

Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- 1 (a)** Expand $\left(1 - \frac{1}{2x}\right)^2$. [1]

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- (b)** Find the first four terms in the expansion, in ascending powers of x , of $(1 + 2x)^6$. [2]

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- (c)** Hence find the coefficient of x in the expansion of $\left(1 - \frac{1}{2x}\right)^2 (1 + 2x)^6$. [2]

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- 2** A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3 Solve, by factorising, the equation

$$6 \cos \theta \tan \theta - 3 \cos \theta + 4 \tan \theta - 2 = 0,$$

for $0^\circ \leq \theta \leq 180^\circ$.

[4]

[illegible]

- 4 The first term of an arithmetic progression is a and the common difference is -4 . The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a .

[4]

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The k th term of the arithmetic progression is zero.

(b) Find the value of k .

[2]

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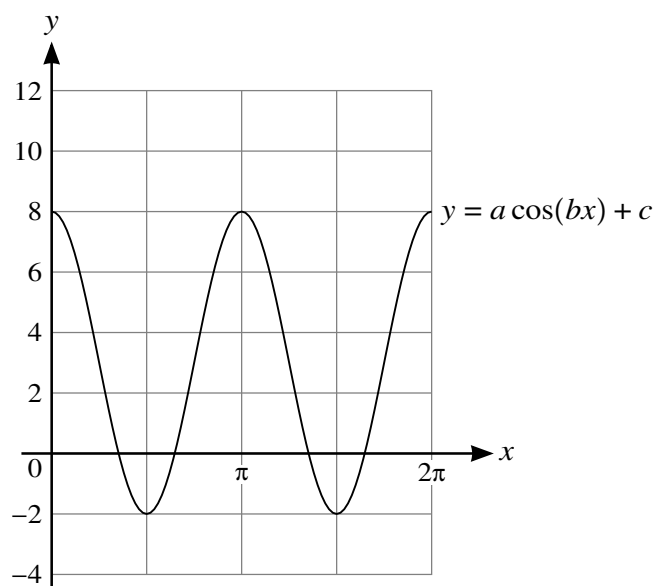
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The diagram shows part of the graph of $y = a \cos(bx) + c$.

- (a) Find the values of the positive integers a , b and c . [3]

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- (b) For these values of a , b and c , use the given diagram to determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for each of the following equations.

(i) $a \cos(bx) + c = \frac{6}{\pi}x$ [1]

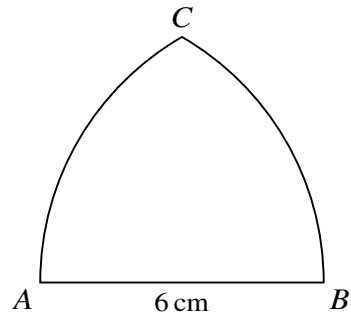
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(ii) $a \cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

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The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC . The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

- (a) Find the perimeter of the plate, giving your answer in terms of π . [3]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

[Turn over

- 7 A circle with centre $(5, 2)$ passes through the point $(7, 5)$.

(a) Find an equation of the circle. [2]

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The line $y = 5x - 10$ intersects the circle at A and B .

(b) Find the exact length of the chord AB . [7]

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- 8 (a) Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

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The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leq k$.

- (b) State the largest possible value of the constant k . [1]

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It is now given that $k = -1$.

- (c) State the range of f . [1]

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- (d) Find an expression for $f^{-1}(x)$. [3]

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The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

- (e) Express $g(x)$ in the form $px^2 + qx + r$, where p , q and r are constants. [3]

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- 9 A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \frac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Find the coordinates of the stationary points on the curve. [5]

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- (c) Find $f''(x)$. [1]

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- (d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

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10 (a) Find $\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}} dx$.

[4]

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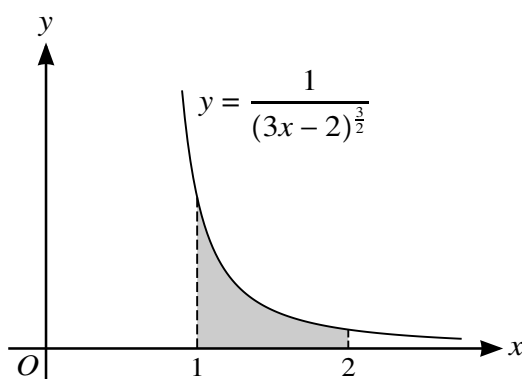
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The diagram shows the curve with equation $y = \frac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x -axis.

(b) Find the volume of revolution.

