
MATHEMATICS

9709/11

Paper 1

October/November 2018

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **14** printed pages.



Cambridge Assessment
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GENERIC MARKING PRINCIPLE 1:

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

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GENERIC MARKING PRINCIPLE 6:

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$(4x^{1/2} - 3)(x^{1/2} - 2)$ oe soi Alt: $4x + 6 = 11\sqrt{x} \Rightarrow 16x^2 - 73x + 36$	M1	Attempt solution for $x^{1/2}$ or sub $u = x^{1/2}$
	$x^{1/2} = 3/4$ or 2 $(16x - 9)(x - 4)$	A1	Reasonable solutions for $x^{1/2}$ implies M1 ($x = 2, 3/4$, M1A0)
	$x = 9/16$ oe or 4	A1	Little or no working shown scores SCB3, spotting one solution, B0
		3	

Question	Answer	Marks	Guidance
2	$x^2 + bx + 5 = x + 1 \rightarrow x^2 + x(b - 1) + 4 (= 0)$	M1	Eliminate x or y with all terms on side of an equation
	$(b^2 - 4ac =) (b - 1)^2 - 16$	M1	
	b associated with -3 & $+5$ or $b - 1$ associated with ± 4	A1	$(x - 2)^2 = 0$ or $(x + 2)^2 = 0, x = \pm 2, b - 1 = \pm 4$ (M1A1) Association can be an equality or an inequality
	$b \geq 5, b \leq -3$	A1	
		4	

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Question	Answer	Marks	Guidance
3(i)	Gradient of $AB = -3/4$	B1	Accept $-3a/4a$
	$y = -\frac{3}{4}x$ oe	B1FT	Answer must not include a . Ft on <i>their</i> <u>numerical</u> gradient
		2	
3(ii)	$(4a)^2 + (3a)^2 = (10/3)^2$ soi	M1	May be unsimplified
	$25a^2 = 100/9$ oe	A1	
	$a = 2/3$	A1	
		3	

Question	Answer	Marks	Guidance
4(i)	$S_{80} = \frac{80}{2}[12 + 79 \times (-4)]$ or $\frac{80}{2}[6 + l], l = -310$	M1A1	Correct formula (M1). Correct a , d and n (A1) .
	$-12\ 160$	A1	
		3	
4(ii)	$S_{\infty} = \frac{6}{1 - \frac{1}{3}} = 9$	M1A1	Correct formula with $ r < 1$ for M1
		2	

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Question	Answer	Marks	Guidance
5(i)	$\frac{(\cos \theta - 4)(5 \cos \theta - 2) - 4 \sin^2 \theta}{\sin \theta (5 \cos \theta - 2)} (= 0)$	M1	Accept numerator only
	$\frac{5 \cos^2 \theta - 22 \cos \theta + 8 - 4(1 - \cos^2 \theta)}{\sin \theta (5 \cos \theta - 2)} (= 0)$	M1	Simplify numerator and use $s^2 = 1 - c^2$. Accept numerator only
	$9 \cos^2 \theta - 22 \cos \theta + 4 = 0$ www AG	A1	
		3	
5(ii)	Attempt to solve for $\cos \theta$, (formula, completing square expected)	M1	Expect $\cos \theta = 0.1978$. Allow 2.247 in addition
	$\theta = 78.6^\circ, 281.4^\circ$ (only, second solution in the range)	A1A1FT	Ft for $(360^\circ - 1^{\text{st}} \text{ solution})$
		3	

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Question	Answer	Marks	Guidance
6(i)	$0 = 9a + 3a^2$	M1	Sub $\frac{dy}{dx} = 0$ and $x = 3$
	$a = -3$ only	A1	
		2	
6(ii)	$\frac{dy}{dx} = -3x^2 + 9x \rightarrow y = -x^3 + \frac{9x^2}{2} (+c)$	M1A1FT	Attempt integration. $\frac{1}{3}ax^3 + \frac{1}{2}a^2x^2$ scores M1. Ft on <i>their a</i> .
	$9\frac{1}{2} = -27 + 40\frac{1}{2} + c$	DM1	Sub $x = 3, y = 9\frac{1}{2}$. Dependent on c present
	$c = -4$	A1	Expect $y = -x^3 + \frac{9x^2}{2} - 4$
		4	
6(iii)	$\frac{d^2y}{dx^2} = -6x + 9$	M1	$2ax + a^2$ scores M1
	At $x = 3, \frac{d^2y}{dx^2} = -9 < 0$ MAX www	A1	Requires at least one of -9 or < 0 . Other methods possible.
		2	

