

Cambridge International AS & A Level

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MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2

October/November 2021

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1** Find the exact value of $\int_{-1}^2 (4e^{2x} - 2e^{-x}) \, dx$.

[4]

[illegible]

- 2 (a) Sketch, on the same diagram, the graphs of $y = 3x$ and $y = |x - 3|$. [2]

- (b) Find the coordinates of the point where the two graphs intersect. [3]

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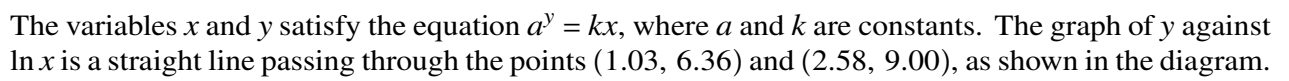
- (c) Deduce the solution of the inequality $3x < |x - 3|$. [1]

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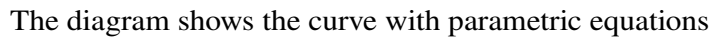
- 4** The curve with equation $y = xe^{2x} + 5e^{-x}$ has a minimum point M .

- (a) Show that the x -coordinate of M satisfies the equation $x = \frac{1}{3} \ln 5 - \frac{1}{3} \ln(1 + 2x)$. [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Use an iterative formula, based on the equation in part **(a)**, to find the x -coordinate of M correct to 3 significant figures. Use an initial value of 0.35 and give the result of each iteration to 5 significant figures. [3]

[illegible]



The curve crosses the y -axis at the point A and the x -axis at the point B .

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- This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the gradient of the curve at A .

[2]

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(c) Find the gradient of the curve at B .

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- 6 The polynomials $f(x)$ and $g(x)$ are defined by

$$f(x) = 4x^3 + ax^2 + 8x + 15 \quad \text{and} \quad g(x) = x^2 + bx + 18,$$

where a and b are constants.

- (a) Given that $(x + 3)$ is a factor of $f(x)$, find the value of a . [2]

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- (b) Given that the remainder is 40 when $g(x)$ is divided by $(x - 2)$, find the value of b . [2]

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- (c) When a and b have these values, factorise $f(x) - g(x)$ completely. [3]

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- (d) Hence solve the equation $f(\operatorname{cosec} \theta) - g(\operatorname{cosec} \theta) = 0$ for $0 < \theta < 2\pi$. [3]

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- 7 (a) By first expanding $\cos(2\theta + \theta)$, show that $\cos 3\theta \equiv 4\cos^3 \theta - 3\cos \theta$. [3]

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- (b) Find the exact value of $2\cos^3\left(\frac{5}{18}\pi\right) - \frac{3}{2}\cos\left(\frac{5}{18}\pi\right)$. [2]

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(c) Find $\int (12 \cos^3 x - 4 \cos^3 3x) dx$.

[4]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal dashed lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

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MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

October/November 2021

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

1 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 + bx - 10,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $p(x)$ and that the remainder is -55 when $p(x)$ is divided by $(x + 3)$.

(a) Find the values of a and b .

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Hence factorise $p(x)$ completely.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 2 (a) Sketch, on the same diagram, the graphs of $y = x + 3$ and $y = |2x - 1|$. [2]

- (b) Solve the equation $x + 3 = |2x - 1|$. [3]

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- (c) Find the value of y such that $5^{\frac{1}{2}y} + 3 = |2 \times 5^{\frac{1}{2}y} - 1|$. Give your answer correct to 3 significant figures. [2]

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3 The curve with equation

$$y = 5x - 2 \tan 2x$$

has exactly one stationary point in the interval $0 \leq x < \frac{1}{4}\pi$.

Find the coordinates of this stationary point, giving each coordinate correct to 3 significant figures.

