

CANDIDATE  
NAME

|  |
|--|
|  |
|--|

CENTRE  
NUMBER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

CANDIDATE  
NUMBER

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

**MATHEMATICS**

**9709/11**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2019**

**1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **20** printed pages.



[illegible]

- 2** An increasing function,  $f$ , is defined for  $x > n$ , where  $n$  is an integer. It is given that  $f'(x) = x^2 - 6x + 8$ . Find the least possible value of  $n$ . [3]

[illegible]

- 3** The line  $y = ax + b$  is a tangent to the curve  $y = 2x^3 - 5x^2 - 3x + c$  at the point  $(2, 6)$ . Find the values of the constants  $a$ ,  $b$  and  $c$ . [5]

[illegible]

- 4 A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run  $x$  km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

- (i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Find the total distance she runs over the 21 days. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



**PapaCambridge**



- 6** A straight line has gradient  $m$  and passes through the point  $(0, -2)$ . Find the two values of  $m$  for which the line is a tangent to the curve  $y = x^2 - 2x + 7$  and, for each value of  $m$ , find the coordinates of the point where the line touches the curve. [7]

[illegible]



$$g : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Find an expression for  $fg(x)$ , giving your answer in the form  $\frac{ax}{bx+c}$ , where  $a$ ,  $b$  and  $c$  are integers. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Find an expression for  $(fg)^{-1}(x)$ , giving your answer in the same form as for part (ii). [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

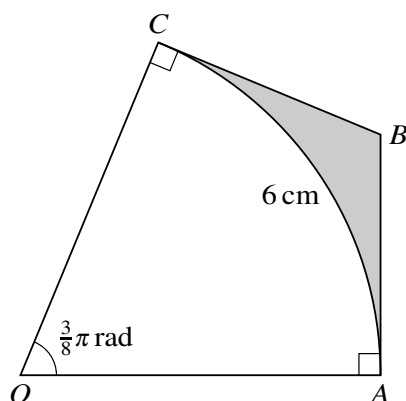
.....

.....

.....

.....

.....



The diagram shows a sector  $OAC$  of a circle with centre  $O$ . Tangents  $AB$  and  $CB$  to the circle meet at  $B$ . The arc  $AC$  is of length 6 cm and angle  $AOC = \frac{3}{8}\pi$  radians.

- (i) Find the length of  $OA$  correct to 4 significant figures. [2]

.....

.....

.....

.....

.....

- (ii) Find the perimeter of the shaded region. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find the area of the shaded region.

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 9** A curve for which  $\frac{dy}{dx} = (5x - 1)^{\frac{1}{2}} - 2$  passes through the point  $(2, 3)$ .

(i) Find the equation of the curve.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(ii) Find  $\frac{d^2y}{dx^2}$ .

[2]

.....

.....

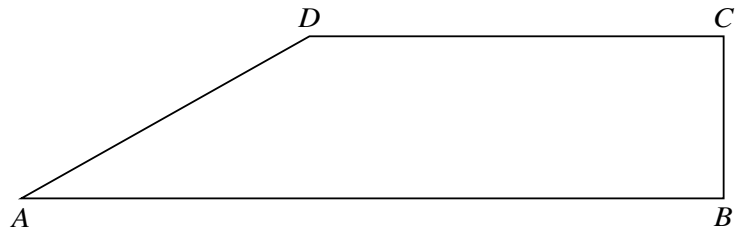
.....

.....

**(iii)** Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

10



Relative to an origin  $O$ , the position vectors of the points  $A$ ,  $B$ ,  $C$  and  $D$ , shown in the diagram, are given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}.$$

- (i) Show that  $AB$  is perpendicular to  $BC$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Show that  $ABCD$  is a trapezium. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find the area of  $ABCD$ , giving your answer correct to 2 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

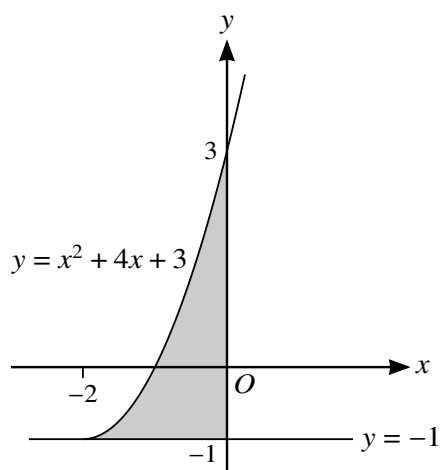
.....