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Question	Answer	Marks	Guidance
7(i)	$2 = k(8 - 28 + 24) \rightarrow k = 1/2$	B1	
		1	
7(ii)	When $x = 5$, $y = [\frac{1}{2}](125 - 175 + 60) = 5$	M1	Or solve $[\frac{1}{2}](x^3 - 7x^2 + 12x) = x \Rightarrow x = 5$ [$x = 0, 2$]
	Which lies on $y = x$, oe	A1	
		2	
7(iii)	$\int [\frac{1}{2}(x^3 - 7x^2 + 12x) - x] dx$	M1	Expect $\int \frac{1}{2}x^3 - \frac{7}{2}x^2 + 5x$
	$\frac{1}{8}x^4 - \frac{7}{6}x^3 + \frac{5}{2}x^2$	B2,1,0FT	Ft on their k
	$2 - 28/3 + 10$	DM1	Apply limits $0 \rightarrow 2$
	$8/3$	A1	
	OR $\frac{1}{8}x^4 - \frac{7}{6}x^3 + 3x^2$	B2,1,0FT	Integrate to find area under curve, Ft on their k
	$2 - 28/3 + 12$	M1	Apply limits $0 \rightarrow 2$. Dep on integration attempted
	Area $\Delta = \frac{1}{2} \times 2 \times 2$ or $\int_0^2 x dx = [\frac{1}{2}x^2] = 2$	M1	
	$8/3$	A1	
		5	

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Question	Answer	Marks	Guidance
8(i)	$\overrightarrow{DF} = -6\mathbf{i} + 2\mathbf{k}$	B1	
		1	
8(ii)	$\overrightarrow{EF} = -6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	B1	
	$ \overrightarrow{EF} = \sqrt{(-6)^2 + (-3)^2 + 2^2}$	M1	Must use <i>their</i> $\overrightarrow{EF}$
	Unit vector = $\frac{1}{7}(-6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$	A1	
		3	
8(iii)	$\overrightarrow{DF} \cdot \overrightarrow{EF} = (-6\mathbf{i} + 2\mathbf{k}) \cdot (-6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 36 + 4 = 40$	M1	
	$ \overrightarrow{DF} = \sqrt{40}, \overrightarrow{EF} = 7$	M1	
	$\cos EFD = \frac{40}{7\sqrt{40}}$ oe	M1	
	$EFD = 25.4^\circ$	A1	Special case: use of cosine rule M1(must evaluate lengths using correct method) A1 only
		4	

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Question	Answer	Marks	Guidance
9	Angle $OAB = \pi / 2 - \pi / 5 = 3\pi / 10$ soi	B1	Allow 54° or 0.9425 rads
	Sector $CAB = \frac{1}{2} \times \left(\text{their } \frac{3\pi}{10} \right) \times 5^2$	M1	Expect 11.78
	$OA = \frac{5}{\sin \frac{\pi}{5}} = 8.507$	M1A1	May be implied by $OC = 3.507$
	Sector $COD = \frac{1}{2} \times (\text{their } 3.507)^2 \times \frac{\pi}{5}$	M1	Expect 3.86
	$\Delta OAB = \frac{1}{2} \times 5 \times (\text{their } 8.507) \sin \frac{3\pi}{10}$	M1	Or $\frac{1}{2} \times 5 \times \frac{5}{\tan \frac{\pi}{5}}$ or $2.5 \times \sqrt{(\text{their } 8.507)^2 - 25}$
	= 17.20 or 17.21	A1	
	Shaded area $17.20 (\text{or } 17.21) - 11.78 - 3.86 = 1.56$ or 1.57	A1	
		8	

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Question	Answer	Marks	Guidance
10(i)(a)	$\frac{dy}{dx} = \left[-\frac{1}{2}(4x-3)^{-2} \right] \times [4]$	B1B1	Can gain this in part (b)(ii)
	When $x=1$, $m=-2$	B1FT	Ft from <i>their</i> $\frac{dy}{dx}$
	Normal is $y - \frac{1}{2} = \frac{1}{2}(x-1)$	M1	Line with gradient $-1/m$ and through A
	$y = \frac{1}{2}x$ soi	A1	Can score in part (b)
		5	
10(i)(b)	$\frac{1}{2(4x-3)} = \frac{x}{2} \rightarrow 2x(4x-3) = 2 \rightarrow (2)(4x^2 - 3x - 1) (=0)$	M1A1	$x/2$ seen on RHS of equation can score <i>previous</i> A1
	$x = -1/4$	A1	Ignore $x=1$ seen in addition
		3	
10(ii)	Use of chain rule: $\frac{dy}{dt} = (their - 2) \times (\pm) 0.3 = 0.6$	M1A1	Allow +0.3 or -0.3 for M1
		2	