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4 Given that $\int_a^{a+14} \frac{1}{3x} \, dx = \ln 2$, find the value of the positive constant a . [5]

[illegible]

5 A curve has equation $x^2 + 4x \cos 3y = 6$.

Find the exact value of the gradient of the normal to the curve at the point $(\sqrt{2}, \frac{1}{12}\pi)$. [6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a) By sketching a suitable pair of graphs on the same diagram, show that the equation

$$\ln x = 2e^{-x}$$

has exactly one root.

[2]

- (b) Verify by calculation that the root lies between 1.5 and 1.6.

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- (c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = e^{2e^{-x_n}}$$

converges, then it converges to the root of the equation in part (a).

[1]

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- (d) Use the iterative formula in part (c) to determine the root correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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- 7 (a) Prove that $4 \sin x \sin\left(x + \frac{1}{6}\pi\right) \equiv \sqrt{3} - \sqrt{3} \cos 2x + \sin 2x$. [3]

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- (b) Find the exact value of $\int_0^{\frac{5}{6}\pi} 4 \sin x \sin\left(x + \frac{1}{6}\pi\right) dx$. [4]

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- (c) Find the smallest positive value of y satisfying the equation

$$4 \sin(2y) \sin\left(2y + \frac{1}{6}\pi\right) = \sqrt{3}.$$

Give your answer in an exact form.

[3]

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[illegible]

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9709/23

October/November 2021

1 hour 15 minutes

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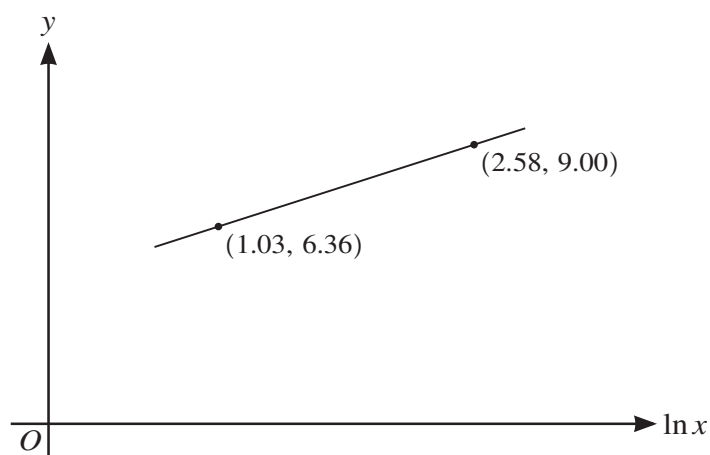
- (c) Deduce the solution of the inequality $3x < |x - 3|$. [1]

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Find the values of a and k , giving each value correct to 2 significant figures. [5]

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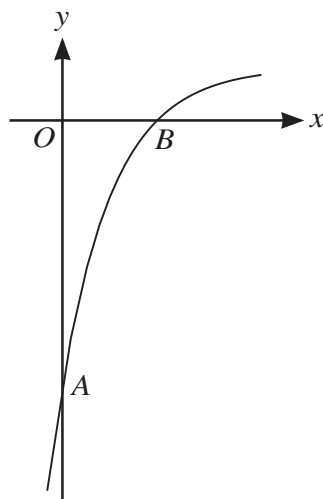
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- (b) Use an iterative formula, based on the equation in part (a), to find the x -coordinate of M correct to 3 significant figures. Use an initial value of 0.35 and give the result of each iteration to 5 significant figures. [3]

[illegible]


$$x = \ln(2t + 3), \quad y = \frac{2t - 3}{2t + 3}.$$

(a) Show that $\frac{dy}{dx} = \frac{6}{2t+3}$. [4]

This image shows a full page of a handwriting practice worksheet. It features ten sets of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

(b) Find the gradient of the curve at A .

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(c) Find the gradient of the curve at B .

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- (b) Find the exact value of $2\cos^3\left(\frac{5}{18}\pi\right) - \frac{3}{2}\cos\left(\frac{5}{18}\pi\right)$. [2]

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