[4]

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The normal to the curve at the point $(1, 1)$ crosses the y -axis at the point A .

(c) Find the y -coordinate of A .

[4]

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[illegible]

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

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- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

- 1** Solve the equation $2 \cos \theta = 7 - \frac{3}{\cos \theta}$ for $-90^\circ < \theta < 90^\circ$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

- (a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

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The point $P(5, 6)$ lies on the transformed curve $y = f(2x) - 3$.

- (b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

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3 The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of $ff(5)$.

[2]

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(b) Find an expression for $f^{-1}(x)$.

[3]

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- 4** A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $(2, 5\frac{2}{3})$.

Find the equation of the curve.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 5 The first, third and fifth terms of an arithmetic progression are $2 \cos x$, $-6\sqrt{3} \sin x$ and $10 \cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x . [3]

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(b) Hence find the exact sum of the first 25 terms of the progression. [3]

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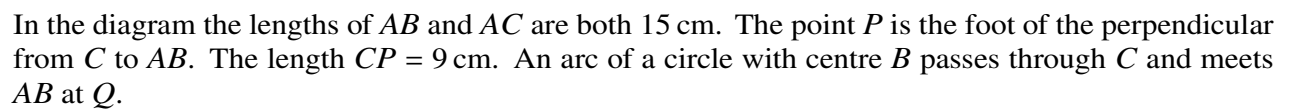
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- 6** The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form.

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



- [illegible]

[illegible]

- 8 (a)** It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15 .

Find the possible values of a .

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- (b)** It is given instead that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is k . It is also given that there is only one value of a which leads to this value of k .

Find the values of k and a .

[4]

This image shows a full page of a document template. It consists of approximately 28 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice or structured note-taking. The background is plain white, and there are no margins, headers, or footers present.

- 9** The volume $V \text{ m}^3$ of a large circular mound of iron ore of radius $r \text{ m}$ is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geq 2$. Iron ore is added to the mound at a constant rate of 1.5 m^3 per second.
- (a)** Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m .
- [3]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 10** The function f is defined by $f(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

- (a)** Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k . [3]

[illegible]

- (b) Determine the nature of the stationary point. [2]

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- (c) Given that this is the only stationary point of the curve, find the range of f . [2]

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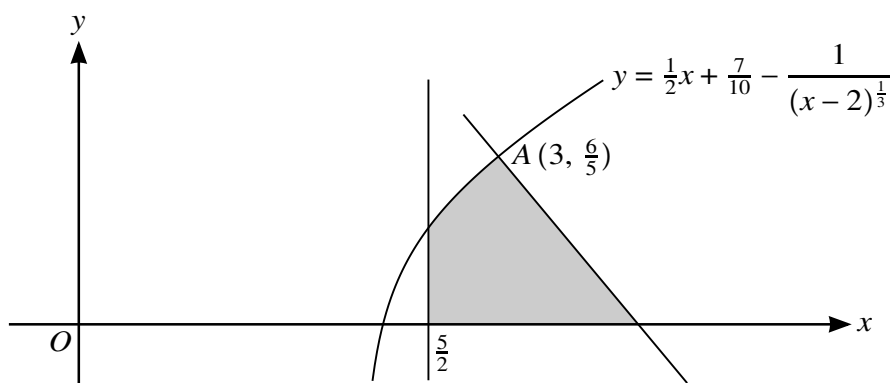
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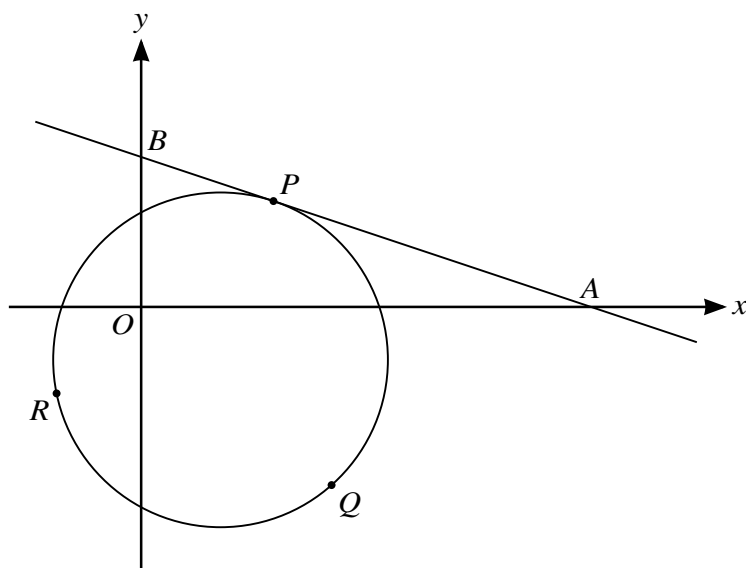
The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point A $(3, \frac{6}{5})$.

