

CANDIDATE
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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

October/November 2017

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



- 1** A curve has equation $y = 2x^{\frac{3}{2}} - 3x - 4x^{\frac{1}{2}} + 4$. Find the equation of the tangent to the curve at the point $(4, 0)$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** A function f is defined by $f : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a . [4]

[illegible]

- 3 (a)** A geometric progression has first term $3a$ and common ratio r . A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n . [3]

[illegible]

- 4 Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm³, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

(i) Show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$. [1]

[illegible]

(ii) Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]

[illegible]

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(iii) Find the maximum volume of a cone that can be made by these machines. [1]

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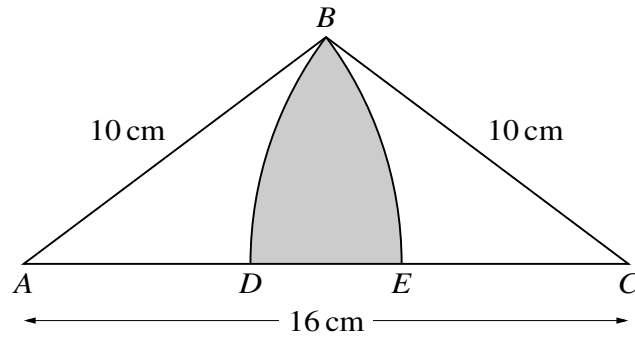
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The diagram shows an isosceles triangle ABC in which $AC = 16$ cm and $AB = BC = 10$ cm. The circular arcs BE and BD have centres at A and C respectively, where D and E lie on AC .

- (i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

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- (ii) Find the area of the shaded region. [5]

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- 6** The points $A(1, 1)$ and $B(5, 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

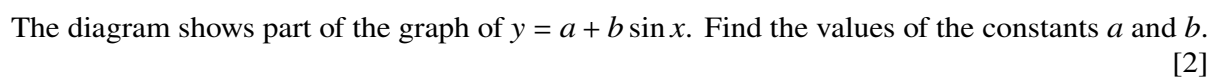
(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$.

[4]

[illegible]

(ii) Find, by calculation, the distance CD , giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) (i) Show that the equation

$$(\sin \theta + 2 \cos \theta)(1 + \sin \theta - \cos \theta) = \sin \theta(1 + \cos \theta)$$

may be expressed as $3 \cos^2 \theta - 2 \cos \theta - 1 = 0$.

[3]

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- (ii) Hence solve the equation

$$(\sin \theta + 2 \cos \theta)(1 + \sin \theta - \cos \theta) = \sin \theta(1 + \cos \theta)$$

for $-180^\circ \leq \theta \leq 180^\circ$.

[4]

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- 8** (a) Relative to an origin O , the position vectors of two points P and Q are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R is such that PQR is a straight line with Q the mid-point of PR . Find the position vector of R in terms of $\mathbf{p}$ and $\mathbf{q}$, simplifying your answer. [3]

[illegible]

- (b) The vector $6\mathbf{i} + a\mathbf{j} + b\mathbf{k}$ has magnitude 21 and is perpendicular to $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Find the possible values of a and b , showing all necessary working. [6]

[illegible]

- 9 Functions f and g are defined for $x > 3$ by

$$f : x \mapsto \frac{1}{x^2 - 9},$$

$$g : x \mapsto 2x - 3.$$

- (i) Find and simplify an expression for $gg(x)$. [2]

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- (ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

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(iii) Solve the equation $fg(x) = \frac{1}{7}$.

[4]

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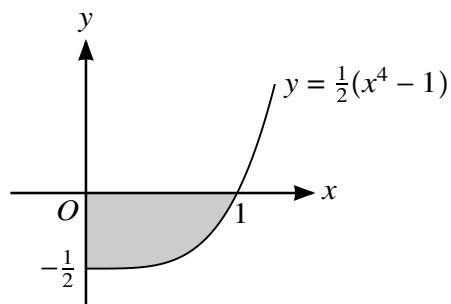
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The diagram shows part of the curve $y = \frac{1}{2}(x^4 - 1)$, defined for $x \geq 0$.

- (i) Find, showing all necessary working, the area of the shaded region. [3]

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- (ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x -axis. [4]

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- (iii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the y-axis. [5]

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1 (P1)

October/November 2017

1 hour 45 minutes

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- 1** Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

[illegible]

2 A function f is defined by $f : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

- (i) Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

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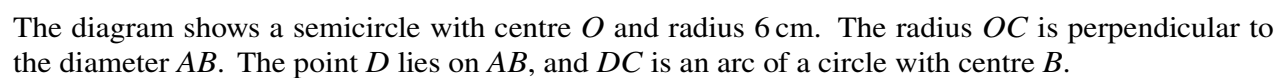
- (ii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

- 3 (a)** Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

[illegible]

- (b)** The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n + 7)$. Find the 1st term and the common difference of the progression. [4]

[illegible]



- [illegible]

5 (i) Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2 \cos^2 2x + 3 \cos 2x + 1 = 0. \quad [3]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[Turn over

- 6** (a) The function f , defined by $f : x \mapsto a + b \sin x$ for $x \in \mathbb{R}$, is such that $f(\frac{1}{6}\pi) = 4$ and $f(\frac{1}{2}\pi) = 3$.