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Question	Answer	Marks	Guidance
11(a)(i)	[Greatest value of a is] 3	B1	Must be in terms of a . Allow $a < 3$. Allow $a \leq 3$
		1	
11(a)(ii)	Range is $y > -1$	B1	Ft on <i>their</i> a . Accept any equivalent notation
	$y = (x - 3)^2 - 1 \rightarrow (x - 3)^2 = 1 + y \rightarrow x = 3(\pm)\sqrt{1 + y}$	M1	Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{1 + x}$ cao	A1	
		3	
11(b)(i)	$gg(2x) = [(2x - 3)^2 - 3]^2$	B1	
	$(2x - 3)^4 - 6(2x - 3)^2 + 9$	B1	
		2	
11(b)(ii)	$[16x^4 - 96x^3 + 216x^2 - 216x + 81] + [(-24x^2 + 72x - 54) + 9]$ $16x^4 - 96x^3 + 192x^2 - 144x + 36$	B4,3,2,1,0	
		4	

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1	For a correctly selected term in $\frac{1}{x^2} : (3x)^4$ or 3^4	B1	Components of coefficient added together 0/4 B1 expect 81
	$\times \left(\frac{2}{3x^2}\right)^3$ or $(2/3)^3$	B1	B1 expect 8/27
	$\times {}_7C_3$ or ${}_7C_4$	B1	B1 expect 35
	$\rightarrow \mathbf{840}$ or $\frac{840}{x^2}$	B1	All of the first three marks can be scored if the correct term is seen in an expansion <u>and it is selected</u> but then wrongly simplified.
			SC: A completely correct unsimplified term seen in an expansion but not correctly selected can be awarded B2.
		4	

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Question	Answer	Marks	Guidance
2	Integrate $\rightarrow \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} (+C)$	B1 B1	B1 for each term correct – allow unsimplified. C not required.
	$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^4 \rightarrow \frac{40}{3} - \frac{14}{3}$	M1	Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1.
	$= \frac{26}{3}$ oe	A1	Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working.
		4	

Question	Answer	Marks	Guidance
3(i)	$P \text{ is } (t, 5t) \ Q \text{ is } (t, t(9 - t^2)) \rightarrow 4t - t^3$	B1 B1	B1 for both y coordinates which can be implied by subsequent working. B1 for PQ allow $ 4t - t^3 $ or $ t^3 - 4t $. Note: $4x - x^3$ from equating line and curve 0/2 even if x then replaced by t.
		[2]	

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Question	Answer	Marks	Guidance
3(ii)	$\frac{d(PQ)}{dt} = 4 - 3t^2$	B1FT	B1FT for differentiation of their PQ , which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from (i) but not the equation of the curve.
	$= 0 \rightarrow t = + \frac{2}{\sqrt{3}}$	M1	Setting their differential of PQ to 0 and attempt to solve for t or x.
	$\rightarrow \text{Maximum } PQ = \frac{16}{3\sqrt{3}} \text{ or } \frac{16\sqrt{3}}{9}$	A1	Allow 3.08 awrt. If answer comes from wrong method in (i) award A0. Correct answer from correct expression by T&I scores 3/3.
		3	

Question	Answer	Marks	Guidance
4(i)	$fg(x) = 2 - 3\cos(\frac{1}{2}x)$	B1	Correct fg
	$2 - 3\cos(\frac{1}{2}x) = 1 \rightarrow \cos(\frac{1}{2}x) = \frac{1}{3} \rightarrow \left(\frac{1}{2}x\right) = \cos^{-1}\left(\text{their } \frac{1}{3}\right)$	M1	M1 for correct order of operations to solve their $fg(x) = 1$ as far as using inverse cos expect 1.23, (or 70.5°) condone $x =$.
	$x = 2.46 \text{ awrt or } \frac{4.7\pi}{6} (0.784\pi \text{ awrt})$	A1	One solution only in the given range, ignore answers outside the range. Answer in degrees A0.
			Alternative: Solve $f(y) = 1 \rightarrow y = 1.23 \rightarrow \frac{1}{2}x = 1.23$ B1M1 $\rightarrow x = 2.46$ A1
		3	

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Question	Answer	Marks	Guidance
4(ii)		B1	One cycle of $\pm \cos$ curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted “V”. If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark $0 \rightarrow 2\pi$.
		B1	Start and finish at roughly the same negative y value. Significantly more above the x axis than below or correct range implied by labels .
		B1	Fully correct. Curves not lines. Must be a reasonable curve clearly turning at both ends. Labels not required but must be appropriate if present.
		3	

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Question	Answer	Marks	Guidance
5(i)	From the AP: $x - 4 = y - x$	B1	Or equivalent statement e.g. $y = 2x - 4$ or $x = \frac{y+4}{2}$.
	From the GP: $\frac{y}{x} = \frac{18}{y}$	B1	Or equivalent statement e.g. $y^2 = 18x$ or $x = \frac{y^2}{18}$.
	Simultaneous equations: $y^2 - 9y - 36 = 0$ or $2x^2 - 17x + 8 = 0$	M1	Elimination of either x or y to give a three term quadratic ($= 0$)
	OR		
	$4+d=x, 4+2d=y \rightarrow \frac{4+2d}{4+d} = r$ oe	B1	
	$(4+d)\left(\frac{4+2d}{4+d}\right)^2 = 18 \rightarrow 2d^2 - d - 28 = 0$	M1	Uses $ar^2 = 18$ to give a three term quadratic ($= 0$)
	$d = 4$	B1	Condone inclusion of $d = \frac{-7}{2}$ oe

