



Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 1 Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

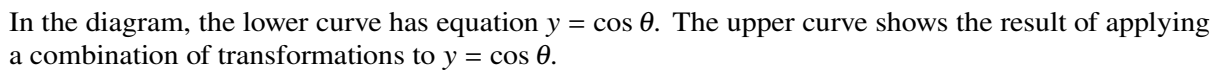
[4]

[illegible]

- 3 Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50 \text{ cm}^3 \text{ s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

[illegible]

5 In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a .

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$.

[1]

.....

.....

.....

.....

.....

- 6** The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

[illegible]

7 (a) Show that $\frac{\sin \theta}{1 - \sin \theta} - \frac{\sin \theta}{1 + \sin \theta} \equiv 2 \tan^2 \theta$. [3]

[illegible]

[Turn over

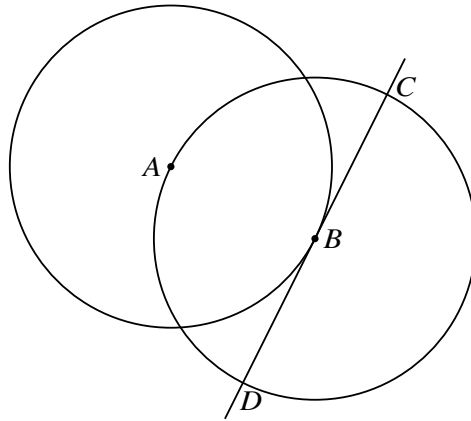
- 8 A geometric progression has first term a , common ratio r and sum to infinity S . A second geometric progression has first term a , common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$.

[3]

[illegible]

[illegible]



The diagram shows a circle with centre A passing through the point B . A second circle has centre B and passes through A . The tangent at B to the first circle intersects the second circle at C and D .

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

- (a) Find the equation of the tangent CBD . [2]

[illegible]

- (b) Find an equation of the circle with centre B . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find, by calculation, the x -coordinates of C and D . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

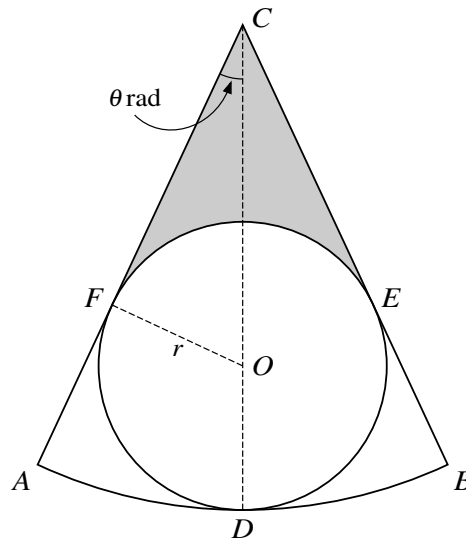
.....

.....

.....

.....

.....



The diagram shows a sector CAB which is part of a circle with centre C . A circle with centre O and radius r lies within the sector and touches it at D , E and F , where COD is a straight line and angle ACD is θ radians.

- (a) Find CD in terms of r and $\sin \theta$. [3]

[illegible]

It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

- (b) Find the perimeter of sector CAB in terms of π . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11 The functions f and g are defined by

$$\begin{aligned} f(x) &= x^2 + 3 \quad \text{for } x > 0, \\ g(x) &= 2x + 1 \quad \text{for } x > -\frac{1}{2}. \end{aligned}$$

(a) Find an expression for $fg(x)$. [1]

.....

.....

.....

.....

.....

(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

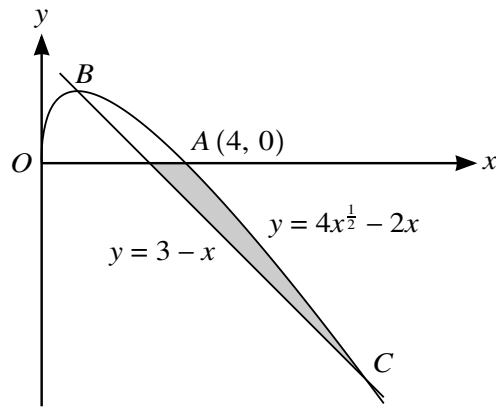
[illegible]

(c) Solve the equation $fg(x) - 3 = gf(x)$.

