



Cambridge International AS & A Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2021

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$1 - \frac{1}{x} + \frac{1}{4x^2}$	B1	OE. Multiply or use binomial expansion. Allow unsimplified.
		1	
1(b)	$1 + 12x + 60x^2 + 160x^3$	B2, 1, 0	Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion.
		2	
1(c)	$their(1 \times 12) + their(-1 \times 60) + their(\frac{1}{4} \times 160)$	M1	Attempts at least 2 products where each product contains one term from each expansion.
	$[12 - 60 + 40 =] -8$	A1	Allow $-8x$.
		2	

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2	$kx^2 + 2x - k = kx - 2$ leading to $kx^2 + (-k + 2)x - k + 2 [= 0]$	*M1	Eliminate y and form 3-term quadratic. Allow 1 error.
	$(-k + 2)^2 - 4k(-k + 2)$	DM1	Apply $b^2 - 4ac$; allow 1 error but a , b and c must be correct for <i>their</i> quadratic.
	$5k^2 - 12k + 4$ or $(-k + 2)(-k + 2 - 4k)$	A1	May be shown in quadratic formula.
	$(-k + 2)(-5k + 2)$	DM1	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give <i>their</i> coefficient of k^2 .
	$\frac{2}{5} < k < 2$	A1	WWW, accept two separate correct inequalities. If M0 for solving quadratic, SC B1 can be awarded for correct final answer.
		5	

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Question	Answer	Marks	Guidance
3	$3 \cos \theta (2 \tan \theta - 1) + 2(2 \tan \theta - 1) [= 0]$	M1	Or similar partial factorisation; condone sign errors.
	$(2 \tan \theta - 1)(3 \cos \theta + 2) [= 0]$ [leading to $\tan \theta = \frac{1}{2}$, $\cos \theta = -\frac{2}{3}$]	M1	OE. At least 2 out of 4 products correct.
	26.6°, 131.8°	A1 A1	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. SC B1 No factorisation: Division by $2 \tan \theta - 1$ leading to 131.8° or division by $3 \cos \theta + 2$ or similar leading to 26.6°.
	Alternative method for question 3		
	$6 \cos \theta \left(\frac{\sin \theta}{\cos \theta} \right) - 3 \cos \theta + 4 \left(\frac{\sin \theta}{\cos \theta} \right) - 2 [= 0]$ $6 \cos \theta \sin \theta - 3 \cos^2 \theta + 4 \sin \theta - 2 \cos \theta [= 0]$ $2 \sin \theta (3 \cos \theta + 2) - \cos \theta (3 \cos \theta + 2) [= 0]$	M1	Using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and reaching a partial factorisation; condone sign errors.
	$(2 \sin \theta - \cos \theta)(3 \cos \theta + 2) [= 0]$ [leading to $\tan \theta = \frac{1}{2}$, $\cos \theta = -\frac{2}{3}$]	M1	At least 2 out of 4 products correct.
	26.6°, 131.8°	A1 A1	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. SC B1 No factorisation: Division by $2 \tan \theta - 1$ leading to 131.8° or division by $3 \cos \theta + 2$ or similar leading to 26.6°.
		4	

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Question	Answer	Marks	Guidance
4(a)	$\frac{5a}{1 - (\pm \frac{1}{4})}$	B1	Use of correct formula for sum to infinity.
	$\frac{8}{2}[2a + 7(-4)]$	*M1	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a = 8a - 112$ leading to $a = [28]$	DM1	Solve equation to reach a value of a .
	$a = 28$	A1	Correct value.
		4	
4(b)	$their\ 28 + (k - 1)(-4) = 0$	M1	Use of correct method with <i>their a</i> .
	$[k =]8$	A1	
		2	

Question	Answer	Marks	Guidance
5(a)	$a = 5$	B1	
	$b = 2$	B1	
	$c = 3$	B1	
		3	

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Question	Answer	Marks	Guidance
5(b)(i)	3	B1	
		1	
5(b)(ii)	2	B1	
		1	

Question	Answer	Marks	Guidance
6(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	B1	SOI; allow 60° .
	One arc $6 \times \text{their } \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times \text{their } \frac{\pi}{3}$	M1	SOI e.g. may see 2π or 4π . Use of correct formula for length of arc and multiply by 2.
	Perimeter = $6 + 4\pi$	A1	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle = 12π	B1	
	One arc $\frac{1}{6} \times 12\pi$ leading to two arcs $2 \times \frac{1}{6} \times 12\pi$	M1	SOI e.g. may see 2π or 4π .
	Perimeter = $6 + 4\pi$	A1	Must be exact value.
		3	

