} \times \frac{p-3}{2p-10} = -1$	M1	Use of $m_1 m_2 = -1$ with their m_{AC} and m_{BC} .
	$p^2 - p - 6 = -(4p^2 - 30p + 50) \rightarrow 5p^2 - 31p + 44 (=0)$	A1	
	$p = 4$ (Ignore $p = \frac{11}{5}$)	A1	Factors $(p-4)(5p-11)$, or formula or completing square must be seen.
		4	
11(b)(ii)	Mid-point of $AB = (7\frac{1}{2}, \frac{1}{2})$	B1	SOI
	$r^2 = 2\frac{1}{2}^2 + 2\frac{1}{2}^2 \left[= \frac{50}{4} \right]$ or $r = \sqrt{(2\frac{1}{2}^2 + 2\frac{1}{2}^2)} \left[= \frac{5\sqrt{2}}{2} \right]$	*M1	Or $r^2 = \frac{1}{4}(5^2 + 5^2) \left[= \frac{50}{4} \right]$ etc.
	Equation of circle is $(x - \text{their } 7\frac{1}{2})^2 + (y - \text{their } \frac{1}{2})^2 = \text{their } \frac{50}{4}$	DM1	Must use r^2 not r or d or d^2
	$x^2 + y^2 - 15x - y + 44 = 0$	A1	CAO
		4	