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Question	Answer	Marks	Guidance
5(i)	OR		
	From the GP $\frac{y}{x} = \frac{18}{y}$	B1	
	$\rightarrow x = \frac{y^2}{18} \rightarrow 4 + d = \frac{y^2}{18} \rightarrow d = \frac{y^2}{18} - 4$	B1	
	$4 + 2\left(\frac{y^2}{18} - 4\right) = y \rightarrow y^2 - 9y - 36 = 0$	M1	
	$x = 8, y = 12$.	A1	Needs both x and y . Condone $\left(\frac{1}{2}, -3\right)$ included in final answer. Fully correct answer www 4/4.
		4	
5(ii)	AP 4th term = 16	B1	Condone inclusion of $\frac{-13}{2}$ oe
	GP 4th term = $8 \times \left(\frac{12}{8}\right)^3$	M1	A valid method using their x and y from (i).
	= 27	A1	Condone inclusion of -108
			Note: Answers from fortuitous $x = 8, y = 12$ in (i) can only score M1. Unidentified correct answer(s) with no working seen after valid $x = 8, y = 12$ to be credited with appropriate marks.
		3	

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Question	Answer	Marks	Guidance
6(i)	In $\triangle ABD$, $\tan\theta = \frac{9}{BD} \rightarrow BD = \frac{9}{\tan\theta}$ or $9\tan(90 - \theta)$ or $9 \cot\theta$ or $\sqrt{(20 \tan\theta)^2 - 9^2}$ (Pythag) or $\frac{9\sin(90 - \theta)}{\sin\theta}$ (Sine rule)	B1	Both marks can be gained for correct equated expressions.
	In $\triangle DBC$, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$	B1	
	$20\sin\theta = \frac{9}{\tan\theta}$	M1	Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary.
	$\rightarrow 20\sin^2\theta = 9\cos\theta$ AG	A1	Correct manipulation of their expression to arrive at given answer.
			SC: In $\triangle DBC$, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ B1 In $\triangle ABD$, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$ $\cos\theta = \frac{20\sin\theta}{9 / \sin\theta} \rightarrow \cos\theta = \frac{20\sin^2\theta}{9}$ M1 $\rightarrow 20\sin^2\theta = 9\cos\theta$ A1 Scores 3/4
		4	
6(ii)	Uses $s^2 + c^2 = 1 \rightarrow 20\cos^2\theta + 9\cos\theta - 20 (= 0)$	M1	Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$
	$\rightarrow \cos\theta = 0.8$	A1	www
	$\rightarrow \theta = 36.9^\circ$ awrt	A1	www. Allow 0.644° awrt. Ignore 323.1° or 2.50° . Note: correct answer without working scores 0/3.
		3	

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Question	Answer	Marks	Guidance
7	$\overrightarrow{PN} = 8\mathbf{i} - 8\mathbf{k}$	B1	
	$\overrightarrow{PM} = 4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$	B2,1,0	Loses 1 mark for each component incorrect
			SC: $\overrightarrow{PN} = -8\mathbf{i} + 8\mathbf{k}$ <u>and</u> $\overrightarrow{PM} = -4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ scores 2/3.
	$\overrightarrow{PN} \cdot \overrightarrow{PM} = 32 + 0 + 48 = 80$	M1	Evaluates $x_1x_2 + y_1y_2 + z_1z_2$ for correct vectors or one or both reversed.
	$ \overrightarrow{PN} \times \overrightarrow{PM} = \sqrt{128} \times \sqrt{68} (= 16\sqrt{34})$	M1	Product of their moduli – may be seen in cosine rule
	$\sqrt{128} \times \sqrt{68} \cos \hat{M}PN = 80$	M1	All linked correctly.
	Angle $\hat{M}PN = \mathbf{31.0^\circ}$ awrt	A1	Answer must come directly from +ve cosine ratio. Cosine rule not accepted as a complete method. Allow 0.540° awrt. Note: Correct answer from incorrect vectors scores A0 (XP)
		7	