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Question	Answer	Marks	Guidance
6(b)	$\text{Sector} = \frac{1}{2} \times 6^2 \times \text{their} \left(\frac{\pi}{3} \right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π .
	$\frac{1}{2} \times (6^2) \times \text{their} \left(\frac{\pi}{3} \right) - \frac{1}{2} \times (6^2) \times \sin \left(\text{their} \left(\frac{\pi}{3} \right) \right) + 6\pi \left[= 6\pi - 9\sqrt{3} + 6\pi \right]$	M1 A1	M1 for attempt at strategy with values substituted: area of segment + area of sector A1 if correct (unsimplified).
	$\text{Area} = 12\pi - 9\sqrt{3}$	A1	Must be simplified exact value.
	Alternative method for question 6(b)		
	$\text{Sector} = \frac{1}{2} \times 6^2 \times \text{their} \left(\frac{\pi}{3} \right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π .
	$2 \times \left(\frac{1}{2} \times 6^2 \times \text{their} \left(\frac{\pi}{3} \right) \right) - \frac{1}{2} \times (6^2) \times \sin \left(\text{their} \left(\frac{\pi}{3} \right) \right)$	M1 A1	M1 for attempt at strategy with values substituted: 2 × sector – triangle A1 if correct (unsimplified).
	$\text{Area} = 12\pi - 9\sqrt{3}$	A1	Must be simplified exact value.
	Alternative method for question 6(b)		
	$\text{Sector} = \frac{1}{2} \times 6^2 \times \text{their} \left(\frac{\pi}{3} \right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π .
	$2 \times \left(\frac{1}{2} \times (6^2) \times \text{their} \left(\frac{\pi}{3} \right) - \frac{1}{2} \times (6^2) \times \sin \left(\text{their} \left(\frac{\pi}{3} \right) \right) \right) + \frac{1}{2} \times (6^2) \times \sin \left(\text{their} \left(\frac{\pi}{3} \right) \right) \left[= 12\pi - 18\sqrt{3} + 9\sqrt{3} \right]$	M1 A1	M1 for attempt at strategy with values substituted: 2 × segment + triangle A1 if correct (unsimplified).
	$\text{Area} \left[= 6\pi - 9\sqrt{3} + 6\pi \right] = 12\pi - 9\sqrt{3}$	A1	Must be simplified exact value.
		4	

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Question	Answer	Marks	Guidance
7(a)	$r^2 \left[= (5-2)^2 + (7-5)^2 \right] = 13$	B1	$r^2 = 13$ or $r = \sqrt{13}$
	Equation of circle is $(x-5)^2 + (y-2)^2 = 13$	B1 FT	OE. FT on <i>their</i> 13 but LHS must be correct.
		2	
7(b)	$(x-5)^2 + (5x-10-2)^2 = 13$	M1	Substitute $y = 5x - 10$ into <i>their</i> equation.
	$26x^2 - 130x + 156 \quad [= 0]$	A1 FT	OE 3-term quadratic with all terms on one side. FT on <i>their</i> circle equation.
	$[26](x-2)(x-3) \quad [= 0]$	M1	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give <i>their</i> coefficient of x^2 .
	(2, 0), (3, 5)	A1 A1	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, SC B2 can be awarded for correct coordinates, SC B1 if two x or y values only.
	$(AB)^2 = (3-2)^2 + (5-0)^2$	M1	SOI. Using <i>their</i> points to find length of AB .
	$AB = \sqrt{26}$	A1	ISW. Dependent on final M1 only.

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Question	Answer	Marks	Guidance
7(b)	Alternative method for question 7(b)		
	$\left(\frac{y+10}{5}-5\right)^2 + (y-2)^2 = 13$	M1	Substitute $x = \frac{y+10}{5}$ into <i>their</i> equation.
	$\frac{26y^2}{25} - \frac{26y}{5} [= 0]$	A1 FT	OE 2-term quadratic with all terms on one side. FT on <i>their</i> circle equation.
	$[26]y(y-5) [= 0]$	M1	Solve 2-term quadratic in y by factorising, using formula or completing the square. Factors must expand to give <i>their</i> coefficient of y^2 .
	$(2, 0), (3, 5)$	A1 A1	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, SC B2 can be awarded for correct coordinates, SC B1 if two x or y values only.
	$(AB)^2 = (3-2)^2 + (5-0)^2$	M1	SOI. Using <i>their</i> points to find length of AB .
	$AB = \sqrt{26}$	A1	ISW. Dependent on final M1 only.
		7	

Question	Answer	Marks	Guidance
8(a)	$\{-3(x-2)^2\} \quad \{+14\}$	B1 B1	B1 for each correct term; condone $a = 2, b = 14$.
		2	
8(b)	$[k =] 2$	B1	Allow $[x] \leq 2$.
		1	

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Question	Answer	Marks	Guidance
8(c)	[Range is] $[y] \leq -13$	B1	Allow $[f(x)] \leq -13$, $[f] \leq -13$ but NOT $x \leq -13$.
		1	
8(d)	$y = -3(x-2)^2 + 14$ leading to $(x-2)^2 = \frac{14-y}{3}$	M1	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$x = 2(\pm)\sqrt{\frac{14-y}{3}}$	A1	Allow $\frac{y-14}{-3}$.
	$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only</u> .
	Alternative method for question 8(d)		
	$x = -3(y-2)^2 + 14$ leading to $(y-2)^2 = \frac{14-x}{3}$	M1	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$= 2(\pm)\sqrt{\frac{14-x}{3}}$	A1	Allow $\frac{x-14}{-3}$.
	$[f^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only</u> .
		3	

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Question	Answer	Marks	Guidance
8(e)	$[g(x) =] \{-3(x+3-2)^2\} + \{14+1\}$	B2, 1, 0	OR $\{-3(x+3)^2\} + \{12(x+3)\} + \{3\}$
	$g(x) = -3x^2 - 6x + 12$	B1	
		3	

Question	Answer	Marks	Guidance
9(a)	$f(x) = \frac{2}{3}x^3 - 7x + 4x^{-1} [+c]$	B2, 1, 0	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3} = \frac{2}{3} - 7 + 4 + c$ leading to $c = [2]$	M1	Substitute $f(1) = -\frac{1}{3}$ into an integrated expression and evaluate c .
	$f(x) = \frac{2}{3}x^3 - 7x + 4x^{-1} + 2$	A1	OE.
		4	