[4]

[illegible]

12



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geq 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x -axis at $A(4, 0)$ and crosses the straight line at B and C .

- (a) Find, by calculation, the x -coordinates of B and C . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Show that B is a stationary point on the curve. [2]

.....

.....

.....

.....

.....

.....

[illegible]

[illegible]

9709/11/O/N/20

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

BLANK PAGE

- 1** The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k .

[4]

[illegible]

- 2** The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression.

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 3** The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m , the line intersects the curve at two distinct points. [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 4** The sum, S_n , of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The k th term in the progression is greater than 200.

Find the smallest possible value of k .

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R}, x \neq -1.$$

(a) Find the value of $fg(7)$.

[1]

(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$.

[5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2 \tan^2 x$ for $0^\circ \leq x \leq 180^\circ$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

- (a) A point moves along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y -coordinate when $x = 4$.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the equation of the curve.

[4]

.....

.....

.....

.....

.....

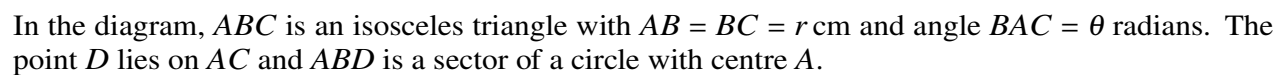
.....

.....

.....

.....

.....



-
- This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

© UCLES 2020

- 9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

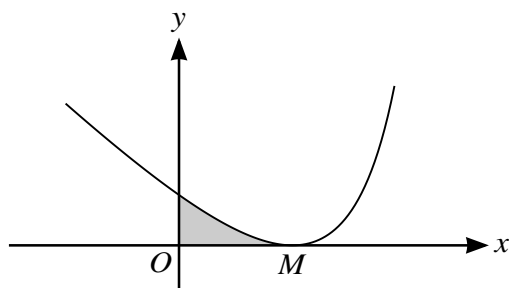
.....

.....

.....

[2]

[illegible]



(a) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y \, dx$. [6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (b) Find, by calculation, the x -coordinate of M . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11 A curve has equation $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$.

(a) State the greatest and least values of y . [2]

.....

.....

.....

.....

(b) Sketch the graph of $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3 \cos 2x + 2 = kx$ for $0 \leq x \leq \pi$ in each of the following cases.

(i) $k = -3$ [1]

.....

.....

(ii) $k = 1$ [1]

.....

.....

(iii) $k = 3$ [1]

.....

.....

Functions f , g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3 \cos 2x + 2,$$

$$g(x) = f(2x) + 4,$$

$$h(x) = 2f\left(x + \frac{1}{2}\pi\right).$$

- (d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

- 1 (a)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b)** The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.
Describe fully the transformation(s) involved. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 2 The function f is defined by $f(x) = \frac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\int_1^{\infty} f(x) \, dx$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) The equation of a curve is such that $\frac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 3** Solve the equation $3 \tan^2 \theta + 1 = \frac{2}{\tan^2 \theta}$ for $0^\circ < \theta < 180^\circ$. [5]

[illegible]

- 4** A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection.

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 5** In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x , x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$.

[5]

[illegible]

- 6 The function f is defined by $f(x) = \frac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) State the range of f . [1]

.....

.....

.....

- 7** The first and second terms of an arithmetic progression are $\frac{1}{\cos^2 \theta}$ and $-\frac{\tan^2 \theta}{\cos^2 \theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4 \theta}$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[Turn over

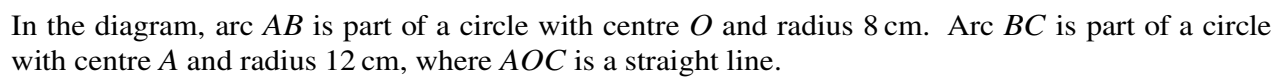
- 8** The equation of a curve is $y = 2x + 1 + \frac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



-
- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Find the area of the shaded region.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the perimeter of the shaded region.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 10** A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

- (a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k .

[4]

[illegible]

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

11 A circle with centre C has equation $(x - 8)^2 + (y - 4)^2 = 100$.

(a) Show that the point $T(-6, 6)$ is outside the circle. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45° . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The two tangents touch the circle at A and B .

- (c) Find the equation of the line AB , giving your answer in the form $y = mx + c$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (d) Find the x -coordinates of A and B . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[illegible]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.