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Question	Answer	Marks	Guidance
8(i)	$A \hat{B} C$ using cosine rule giving $\cos^{-1}\left(\frac{-1}{8}\right)$ or $2\sin^{-1}\left(\frac{3}{4}\right)$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ or $B \hat{A} C = \cos^{-1}\left(\frac{3}{4}\right)$ or $B \hat{A} C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B \hat{A} C = \tan^{-1}\frac{\sqrt{7}}{3}$	M1	Correct method for $A \hat{B} C$, expect 1.696°awrt Or for $B \hat{A} C$, expect 0.723°awrt
	$C \hat{B} Y = \pi - A \hat{B} C$ or $2 \times C \hat{A} B$	M1	For attempt at $C \hat{B} Y = \pi - A \hat{B} C$ or $C \hat{B} Y = 2 \times C \hat{A} B$
	OR		
	Find CY from ΔACY using Pythagoras or similar Δ s	M1	Expect $4\sqrt{7}$
	$C \hat{B} Y = \cos^{-1}\left(\frac{8^2 + 8^2 - (\text{their } CY)^2}{2 \times 8 \times 8}\right)$	M1	Correct use of cosine rule
	$C \hat{B} Y = 1.445^\circ$ AG	A1	Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. Need 82.8° awrt converted to radians for A1. Identification of angles must be consistent for A1.
		3	
8(ii)	Arc $CY = 8 \times 1.445$	B1	Use of $s = r\theta$ for arc CY , Expect 11.56
	$B \hat{A} C = \frac{1}{2}(\pi - A \hat{B} C)$ or $\cos^{-1}\left(\frac{3}{4}\right)$	*M1	For a valid attempt at $B \hat{A} C$, may be from (i). Expect 0.7227°
	Arc $XC = 12 \times (\text{their } B \hat{A} C)$	DM1	Expect 8.673
	Perimeter = $11.56 + 8.673 + 4 = 24.2$ cm awrt www	A1	Omission of '+4' only penalised here.
		4	

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Question	Answer	Marks	Guidance
9(i)	$2x^2 - 12x + 7 = 2(x - 3)^2 - 11$	B1 B1	Mark full expression if present: B1 for $2(x - 3)^2$ and B1 for $- 11$. If no clear expression award $a = - 3$ and $b = - 11$.
		2	
9(ii)	Range (of f or y) $\geq$ ‘their – 11’	B1FT	FT for their ‘b’ or start again. Condone $>$. Do NOT accept $x >$ or $\geq$
		1	
9(iii)	$(k =)$ – “their a” also allow x or $k \leq 3$	B1FT	FT for their “a” or start again using $\frac{dy}{dx} = 0$. Do NOT accept $x = 3$.
		1	
9(iv)	$y = 2(x - 3)^2 - 11 \rightarrow y + 11 = 2(x - 3)^2$ $\frac{y + 11}{2} = (x - 3)^2$	*M1	Isolating their $(x - 3)^2$, condone $- 11$.
	$x = 3 + \sqrt{\left(\frac{y + 11}{2}\right)}$ or $3 - \sqrt{\left(\frac{y + 11}{2}\right)}$	DM1	Other operations in correct order, allow $\pm$ at this stage. Condone $- 3$.
	$(g^{-1}(x) \text{ or } y) = 3 - \sqrt{\left(\frac{x + 11}{2}\right)}$	A1	needs ‘–’. x and y could be interchanged at the start.
		3	

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Question	Answer	Marks	Guidance
10(i)	$2x + \frac{12}{x} = k - x$ or $y = 2(k - y) + \frac{12}{k - y} \rightarrow 3$ term quadratic.	*M1	Attempt to eliminate y (or x) to form a 3 term quadratic. Expect $3x^2 - kx + 12$ or $3y^2 - 5ky + (2k^2 + 12) (= 0)$
	Use of $b^2 - 4ac \rightarrow k^2 - 144 < 0$	DM1	Using the discriminant, allow $\leq$, $= 0$; expect 12 and -12
	$-12 < k < 12$	A1	Do NOT accept $\leq$. Separate statements OK.
		3	
10(ii)	Using $k = 15$ in their 3 term quadratic	M1	From (i) or restart. Expect $3x^2 - 15x + 12$ or $3y^2 - 75y + 462 (= 0)$
	$x = 1, 4$ or $y = 11, 14$	A1	Either pair of x or y values correct..
	(1, 14) and (4, 11)	A1	Both pairs of coordinates
		3	
10(iii)	Gradient of $AB = -1 \rightarrow$ Perpendicular gradient $= +1$	B1FT	Use of $m_1m_2 = -1$ to give $+1$ or ft from their A and B .
	Finding their midpoint using their (1, 14) and (4, 11)	M1	Expect $(2\frac{1}{2}, 12\frac{1}{2})$
	Equation: $y - 12\frac{1}{2} = (x - 2\frac{1}{2})$ [$y = x + 10$]	A1	Accept correct unsimplified and isw
		3	

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Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{3}{2} \times (4x+1)^{-\frac{1}{2}} \right] [\times 4] [-2] \left(\frac{6}{\sqrt{4x+1}} - 2 \right)$	B2,1,0	Looking for 3 components
	$\int y dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2} \right] [\div 4] \left[-\frac{2x^2}{2} \right] (+C)$ $\left(= \frac{(4x+1)^{\frac{3}{2}}}{2} - x^2 \right)$	B1 B1 B1	B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for ' $\div 4$ '. B1 for ' $-\frac{2x^2}{2}$ '. Ignore omission of + C. If included isw any attempt at evaluating.
		5	
11(ii)	At M, $\frac{dy}{dx} = 0 \rightarrow \frac{6}{\sqrt{4x+1}} = 2$	M1	Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as $x = k$)
	$x = 2, y = 5$	A1 A1	
		3	