- (a) Find the x -coordinate of the point where the normal to the curve meets the x -axis. [5]

[illegible]

- (b)** Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

[illegible]



(a) The tangent to the circle at P meets the x -axis at A and the y -axis at B .

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[3]

[illegible]

[illegible]

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2021

1 hour 50 minutes

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You will need: List of formulae (MF19)

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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 (a) Find the first three terms, in ascending powers of x , in the expansion of $(1 + ax)^6$. [1]

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- (b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3 , find the possible values of the constant a . [4]

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- 3 (a)** Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

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- (b)** The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

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- 4 The first term of an arithmetic progression is 84 and the common difference is -3 .

(a) Find the smallest value of n for which the n th term is negative. [2]

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It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k . [3]

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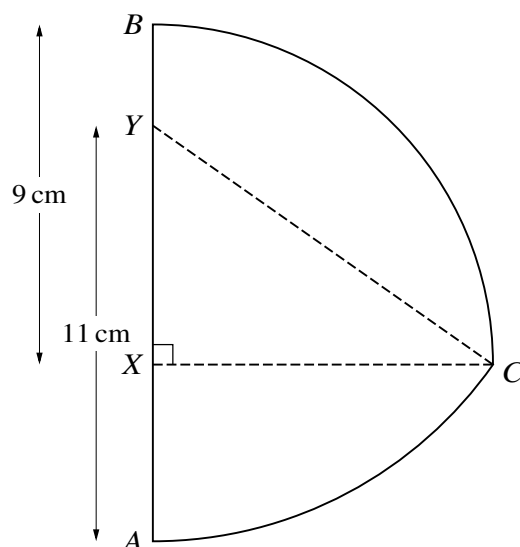
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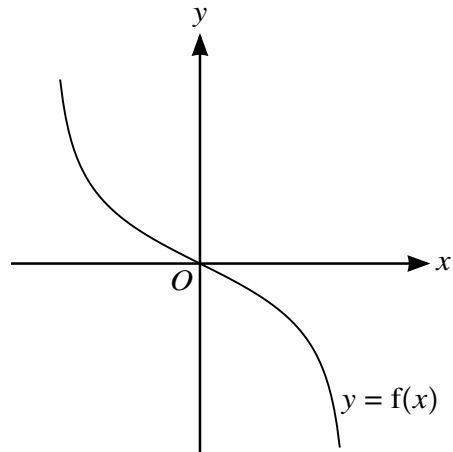
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(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

[illegible]

[Turn over



The diagram shows the graph of $y = f(x)$.

- (a) On this diagram sketch the graph of $y = f^{-1}(x)$. [1]

It is now given that $f(x) = -\frac{x}{\sqrt{4-x^2}}$ where $-2 < x < 2$.

- (b)** Find an expression for $f^{-1}(x)$. [4]

[illegible]

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The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

- (c) State the maximum possible value of a for which fg can be formed. [1]

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- (d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

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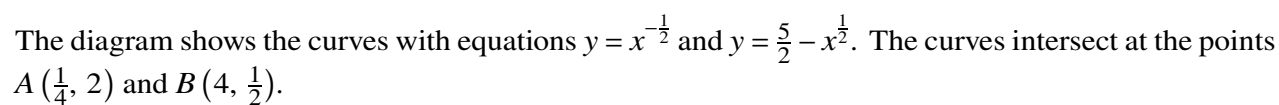
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[Turn over



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- This image shows a full page of a notebook or worksheet template. It consists of approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for handwriting practice. The background is plain white, and there are no margins, text, or other markings present.

- (b)** The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1, 1)$ intersects the y-axis at the point $(0, p)$.

Find the value of p .

[4]

[illegible]

- 9** The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B .

- (a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB .

[7]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

A straight line through the point $(10, 0)$ with gradient m is a tangent to the circle.

(b) Find the two possible values of m .

[5]

[illegible]

- 10** A curve has equation $y = f(x)$ and it is given that

$$f'(x) = \left(\frac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

- (a)** Find $f''(x)$ in terms of k and x , and hence find the set of possible values of k . [3]

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It is now given that $k = -3$ and the minimum point is at $(2, 3\frac{1}{2})$.

- (b)** Find $f(x)$. [4]

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- (c) Find the coordinates of the other stationary point and determine its nature. [4]

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