.....

.....

.....

.....



The diagram shows a shaded region bounded by the y-axis, the line  $y = -1$  and the part of the curve  $y = x^2 + 4x + 3$  for which  $x \geq -2$ .

- (i) Express  $y = x^2 + 4x + 3$  in the form  $y = (x + a)^2 + b$ , where  $a$  and  $b$  are constants. Hence, for  $x \geq -2$ , express  $x$  in terms of  $y$ . [4]

[illegible]

- (ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through  $360^\circ$  about the **y-axis**. [6]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

CANDIDATE  
NAME

|  |
|--|
|  |
|--|

CENTRE  
NUMBER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

CANDIDATE  
NUMBER

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2019**

**1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



**BLANK PAGE**

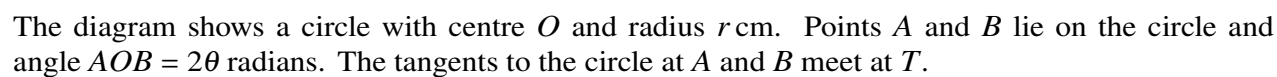
- 1** The coefficient of  $x^2$  in the expansion of  $(4 + ax)\left(1 + \frac{x}{2}\right)^6$  is 3. Find the value of the constant  $a$ . [4]

[illegible]

- [illegible]

- 3** A curve is such that  $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$ , where  $k$  is a constant. The points  $P(1, -1)$  and  $Q(4, 4)$  lie on the curve. Find the equation of the curve. [4]

[illegible]



- [illegible]

- (ii)** In the case where  $r = 5$  and  $\theta = 1.2$ , find the area of the shaded region.

[4]

[illegible]



- [The volume of a cone of radius  $r$  and vertical height  $h$  is  $\frac{1}{3}\pi r^2 h$ .]

[illegible]

[illegible]

- 6** (a) Given that  $x > 0$ , find the two smallest values of  $x$ , in radians, for which  $3 \tan(2x + 1) = 1$ . Show all necessary working. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) The function  $f : x \mapsto 3 \cos^2 x - 2 \sin^2 x$  is defined for  $0 \leq x \leq \pi$ .

(i) Express  $f(x)$  in the form  $a \cos^2 x + b$ , where  $a$  and  $b$  are constants. [1]

.....

.....

.....

.....

.....

.....

(ii) Find the range of  $f$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

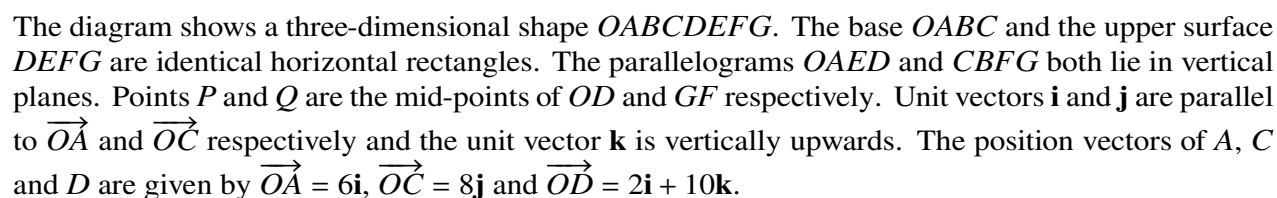
.....

.....

.....

.....

.....



- [illegible]

- (ii) Determine whether  $P$  is nearer to  $Q$  or to  $B$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Use a scalar product to find angle  $BPQ$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 8 (a) Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the total distance she runs in the 21-day period. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) The first, second and third terms of a geometric progression are  $x$ ,  $x - 3$  and  $x - 5$  respectively.