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Question	Answer	Marks	Guidance
11(iii)	Area under the curve = $\left[\frac{1}{2}(4x+1)^{\frac{3}{2}} - x^2 \right]_0^2$	M1	Uses their integral and their '2' and 0 correctly
	$(13.5 - 4) - 0.5$ or $9.5 - 0.5 = 9$	A1	No working implies use of integration function on calculator M0A0.
	Area under the chord = trapezium = $\frac{1}{2} \times 2 \times (3 + 5) = 8$ Or $\left[\frac{x^2}{2} + 3x \right]_0^2 = 8$	M1	Either using the area of a trapezium with their 2, 3 and 5 or $\int (their\ x + 3) dx$ using their '2' and 0 correctly.
	(Shaded area = $9 - 8$) = 1	A1	Dependent on both method marks,
	OR Area between the chord and the curve is:		
	$\int_0^2 3\sqrt{4x+1} - 2x - (x+3) dx$ $= \int_0^2 3\sqrt{4x+1} - 3x - 3 dx$	M1	Subtracts their line from given curve and uses their '2' and 0 correctly.
	$= 3 \left[\frac{1}{6}(4x+1)^{\frac{3}{2}} - \frac{x^2}{2} - x \right]_0^2$	A1	All integration correct and limits 2 and 0.
	$= 3 \left\{ \left(\frac{27}{6} - 2 - 2 \right) - \left(\frac{1}{6} \right) \right\}$	M1	Evidence of substituting their '2' and 0 into their integral.
	$= 3 \left\{ \frac{1}{2} - \frac{1}{6} \right\} = 3 \left\{ \frac{1}{3} \right\} = 1$	A1	No working implies use of a calculator M0A0.
		[4]	

MATHEMATICS

9709/13

Paper 1

October/November 2018

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	$7C5 x^2 (-2/x)^5$ soi	B1	Can appear in an expansion. Allow 7C2
	21×-32 soi	B1	Identified. Allow $(21x^2) \times (-32x^{-5})$. Implied by correct answer
	-672	B1	Allow $\frac{-672}{x^3}$. If 0/3 scored, 672 scores SCB1
		3	

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Question	Answer	Marks	Guidance
2	$f'(x) = 3x^2 + 4x - 4$	B1	
	Factors or crit. values or sub any 2 values ($x \neq -2$) into $f'(x)$ soi	M1	Expect $(x+2)(3x-2)$ or $-2, \frac{2}{3}$ or any 2 subs (excluding $x = -2$).
	For $-2 < x < \frac{2}{3}$, $f'(x) < 0$; for $x > \frac{2}{3}$, $f'(x) > 0$ soi Allow $\leq, \geq$	M1	Or at least 1 specific value ($\neq -2$) in each interval giving opp signs Or $f'(\frac{2}{3}) = 0$ and $f'(\frac{2}{3}) \neq 0$ (i.e. gradient changes sign at $x = \frac{2}{3}$)
	Neither www	A1	Must have 'Neither'
	ALT 1 At least 3 values of $f(x)$	M1	e.g. $f(0) = 7$, $f(1) = 6$, $f(2) = 15$
	At least 3 <u>correct</u> values of $f(x)$	A1	
	At least 3 <u>correct</u> values of $f(x)$ spanning $x = \frac{2}{3}$	A1	
	Shows a decreasing and then increasing pattern. Neither www	A1	Or similar wording. Must have 'Neither'
	ALT 2 $f'(x) = 3x^2 + 4x - 4 = 3(x + \frac{2}{3})^2 - \frac{16}{3}$	B1B1	Do not condone sign errors
	$f'(x) \geq -\frac{16}{3}$	M1	
	$f'(x) < 0$ for some values and > 0 for other values. Neither www	A1	Or similar wording. Must have 'Neither'
		4	

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Question	Answer	Marks	Guidance
3(i)	0.8 oe	B1	
		1	
3(ii)	$BD = 5 \sin \text{their } 0.8$	M1	Expect 3.58(7). Methods using degrees are acceptable
	$DC = 5 - 5 \cos \text{their } 0.8$	M1	Expect 1.51(6)
	Sector = $\frac{1}{2} \times 5^2 \times \text{their } 0.8$ OR Seg = $\frac{1}{2} \times 5^2 \times [\text{their } 0.8 - \sin \text{their } 0.8]$	M1	Expect 10 for sector. Expect 1.03(3) for segment
	Trap = $\frac{1}{2}(5 + \text{their } DC) \times \text{their } BD$ oe OR $\Delta BDC = \frac{1}{2} \text{their } BD \times \text{their } CD$	M1	OR (for last 2 marks) if X is on AB and XC is parallel to BD :
	Shaded area = $11.69 - 10$ OR $2.71(9) - 1.03(3) = 1.69$ cao	A1	$BDCX - (\text{sector} - \Delta AXC) = 5.43(8) - [10 - 6.24(9)] = 1.69$ cao M1A1
		5	