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Question	Answer	Marks	Guidance
9(b)	$2x^4 - 7x^2 - 4 [= 0]$	M1	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2 - 7y - 4 [= 0]$.
	$(2x^2 + 1)(x^2 - 4) [= 0]$	M1	Attempt factors or use formula or complete the square. Allow $\pm$ sign errors. Factors must expand to give <i>their</i> coefficient of x^2 or e.g. y . Must be quartic equation. Accept use of substitution e.g. $(2y + 1)(y - 4)$.
	$x = [\pm]2$	A1	If M0 for solving quadratic, SC B1 can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3 - 7(2) + \frac{4}{2} + 2 \quad \text{leading to} \right] \left(2, -\frac{14}{3} \right)$ $\left[\frac{2}{3}(-2)^3 - 7(-2) + \frac{4}{-2} + 2 \quad \text{leading to} \right] \left(-2, \frac{26}{3} \right)$	B1 B1	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
9(c)	$f''(x) = 4x + 8x^{-3}$	B1	OE
		1	

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Question	Answer	Marks	Guidance
9(d)	$f''(2) = 9 > 0$ MINIMUM at $x = \text{their } 2$	B1 FT	FT on <i>their</i> $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = 2$.
	$f''(-2) = -9 < 0$ MAXIMUM at $x = \text{their } -2$	B1 FT	FT on <i>their</i> $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = -2$. Special case: If values not shown and B0B0 scored, SC B1 for $f''(2) > 0$ MIN and $f''(-2) < 0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x -values either side of 2 and -2	M1	FT on <i>their</i> $x = [\pm]2$
	MINIMUM at $x = \text{their } 2$, MAXIMUM at $x = \text{their } 2$	A1 FT	FT on <i>their</i> $x = [\pm]2$. Must have correct values of $f'(x)$ if shown. Special case: If values not shown and M0A0 scored SC B1 $f'(2) -/0/+$ MIN and $f'(-2) +/0/-$ MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	B1 B1	Need correct coordinates in (b) for this method.
		2	

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Question	Answer	Marks	Guidance
10(a)	$\left\{ \frac{(3x-2)^{-\frac{1}{2}}}{-1/2} \right\} \div \{3\}$	B2, 1, 0	Attempt to integrate
	$-\frac{2}{3}[0-1]$	M1	M1 for applying limits $1 \rightarrow \infty$ to an integrated expression (either correct power or dividing by their power).
	$\frac{2}{3}$	A1	
		4	
10(b)	$[\pi] \int y^2 dx = [\pi] \int (3x-2)^{-3} dx = [\pi] \frac{(3x-2)^{-2}}{-2 \times 3}$	*M1 A1	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi] \left[-\frac{1}{6} \right] \left[\frac{1}{16} - 1 \right]$	DM1	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\frac{5\pi}{32}$	A1	OE
		4	

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Question	Answer	Marks	Guidance
10(c)	$\frac{dy}{dx} = -\frac{3}{2} \times 3(3x-2)^{-\frac{5}{2}}$	M1	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\frac{dy}{dx} = -\frac{9}{2}$	*M1	Substitute $x = 1$ into <i>their</i> differentiated expression; allow 1 error.
	[Equation of normal is] $y - 1 = \frac{2}{9}(x - 1)$ OR evaluates c	DM1	Forms equation of line or evaluates c using (1, 1) and gradient $\frac{-1}{\text{their } \frac{dy}{dx}}$.
	At A , $y = \frac{7}{9}$	A1	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	



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 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
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 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$2\cos^2\theta - 7\cos\theta + 3 [= 0]$	M1	Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow $\pm$ sign errors.
	$(2\cos\theta - 1)(\cos\theta - 3) = 0$	DM1	Solving <i>their</i> 3-term quadratic using factorisation, formula or completing the square.
	$[\cos\theta = \frac{1}{2} \text{ or } \cos\theta = 3 \text{ leading to}] \theta = -60^\circ \text{ or } \theta = 60^\circ$	A1	
	$\theta = -60^\circ \text{ and } \theta = 60^\circ$	A1 FT	FT for $\pm$ same answer between 0° and 90° or 0 and $\frac{\pi}{2}$. $\pm \frac{\pi}{3}$ or ± 1.05 AWRT scores maximum M1M1A0A1FT. Special case: If M1 DM0 scored then SC B1 for $\theta = -60^\circ$ or $\theta = 60^\circ$, and SC B1 FT can be awarded for $\pm$ (<i>their</i> 60°).
		4	

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Question	Answer	Marks	Guidance
2(a)	Stretch with [scale factor] either ± 2 or $\pm \frac{1}{2}$	B1	
	Scale factor $\frac{1}{2}$ in the x -direction	B1	
	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ or translation of 3 units in negative y -direction	B1	
		3	
2(b)	(10, 9)	B1 B1	B1 for each correct co-ordinate.
		2	

Question	Answer	Marks	Guidance
3(a)	$f(5) = [2]$ and $f(\text{their } 2) = [5]$ OR $ff(5) = \left[\frac{2+3}{2-1} \right]$	M1	Clear evidence of applying f twice with $x = 5$.
	OR $\frac{x+3}{x-1} + 3$ $\frac{x+3}{x-1} - 1$ and an attempt to substitute $x = 5$.		
	5	A1	
		2	