(i) Find the value of  $x$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the fourth term of the progression. [2]

.....

.....

.....

.....

.....

(iii) Find the sum to infinity of the progression. [2]

.....

.....

.....

.....

.....

.....

.....

.....

9 Functions  $f$  and  $g$  are defined by

$$f(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$

$$g(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where  $k$  is a constant.

- (i) Find the value of  $k$  for which the line  $y = g(x)$  is a tangent to the curve  $y = f(x)$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) In the case where  $k = -9$ , find the set of values of  $x$  for which  $f(x) < g(x)$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) In the case where  $k = -1$ , find  $g^{-1}f(x)$  and solve the equation  $g^{-1}f(x) = 0$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iv) Express  $f(x)$  in the form  $2(x + a)^2 + b$ , where  $a$  and  $b$  are constants, and hence state the least value of  $f(x)$ . [3]

.....

.....

.....

.....

.....

.....

.....

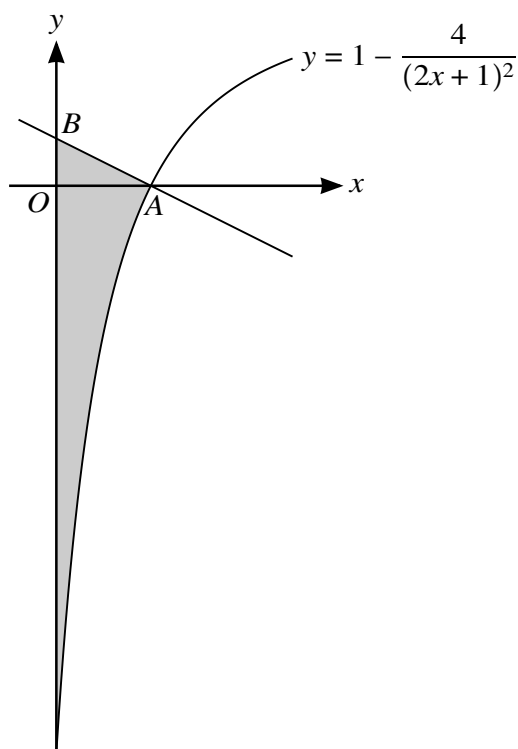
.....

.....

.....

.....

10



The diagram shows part of the curve  $y = 1 - \frac{4}{(2x+1)^2}$ . The curve intersects the  $x$ -axis at  $A$ . The normal to the curve at  $A$  intersects the  $y$ -axis at  $B$ .

- (i) Obtain expressions for  $\frac{dy}{dx}$  and  $\int y \, dx$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the coordinates of  $B$ .

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find, showing all necessary working, the area of the shaded region.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

CANDIDATE  
NAME

|  |
|--|
|  |
|--|

CENTRE  
NUMBER

|  |  |  |  |  |
|--|--|--|--|--|
|  |  |  |  |  |
|--|--|--|--|--|

CANDIDATE  
NUMBER

|  |  |  |  |
|--|--|--|--|
|  |  |  |  |
|--|--|--|--|

**MATHEMATICS**

**9709/13**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2019**

**1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **20** printed pages.



- 1 (i) Expand  $(1 + y)^6$  in ascending powers of  $y$  as far as the term in  $y^2$ . [1]

.....

.....

.....

.....

.....

- (ii) In the expansion of  $(1 + (px - 2x^2))^6$  the coefficient of  $x^2$  is 48. Find the value of the positive constant  $p$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

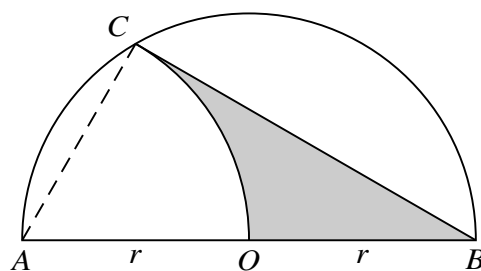
.....