Question	Answer	Marks	Guidance
4(i)	Gradient, m , of $AB = 3/4$	B1	
	Equation of BC is $y - 4 = \frac{-4}{3}(x - 3)$	M1A1	Line through (3, 4) with gradient $\frac{-1}{m}$ (M1). (Expect $y = \frac{-4}{3}x + 8$)
	$x = 6$	A1	Ignore any y coordinate given.
		4	

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Question	Answer	Marks	Guidance
4(ii)	$(AC)^2 = 7^2 + 1^2 \rightarrow AC = 7.071$	M1A1	M mark for $\sqrt{(their\ 6 + / - 1)^2 + 1}$.
		2	

Question	Answer	Marks	Guidance
5	$a + (n - 1)3 = 94$	B1	
	$\frac{n}{2}[2a + (n - 1)3] = 1420$ OR $\frac{n}{2}[a + 94] = 1420$	B1	
	Attempt elimination of a or n	M1	
	$3n^2 - 191n + 2840 (= 0)$ OR $a^2 - 3a - 598 (= 0)$	A1	3-term quadratic (not necessarily all on the same side)
	$n = 40$ (only)	A1	
	$a = -23$ (only)	A1	Award 5/6 if a 2nd pair of solutions (71/3, 26) is given in addition or if given as the only answer.
		6	

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Question	Answer	Marks	Guidance
6	$(\mathbf{BO}) = -8\mathbf{i} - 6\mathbf{j}$	B1	OR $(\mathbf{OB}) = 8\mathbf{i} + 6\mathbf{j}$
	$(\mathbf{BF}) = -6\mathbf{j} - 8\mathbf{i} + 7\mathbf{k} + 4\mathbf{i} + 2\mathbf{j} = -4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$	B1	OR $(\mathbf{FB}) = 4\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$
	$(\mathbf{BF} \cdot \mathbf{BO}) = (-4)(-8) + (-4)(-6)$	M1	OR $(\mathbf{FB} \cdot \mathbf{OB})$ Expect 56. Accept one reversed but award final A0
	$ \mathbf{BF} \times \mathbf{BO} = \sqrt{4^2 + 4^2 + 7^2} \times \sqrt{8^2 + 6^2}$	M1	Expect 90. At least one magnitude <u>methodically</u> correct
	Angle $OBF = \cos^{-1}\left(\frac{\text{their } 56}{\text{their } 90}\right) = \cos^{-1}\left(\frac{56}{90}\right)$ or $\cos^{-1}\left(\frac{28}{45}\right)$	DM1A1	Or equivalent ‘integer’ fractions. All M marks dependent on use of $(\pm)\mathbf{BO}$ and $(\pm)\mathbf{BF}$. 3rd M mark dep on both preceding M marks
		6	

Question	Answer	Marks	Guidance
7(i)	$\frac{(\tan \theta + 1)(1 - \cos \theta) + (\tan \theta - 1)(1 + \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$ soi	M1	
	$\frac{\tan \theta - \tan \theta \cos \theta + 1 - \cos \theta + \tan \theta - 1 + \tan \theta \cos \theta - \cos \theta}{1 - \cos^2 \theta}$ www	A1	
	$\frac{2(\tan \theta - \cos \theta)}{\sin^2 \theta}$ www AG	A1	
		3	

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Question	Answer	Marks	Guidance
7(ii)	$(2)(\tan \theta - \cos \theta) (=0) \rightarrow (2)\left(\frac{\sin \theta}{\cos \theta} - \cos \theta\right) (=0)$ soi	M1	Equate numerator to zero and replace $\tan \theta$ by $\sin \theta / \cos \theta$
	$(2)(\sin \theta - (1 - \sin^2 \theta)) (=0)$	DM1	Multiply by $\cos \theta$ and replace $\cos^2 \theta$ by $1 - \sin^2 \theta$
	$\sin \theta = 0.618(0)$ soi	A1	Allow $(\sqrt{5}-1)/2$
	$\theta = 38.2^\circ$	A1	Apply penalty -1 for extra solutions in range
		4	

Question	Answer	Marks	Guidance
8(i)	$y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x (+c)$	B1	
	$11 = 0 + 0 + 0 + c$	M1	Sub $x = 0, y = 11$ into an integrated expression. c must be present
	$y = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 - 4x + 11$	A1	
		3	
8(ii)	$4a + 2b - 4 = 0$	M1	Sub $x = 2, dy/dx = 0$
	$\frac{1}{3}(8a) + 2b - 8 + 11 = 3$	M1	Sub $x = 2, y = 3$ into an integrated expression. Allow if 11 missing
	Solve simultaneous equations	DM1	Dep. on both M marks
	$a = 3, b = -4$	A1A1	Allow if no working seen for simultaneous equations
		5	