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Question	Answer	Marks	Guidance
3(b)	$\frac{x+3}{x-1} = y \Rightarrow x+3 = xy - y$ OR $\frac{y+3}{y-1} = x \Rightarrow y+3 = xy - x$	*M1	Setting $f(x) = y$ or swapping x and y , clearing of fractions and expanding brackets. Allow $\pm$ sign errors.
	$xy - x = y + 3 \Rightarrow x = \frac{y+3}{y-1}$ OE OR $y + 3 = xy - x \Rightarrow y = \left[\frac{x+3}{x-1} \right]$ OE	DM1	Finding x or $y =$. Allow $\pm$ sign errors.
	$[f^{-1}(x) \text{ or } y] = \frac{x+3}{x-1}$	A1	OE e.g. $1 + \frac{4}{x-1}$ etc. Must be a function of x , cannot be $x =$.
		3	

Question	Answer	Marks	Guidance
4	$y = -\frac{\frac{8}{3}}{(3x+2)}[+c]$	*B1	For $(3x+2)^{-1}$
		DB1	For $-\frac{8}{3}$
	$5\frac{2}{3} = -\frac{\frac{8}{3}}{(3 \times 2 + 2)} + c$	M1	Substituting $\left(2, 5\frac{2}{3}\right)$ into <i>their</i> integrated expression – defined by power = -1, or dividing by their power. + c needed
	$y = -\frac{8}{3(3x+2)} + 6$	A1	OE e.g. $y = -\frac{8}{3}(3x+2)^{-1} + 6$
		4	

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Question	Answer	Marks	Guidance
5(a)	$[(3^{\text{rd}} \text{ term} - 1^{\text{st}} \text{ term}) = (5^{\text{th}} \text{ term} - 3^{\text{rd}} \text{ term}) \text{ leading to...}]$ $-6\sqrt{3} \sin x - 2 \cos x = 10 \cos x + 6\sqrt{3} \sin x$ $[\text{leading to } -12\sqrt{3} \sin x = 12 \cos x]$ OR $[(1^{\text{st}} \text{ term} + 5^{\text{th}} \text{ term}) = 2 \times 3^{\text{rd}} \text{ term leading to...}] \quad 12 \cos x = -12\sqrt{3} \sin x$	*M1	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving $\sin x$ and $\cos x$.
	Elimination of $\sin x$ and $\cos x$ to give an expression in $\tan x$ $\left[\tan x = -\frac{1}{\sqrt{3}} \right]$	DM1	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x =] \frac{5\pi}{6} \text{ only}$	A1	CAO. Must be exact.
		3	
5(b)	$d = 2 \cos x$ or $d = 2 \cos(\text{their } x)$	B1 FT	Or an equivalent expression involving $\sin x$ and $\cos x$ e.g. $-3\sqrt{3} \sin(\text{their } x) - \cos(\text{their } x) \quad [= -\sqrt{3}]$ FT for <i>their x</i> from (a) only. If not $\pm \sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2} (2 \times (2 \cos(\text{their } x)) + (25 - 1) \times (\text{their } d))$ $\left[= 12.5 (2 \times (-\sqrt{3}) + 24(-\sqrt{3})) \right]$	M1	Using the correct sum formula with $\frac{25}{2}$, $(25 - 1)$ and with a replaced by either $2(\cos(\text{their } x))$ or $\pm \sqrt{3}$ and d replaced by either $2(\cos(\text{their } x))$ or $\pm \sqrt{3}$.
	$-325\sqrt{3}$	A1	Must be exact.
		3	

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Question	Answer	Marks	Guidance
6	$ar = 54$ and $\frac{a \text{ or } their a}{1-r} = 243$	B1	SOI
	$\frac{54}{r} = 243(1-r)$ leading to $243r^2 - 243r + 54 [= 0]$ [$9r^2 - 9r + 2 = 0$] OR $a^2 - 243a + 13122 [= 0]$	*M1	Forming a 3-term quadratic expression in r or a using <i>their</i> 2nd term and S_{∞} . Allow $\pm$ sign errors.
	$k(3r-2)(3r-1) [= 0]$ OR $(a-81)(a-162) [= 0]$	DM1	Solving <i>their</i> 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give $\pm$ <i>their</i> coefficient of r^2 .
	$54 \div \left(their \frac{2}{3} \right) = a$ OR $54 \div (their 81) = r$	DM1	May be implied by final answer.
	Tenth term $= \frac{512}{243} \left[\text{OR } 81 \times \left(\frac{2}{3} \right)^9 \text{ OR } 54 \times \left(\frac{2}{3} \right)^8 \right]$	A1	OE. Must be exact. Special case: If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
		5	