- 2** The function  $g$  is defined by  $g(x) = x^2 - 6x + 7$  for  $x > 4$ . By first completing the square, find an expression for  $g^{-1}(x)$  and state the domain of  $g^{-1}$ . [5]

[illegible]

- 3** The equation of a curve is  $y = x^3 + x^2 - 8x + 7$ . The curve has no stationary points in the interval  $a < x < b$ . Find the least possible value of  $a$  and the greatest possible value of  $b$ . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.



(i) Express angle  $CAO$  in radians in terms of  $\pi$ . [1]

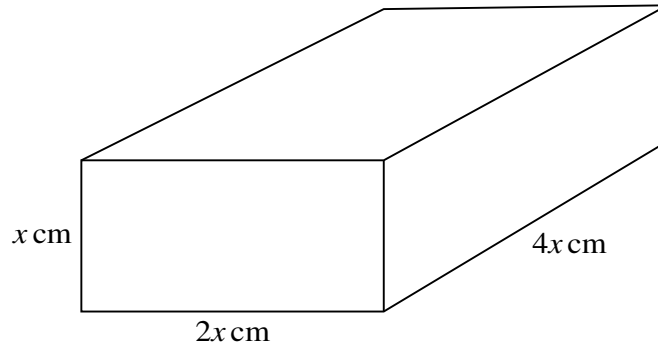
.....

.....

.....

**(ii)** Find the area of the shaded region in terms of  $r$ ,  $\pi$  and  $\sqrt{3}$ , simplifying your answer. [4]

[illegible]



The dimensions of a cuboid are  $x$  cm,  $2x$  cm and  $4x$  cm, as shown in the diagram.

- (i) Show that the surface area  $S \text{ cm}^2$  and the volume  $V \text{ cm}^3$  are connected by the relation

$$S = 7V^{\frac{2}{3}}. \quad [3]$$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- (ii) When the volume of the cuboid is  $1000 \text{ cm}^3$  the surface area is increasing at  $2 \text{ cm}^2 \text{ s}^{-1}$ . Find the rate of increase of the volume at this instant. [4]

[illegible]

- 6** A line has equation  $y = 3kx - 2k$  and a curve has equation  $y = x^2 - kx + 2$ , where  $k$  is a constant.

(i) Find the set of values of  $k$  for which the line and curve meet at two distinct points. [4]

[illegible]

[illegible]

- 7 (i) Show that the equation  $3 \cos^4 \theta + 4 \sin^2 \theta - 3 = 0$  can be expressed as  $3x^2 - 4x + 1 = 0$ , where  $x = \cos^2 \theta$ . [2]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

© UCLES 2019

- 8** A function  $f$  is defined for  $x > \frac{1}{2}$  and is such that  $f'(x) = 3(2x - 1)^{\frac{1}{2}} - 6$ .

**(i)** Find the set of values of  $x$  for which  $f$  is decreasing.

[4]

[illegible]

(ii) It is now given that  $f(1) = -3$ . Find  $f(x)$ .

[4]

[illegible]

- 9 The first, second and third terms of a geometric progression are  $3k$ ,  $5k - 6$  and  $6k - 4$ , respectively.

(i) Show that  $k$  satisfies the equation  $7k^2 - 48k + 36 = 0$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of  $k$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

$$\overrightarrow{OA} = \begin{pmatrix} -8 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 10 \\ 2 \\ 11 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OX} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}.$$

- (i) Find  $\overrightarrow{AX}$  and show that  $AXB$  is a straight line. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

The position vector of a point  $C$  is given by  $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -8 \\ 3 \end{pmatrix}$ .

(ii) Show that  $CX$  is perpendicular to  $AX$ .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find the area of triangle  $ABC$ .

[3]

.....

.....

.....

.....

.....

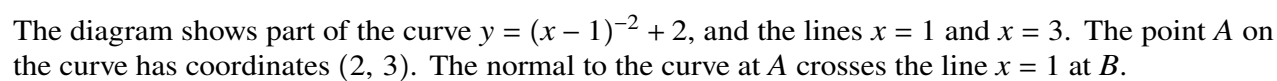
.....

.....

.....

.....

.....



- [illegible]

- (ii)** Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through  $360^\circ$  about the  $x$ -axis. [8]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.