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Question	Answer	Marks	Guidance
9(i)	For <i>their</i> 3-term quad a recognisable application of $b^2 - 4ac$	M1	Expect $2x^2 - x(3+k) + 1 - k^2 (=0)$ oe for the 3-term quad.
	$(b^2 - 4ac =) (3+k)^2 - 4(2)(1-k^2)$ oe	A1	Must be correct. Ignore any RHS
	$9k^2 + 6k + 1$	A1	Ignore any RHS
	$(3k+1)^2 \geq 0$ Do not allow > 0 . Hence curve and line meet. AG	A1	Allow $(9)\left(k + \frac{1}{3}\right)^2 \geq 0$. Conclusion required.
	ALT Attempt solution of 3-term quadratic	M1	
	Solutions $x = k+1, \frac{1}{2}(1-k)$	A1A1	
	Which exist for all values of k . Hence curve and line meet. AG	A1	
		4	

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Question	Answer	Marks	Guidance
9(ii)	$k = -1/3$	B1	ALT $dy/dx = 4x - 3 \Rightarrow 4x - 3 = k$
	Sub (one of) <i>their</i> $k = -1/3$ into either line 1 $\rightarrow 2x^2 - \frac{8}{3}x + \frac{8}{9} (=0)$ Or into the derivative of line 1 $\rightarrow 4x - (3 + k)(=0)$	M1	Sub $k = 4x - 3$ into line 1 $\rightarrow 2x^2 - x(4x) + 1 - (4x - 3)^2 (=0)$
	$x = 2/3$ Do not allow unsubstantiated $\left(\frac{2}{3}, -\frac{1}{9}\right)$ following $k = -\frac{1}{3}$	A1	$x = 2/3, y = -1/9$ (both required) [from $-18x^2 + 24x - 8 (=0)$ oe]
	$y = -1/9$ Do not allow unsubstantiated $\left(\frac{2}{3}, -\frac{1}{9}\right)$ following $k = -\frac{1}{3}$	A1	$k = -1/3$
		4	

Question	Answer	Marks	Guidance
10(i)	$V = 4(\pi) \int (3x-1)^{-2/3} dx = 4(\pi) \left[\frac{(3x-1)^{1/3}}{1/3} \right] [\div 3]$	M1A1A1	Recognisable integration of y^2 (M1) Independent A1, A1 for [] []
	$4(\pi)[2-1]$	DM1	Expect $4(\pi)(3x-1)^{1/3}$
	4π or 12.6	A1	Apply limits $2/3 \rightarrow 3$. Some working must be shown.
		5	

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Question	Answer	Marks	Guidance
10(ii)	$\frac{dy}{dx} = (-2/3)(3x-1)^{-4/3} \times 3$	B1	Expect $-2(3x-1)^{-4/3}$
	When $x = 2/3$, $y = 2$ so $\frac{dy}{dx} = -2$	B1B1	2nd B1 dep. on correct expression for $\frac{dy}{dx}$
	Equation of normal is $y - 2 = \frac{1}{2}(x - \frac{2}{3})$	M1	Line through $(\frac{2}{3}, \text{their } 2)$ and with grad $-1/m$. Dep on m from diffn
	$y = \frac{1}{2}x + \frac{5}{3}$	A1	
		5	

Question	Answer	Marks	Guidance
11(i)	$[2] [(x-3)^2] [-7]$	B1B1B1	
		3	
11(ii)	Largest value of k is 3. Allow $(k =) 3$.	B1	Allow $k \leq 3$ but not $x \leq 3$ as final answer.
		1	

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Question	Answer	Marks	Guidance
11(iii)	$y = 2(x-3)^2 - 7 \rightarrow (x-3)^2 = \frac{1}{2}(y+7)$ or with x/y transposed	M1	Ft <i>their</i> a, b, c . Order of operations correct. Allow sign errors
	$x = 3 \pm \sqrt{\frac{1}{2}(y+7)}$ Allow $3 + \sqrt{\quad}$ or $3 - \sqrt{\quad}$ or with x/y transposed	DM1	Ft <i>their</i> a, b, c . Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{\frac{1}{2}(x+7)}$	A1	
	(Domain is $x \geq \text{their} - 7$)	B1FT	Allow other forms for interval but if variable appears must be x
		4	
11(iv)	$x + 3 \leq 1$. Allow $x + 3 = 1$	M1	Allow $x + 3 \leq k$
	largest p is -2 . Allow $(p =) -2$	A1	Allow $p \leq -2$ but not $x \leq -2$ as final answer.
	$fg(x) = f(x+3) = 2x^2 - 7$ cao	B1	
		3	