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Question	Answer	Marks	Guidance
7(a)	EITHER By using trigonometry: $\hat{BAC} = 0.6435\dots$ and $\hat{ABC} = \frac{\pi - 0.6435}{2}$ OR By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan \hat{ABC} = \frac{9}{3}$ OR Using $\triangle PBC$ and either the sine or cosine rule $\sin \hat{ABC} = \frac{3}{\sqrt{10}}$ or $\cos \hat{ABC} = \frac{\sqrt{10}}{10}$	M1	$\frac{3}{\sqrt{10}} = 0.9486\dots$ $\frac{\sqrt{10}}{10} = 0.3162\dots$
	$\hat{ABC} = \frac{\pi - 0.6435}{2}$ or $\tan^{-1} \frac{9}{3}$ or $\sin^{-1} \frac{3}{\sqrt{10}}$ or $\cos^{-1} \frac{\sqrt{10}}{10}$ or $1.249(04\dots)$ or $71.56^\circ = 1.25$ radians (3 sf)	A1	AG. Final answer must be 1.25, more accurate value 1.24904... with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
7(b)	$BC = \sqrt{(their\ 3)^2 + 9^2}$ or $\frac{9}{\sin 1.25}$ [= $\sqrt{90}$, $3\sqrt{10}$ or 9.48697...]	M1	Using correct method(s) to find BC .
	Area of sector = $\frac{1}{2} \times (their\ BC)^2 \times \tan^{-1} 3$ [= 56.207 or 56.25]	M1	Using $\tan^{-1} 3$ or 1.25 and <i>their</i> BC , but not 9 or 15, in correct area of sector formula.
	Area of triangle $PBC = 13.4$ to 13.6 or $\frac{1}{2} \times 9 \times 3$	B1	
	[Area = (56.207 or 56.25) – <i>their</i> 13.5 =] 42.7 or 42.8	A1	AWRT
		4	

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Question	Answer	Marks	Guidance
8(a)	Terms required for x^2 : $-5 \times 2^4 \times ax + 10 \times 2^3 \times a^2 x^2 \left[= -80ax + 80a^2 x^2 \right]$	B1	Can be seen as part of an expansion or in correct products.
	$2 \times (\pm \text{their coefficient of } x) + 4 \times (\pm \text{their coefficient of } x^2)$	*M1	
	x^2 coefficient is $320a^2 - 160a = -15$ $\Rightarrow 64a^2 - 32a + 3 \Rightarrow (8a - 3)(8a - 1)$	DM1	Forming a 3-term quadratic in a , with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give <i>their</i> coefficient of a^2 .
	$a = \frac{1}{8}$ or $a = \frac{3}{8}$	A1	OE. Special case: If DM0 for solving quadratic, SC B1 can be awarded for correct final answers.
		4	

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Question	Answer	Marks	Guidance
8(b)	$320a^2 - 160a = k \Rightarrow 320a^2 - 160a - k = 0$	M1	Forming a 3-term quadratic in a with all terms on the same side. Allow $\pm$ sign errors.
	<i>Their</i> $b^2 - 4ac = 0$, $[160^2 - 4 \times 320 \times (-k) = 0]$	M1	Any use of discriminant on a 3-term quadratic.
	$k = -20$	A1	
	$a = \frac{1}{4}$	B1	Condone $a = \frac{1}{4}$ from $k = 20$.
	Alternative method for question 8(b)		
	$320a^2 - 160a = k$ and divide by 320 $\left[a^2 - \frac{a}{2} = \frac{k}{320} \right]$	M1	Allow $\pm$ sign errors.
	Attempt to complete the square $\left[\left(a - \frac{1}{4} \right)^2 - \frac{1}{16} = \frac{k}{320} \right]$	M1	Must have $\left(a - \frac{1}{4} \right)^2$
	$a = \frac{1}{4}$	A1	
	$k = -20$	B1	

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Question	Answer	Marks	Guidance
8(b) cont'd	Alternative method for question 8(b)		
	$320a^2 - 160a = k$ and attempt to differentiate LHS $[640a - 160]$	M1	Allow $\pm$ sign errors.
	Setting <i>their</i> $(640a - 160) = 0$ and attempt to solve.	M1	
	$a = \frac{1}{4}$	A1	
	$k = -20$	B1	
		4	

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Question	Answer	Marks	Guidance
9(a)	$\left[\frac{dV}{dr} = \right] \frac{9}{2} \left(r - \frac{1}{2} \right)^2$	B1	OE. Accept unsimplified.
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{1.5}{\text{their } \frac{dV}{dr}} \left[= \frac{1.5}{\frac{9}{2} \left(5.5 - \frac{1}{2} \right)^2} = \frac{1.5}{112.5} \right]$	M1	Correct use of chain rule with 1.5, <i>their</i> differentiated expression for $\frac{dV}{dr}$ and using $r = 5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
9(b)	$\frac{dV}{dr}$ or <i>their</i> $\frac{dV}{dr} = \frac{1.5}{0.1}$ or 15 OR $0.1 = \frac{1.5}{\text{their } \frac{dV}{dr}} \left[= \frac{2 \times 1.5}{9 \left(r - \frac{1}{2} \right)^2} \text{OE} \right]$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or <i>their</i> $\frac{dV}{dr}$, 1.5 and 0.1.
	$\left[\frac{9}{2} \left(r - \frac{1}{2} \right)^2 = 15 \Rightarrow \right] r = \frac{1}{2} + \sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	B1	OE e.g. $\frac{-3 + 5\sqrt{30}}{3}$. CAO.
		3	

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Question	Answer	Marks	Guidance
10(a)	$[f'(x) =] 2x - \frac{k}{x^2}$	B1	
	$f'(2) = 0 \left[2 \times 2 - \frac{k}{2^2} = 0 \right] \Rightarrow k = \dots$	M1	Setting <i>their</i> 2-term $f'(2) = 0$, at least one term correct and attempting to solve as far as $k =$.
	$k = 16$	A1	
		3	
10(b)	$f''(2) = \text{e.g. } 2 + \frac{2k}{2^3}$	M1	Evaluate a two term $f''(2)$ with at least one term correct. Or other valid method.
	$\left[2 + \frac{2k}{2^3} \right] > 0 \Rightarrow \text{minimum or } = 6 \Rightarrow \text{minimum}$	A1 FT	WWW. FT on positive k value.
		2	
10(c)	When $x = 2$, $f(x) = 14$	B1	SOI
	$[\text{Range is or } y \text{ or } f(x)] \geq \text{their } f(2)$	B1 FT	Not $x \geq \text{their } f(2)$
		2	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{3(x-2)^{\frac{4}{3}}}$	B1	OE. Allow unsimplified.
	Attempt at evaluating <i>their</i> $\frac{dy}{dx}$ at $x = 3$ $\left[\frac{1}{2} + \frac{1}{3(3-2)^{\frac{4}{3}}} = \frac{5}{6} \right]$	*M1	Substituting $x = 3$ into <i>their</i> differentiated expression – defined by one of 3 original terms with correct power of x .
	Gradient of normal = $\frac{-1}{\text{their } \frac{dy}{dx}}$ $\left[= -\frac{6}{5} \right]$	*DM1	Negative reciprocal of <i>their</i> evaluated $\frac{dy}{dx}$.
	Equation of normal $y - \frac{6}{5} = (\text{their normal gradient})(x - 3)$ $\left[y = -\frac{6}{5}x + 4.8 \Rightarrow 5y = -6x + 24 \right]$	DM1	Using <i>their</i> normal gradient and A in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0,$] $x = 4$	A1	or (4, 0)
		5	

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Question	Answer	Marks	Guidance
11(b)	Area under curve = $\int \left(\frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}} \right) [dx]$	M1	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\frac{1}{4}x^2 + \frac{7}{10}x - \frac{3(x-2)^{\frac{2}{3}}}{2}$	A1	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\frac{9}{4} + 2.1 - \frac{3}{2} \right) - \left(\frac{6.25}{4} + 1.75 - \frac{3 \times 0.5^{\frac{2}{3}}}{2} \right)$	M1	Clear substitution of 3 and 2.5 into <i>their</i> integrated expression (with at least one correct term) and subtracting.
	0.48[24]	A1	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	B1	OE
	[Total area =] 1.08	A1	Dependent on the first M1 and WWW.
		6	

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Question	Answer	Marks	Guidance
12(a)	Centre is (3, – 2)	B1	
	Gradient of radius = $\frac{(their - 2) - 4}{(their 3) - 5} [= 3]$	*M1	Finding gradient using <i>their</i> centre (not (0, 0)) and <i>P</i> (5,4).
	Equation of tangent $y - 4 = -\frac{1}{3}(x - 5)$	DM1	Using <i>P</i> and the negative reciprocal of <i>their</i> gradient to find the equation of <i>AB</i> .
	Sight of $[x =]17$ and $[y =] \frac{17}{3}$	A1	
	$\left[\text{Area} = \frac{1}{2} \times \frac{17}{3} \times 17 = \right] \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
	Alternative method for question 12(a)		
	$2x + 2y \frac{dy}{dx} - 6 + 4 \frac{dy}{dx} = 0$	B1	
	At <i>P</i> : $10 + 8 \frac{dy}{dx} - 6 + 4 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{3}$	*M1	Find the gradient using <i>P</i> (5,4) in <i>their</i> implicit differential (with at least one correctly differentiated <i>y</i> term).
	Equation of tangent $y - 4 = -\frac{1}{3}(x - 5)$	DM1	Using <i>P</i> and <i>their</i> value for the gradient to find the equation of <i>AB</i> .
	Sight of $[x =]17$ and $[y =] \frac{17}{3}$	A1	
	$\left[\text{Area} = \frac{1}{2} \times \frac{17}{3} \times 17 = \right] \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.

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Question	Answer	Marks	Guidance
12(a) cont'd	Alternative method for question 12(a)		
	$\left[y = -2 \pm \left(40 - (x-3)^2 \right)^{\frac{1}{2}} \text{ OE leading to } \frac{dy}{dx} = (3-x)(31+6x-x^2)^{-\frac{1}{2}} \right]$	B1	OE. Correct differentiation of rearranged equation.
	$\frac{dy}{dx} = (3-5)(31+6(5)-(5)^2)^{-\frac{1}{2}} \left[\Rightarrow \frac{dy}{dx} = -\frac{1}{3} \right]$	*M1	Find the gradient using $x = 5$ in <i>their</i> differential (with clear use of chain rule).
	Equation of tangent $y - 4 = -\frac{1}{3}(x - 5)$	DM1	Using P and <i>their</i> value for the gradient to find the equation of AB .
	Sight of $[x =]17$ and $[y =] \frac{17}{3}$	A1	
	$\left[\text{Area} = \frac{1}{2} \times \frac{17}{3} \times 17 = \right] \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
		5	

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Question	Answer	Marks	Guidance
12(b)	Radius of circle = $\sqrt{40}$,	B1	Or $2\sqrt{10}$ or 6.32 AWR or $r^2 = 40$.
	Area of $\triangle CRQ = \frac{1}{2} \times (\text{their } r)^2 \sin 120 \left[= \frac{1}{2} \times 40 \times \frac{\sqrt{3}}{2} \right]$ OR Area of $\triangle CQX = \frac{1}{2} \times \sqrt{40} \cos 30 \times \sqrt{40} \cos 60$ OE $\left[= \frac{1}{2} \times \sqrt{30} \times \sqrt{10} \right]$ OR Area of circle – $3 \times$ Area of segment = $40\pi - 3 \times (40 \frac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120}$ or $2\sqrt{30}$ and area = $\frac{1}{2} QR^2 \sin 60$	M1	Using $\frac{1}{2} r^2 \sin \theta$ with <i>their</i> r and 120 or 60 [$\times 3$] Using $\frac{1}{2} \times \text{base} \times \text{height}$ in a correct right-angled triangle [$\times 6$]. Use of cosine rule and area of large triangle
	$30\sqrt{3}$	A1	AWRT 52[.0] implies B1M1A0.
		3	See diagram for points stated in ‘Answer’ column.



Cambridge International AS & A Level

MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2021

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
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Question	Answer	Marks	Guidance
1	{Reflection} {[in the] x -axis} or {Stretch of scale factor -1} {parallel to y -axis}	*B1 DB1	{ } indicate how the B1 marks should be awarded throughout.
	Then {Translation} $\left\{\begin{pmatrix} 0 \\ 3 \end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the positive y -direction. N.B. If order reversed a maximum of 3 out of 4 marks awarded.
	Alternative method for question 1		
	{Translation} $\left\{\begin{pmatrix} 0 \\ -3 \end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the negative y -direction.
	Then {Reflection} {in the x -axis} or {Stretch of scale factor -1} {parallel to y -axis}	*B1 DB1	N.B. If order reversed a maximum of 3 out of 4 marks awarded.
		4	

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Question	Answer	Marks	Guidance
2(a)	$1 + 6ax + 15a^2x^2$	B1	Terms must be evaluated.
		1	
2(b)	<i>their</i> $15a^2 \pm (3 \times \text{their } 6a)$	*M1	Expect $15a^2 - 18a$.
	$15a^2 - 18a = -3$	A1	
	$(3)(a-1)(5a-1) [= 0]$	DM1	Dependent on 3-term quadratic. Or solve using formula or completing the square.
	$a = 1, \frac{1}{5}$	A1	WWW. If DM0 awarded SC B1 if both answers correct.
		4	

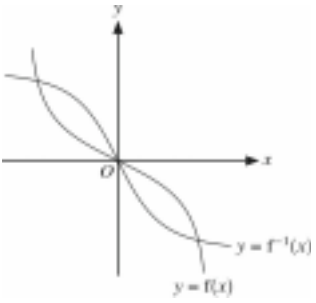
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Question	Answer	Marks	Guidance
3(a)	$\{5(y-3)^2\} \quad \{+5\}$	B1 B1	Accept $a = -3, b = 5$
		2	
3(b)	$[f'(x) =] 5x^4 - 30x^2 + 50$	B1	
	$5(x^2 - 3)^2 + 5$ or $b^2 < 4ac$ and at least one value of $f'(x) > 0$	M1	
	> 0 and increasing	A1	WWW
		3	

Question	Answer	Marks	Guidance
4(a)	$84 - 3(n-1) = 0$	M1	OE, SOI. Allow either $= 0$ or < 0 (to -3).
	Smallest n is 30	A1	SC B2 for answer only $n = 30$ WWW.
		2	
4(b)	$\left(\frac{2k}{2}\right)[168 + (2k-1)(-3)] = \left(\frac{k}{2}\right)[168 + (k-1)(-3)]$	M1 A1	M1 for forming an equation using correct formula. A1 for at least one side correct.
	$k = 19$	A1	
		3	

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Question	Answer	Marks	Guidance
5(a)	$\text{Angle } XYZ = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYZ = \frac{9}{11}$ leading to $XYZ = 0.9582$	B1	AG. OE using cosine rule.
		1	
5(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	*M1 A1	
	$AB = 9 + 11 - \text{their } XY$	B1 FT	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	$\text{Arc } AC = 11 \times 0.9582$	M1	
	$\text{Arc } BC = 9 \times \frac{\pi}{2}$	M1	
	$\text{Perimeter} = [13.6(8) + 10.5(4) + 14.1(4)] = 38.4$	A1	AWRT. Answer must be evaluated as a single decimal.
		6	

Question	Answer	Marks	Guidance
6(a)		B1	A reflection of the given curve in $y = x$ (the line $y = x$ can be implied by position of curve).
		1	

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Question	Answer	Marks	Guidance
6(b)	$y = \frac{-x}{\sqrt{4-x^2}}$ leading to $x^2 = y^2(4-x^2)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
	$x^2(1+y^2) = 4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
	$x = (\pm) \frac{2y}{\sqrt{1+y^2}}$	DM1	$x = (\pm) \sqrt{\frac{4y^2}{(1+y^2)}}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
	$f^{-1}(x) = \frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.
		4	
6(c)	1 or $a=1$	B1	Do not allow $x=1$ or $-1 < x < 1$
		1	
6(d)	$[fg(x) = f(2x)] \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
	$fg(x) = \frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2} \sqrt{1-x^2}$ or $\frac{x}{x^2-1} \sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.
		2	

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Question	Answer	Marks	Guidance
7(a)	$\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$	M1	Use $\tan x = \frac{\sin x}{\cos x}$ and clear fraction.
	$\sin x + 1 - \sin^2 x = k \sin x - k + k \sin^2 x$	*M1	Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine.
	$k \sin^2 x + \sin^2 x + k \sin x - \sin x - k - 1 = 0$	DM1	Gather like terms on one side of the equation.
	$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$	A1	AG. Factorise to obtain answer.
		4	
7(b)	$5\sin^2 x + 3\sin x - 5 = 0$	B1	
	$\sin x = \frac{-3 \pm \sqrt{9+100}}{10}$	M1	Use formula or complete the square.
	$x = 48.1^\circ, 131.9^\circ$	A1 A1 FT	AWRT. Maximum A1 if extra solutions in range. FT for $180 - \text{their answer}$ or $540 - \text{their answer}$ if $\sin x$ is negative If M0 given and correct answers only SCB1B1 available. If answers in radians; 0.839, 2.30 can score SCB1 for both.
		4	

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Question	Answer	Marks	Guidance
8(a)	$\int \left(\frac{5}{2} - x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right) dx$	M1	OR as 2 separate integrals $\int \left(\frac{5}{2} - x^{1/2} \right) dx - \int (x^{-1/2}) dx$
	$\left\{ \frac{5}{2}x - \frac{2}{3}x^{\frac{3}{2}} \right\} \{-\} \left\{ 2x^{\frac{1}{2}} \right\}$	A1 A1 A1	If two separate integrals with no subtraction SC B1 for each correct integral.
	$\left(10 - \frac{16}{3} - 4 \right) - \left(\frac{5}{8} - \frac{1}{12} - 1 \right)$	DM1	Substitute limits $\frac{1}{4} \rightarrow 4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	A1	WWW. Cannot be awarded if π appears in any integral.
		6	
8(b)	$\left[\frac{dy}{dx} = \right] -\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	When $x = 1$, $m = -\frac{1}{2}$	M1	Substitute $x = 1$ into a differential.
	[Equation of normal is] $y - 1 = 2(x - 1)$	M1	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1} = 2$
	[When $x = 0$,] $p = -1$	A1	WWW
		4	

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Question	Answer	Marks	Guidance
9(a)	$x^2 + (2x + 5)^2 = 20$ leading to $x^2 + 4x^2 + 20x + 25 = 20$	M1	Substitute $y = 2x + 5$ and expand bracket.
	$(5)(x^2 + 4x + 1)[= 0]$	A1	3-term quadratic.
	$x = \frac{-4 \pm \sqrt{16 - 4}}{2}$	M1	OE. Apply formula or complete the square.
	$A = (-2 + \sqrt{3}, 1 + 2\sqrt{3})$	A1	Or 2 correct x values.
	$B = (-2 - \sqrt{3}, 1 - 2\sqrt{3})$	A1	Or all values correct. SC B1 all 4 values correct in surd form without working. SC B1 all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2 = \text{their}(x_2 - x_1)^2 + \text{their}(y_2 - y_1)^2$	M1	Using <i>their</i> coordinates in a correct distance formula. Condone one sign error in $x_2 - x_1$ or $y_2 - y_1$
	$[AB^2 = 48 + 12 \text{ leading to}] AB = \sqrt{60}$	A1	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	

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Question	Answer	Marks	Guidance
9(b)	$x^2 + m^2(x - 10)^2 = 20$	*M1	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2 + 1) - 20m^2x + 20(5m^2 - 1) [= 0]$	DM1	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2 - 4ac =] 400m^4 - 80(m^2 + 1)(5m^2 - 1)$	M1	Using correct coefficients from <i>their</i> quadratic equation.
	$400m^4 - 80(5m^4 + 4m^2 - 1) = 0 \rightarrow (-80)(4m^2 - 1) = 0$	A1	OE. Must have ‘=0’ for A1.
	$m = \pm \frac{1}{2}$	A1	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2 = 10^2 - 20$	M1	
	$l = \sqrt{80}$	A1	
	$\tan \alpha = \frac{\sqrt{20}}{\sqrt{80}} = \frac{1}{2}$	M1 A1	Where α is the angle between the tangent and the x -axis.
	$m = \pm \frac{1}{2}$	A1	
		5	

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Question	Answer	Marks	Guidance
10(a)	$f''(x) = -\left(\frac{1}{2}x + k\right)^{-3}$	B1	
	$f''(2) > 0 \Rightarrow -(1+k)^{-3} > 0$	M1	Allow for solving <i>their</i> $f''(2) > 0$
	$k < -1$	A1	WWW
		3	
10(b)	$\left[f(x) = \int \left(\left(\frac{1}{2}x - 3 \right)^{-2} - (-2)^{-2} \right) dx = \right] \left\{ \frac{\left(\frac{1}{2}x - 3 \right)^{-1}}{-1 \times \frac{1}{2}} \right\} \left\{ -\frac{x}{4} \right\}$	B1 B1	Allow $-2 \left(\frac{1}{2}x + k \right)^{-1}$ OE for 1 st B1 and $-(1+k)^{-2}x$ OE for 2 nd B1
	$3\frac{1}{2} = 1 - \frac{1}{2} + c$	M1	Substitute $x = 2, y = 3\frac{1}{2}$ into <i>their</i> integral with c present.
	$f(x) = \frac{-2}{\left(\frac{1}{2}x - 3 \right)} - \frac{x}{4} + 3$	A1	OE
		4	
10(c)	$\left(\frac{1}{2}x - 3 \right)^{-2} - (-2)^{-2} = 0$	M1	Substitute $k = -3$ and set to zero.
	leading to $\left(\frac{1}{2}x - 3 \right)^2 = 4 \quad \left[\frac{1}{2}x - 3 = (\pm)2 \right]$ leading to $x = 10$	A1	
	$(10, -\frac{1}{2})$	A1	Or when $x = 10, y = -1 - 2\frac{1}{2} + 3 = -\frac{1}{2}$
	$f''(10) \left[= -(5-3)^{-3} \rightarrow \right] < 0 \rightarrow \text{MAXIMUM}$	A1	WWW
		4	