(i) Find the values of the constants a and b . [3]

[illegible]

(ii) Evaluate $\text{ff}(0)$. [2]

[illegible]

- (b)** The function g is defined by $g : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leq g(x) \leq 10$. Find the values of the constants c and d . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 7** Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates $(4, 7)$ and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

(i) In the case where L passes through the mid-point of AB , find the value of m . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[4]

[illegible]

8 A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x -coordinate of each of the stationary points of the curve. [2]

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(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

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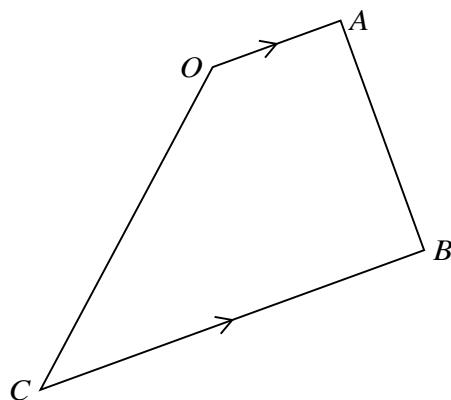
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The diagram shows a trapezium $OABC$ in which OA is parallel to CB . The position vectors of A and B relative to the origin O are given by $\vec{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$.

- (i) Show that angle OAB is 90° . [3]

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The magnitude of $\vec{CB}$ is three times the magnitude of $\vec{OA}$.

- (ii) Find the position vector of C . [3]

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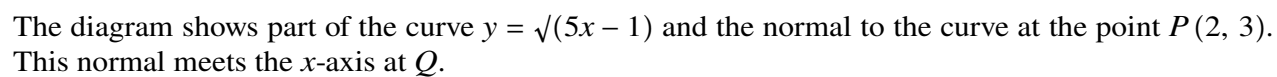
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- (iii) Find the exact area of the trapezium $OABC$, giving your answer in the form $a\sqrt{b}$, where a and b are integers. [3]



- [4]

[illegible]

[illegible]

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MATHEMATICS

9709/13

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- 1** An arithmetic progression has first term -12 and common difference 6 . The sum of the first n terms exceeds 3000 . Calculate the least possible value of n . [4]

[illegible]

- 2** Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

[illegible]

- 3 (i) Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

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- (ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6. \quad [3]$$

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- 4 The function f is such that $f(x) = (2x - 1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

[illegible]

5 (i) Show that the equation $\frac{\cos \theta + 4}{\sin \theta + 1} + 5 \sin \theta - 5 = 0$ may be expressed as $5 \cos^2 \theta - \cos \theta - 4 = 0$. [3]

[illegible]

(ii) Hence solve the equation $\frac{\cos \theta + 4}{\sin \theta + 1} + 5 \sin \theta - 5 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

[illegible]

6 The functions f and g are defined by

$$\begin{aligned} f(x) &= \frac{2}{x^2 - 1} \text{ for } x < -1, \\ g(x) &= x^2 + 1 \text{ for } x > 0. \end{aligned}$$

(i) Find an expression for $f^{-1}(x)$.

[3]

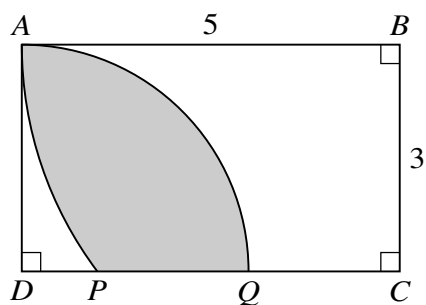
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(ii) Solve the equation $gf(x) = 5$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

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The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B . Point Q lies on DC and AQ is an arc of a circle with centre D .

- (i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

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- (ii) Calculate the areas of the sectors BAP and DAQ . [3]

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(iii) Calculate the area of the shaded region.

[3]

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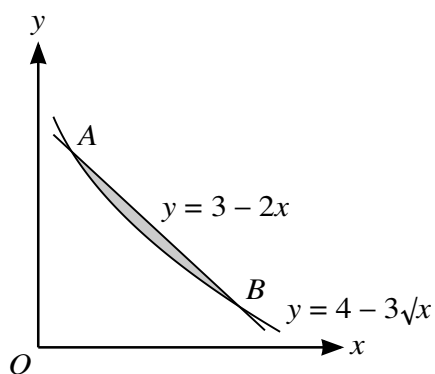
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The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B .

- (i) Find by calculation the x -coordinates of A and B . [3]

[illegible]

- (ii)** Find, showing all necessary working, the area of the shaded region.

[5]

[illegible]

$$\overrightarrow{OA} = \begin{pmatrix} 8 \\ -6 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -10 \\ 3 \\ -13 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}.$$

(i) Find the magnitudes $|\vec{AB}|$, $|\vec{BC}|$ and $|\vec{CD}|$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Given that D is a point lying on the line through B and C , find the two possible position vectors of the point D . [4]

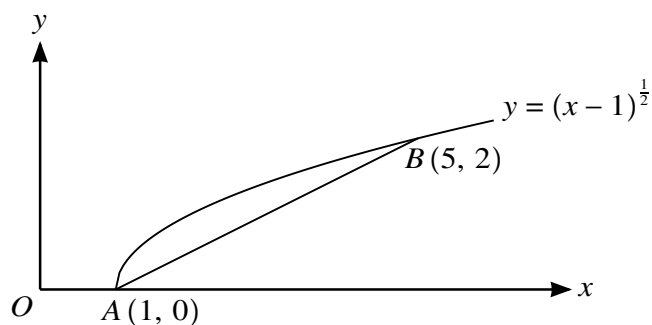
[illegible]

- 10** A curve has equation $y = f(x)$ and it is given that $f'(x) = ax^2 + bx$, where a and b are positive constants.
- (i) Find, in terms of a and b , the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (ii)** It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $f(x)$. [6]

[illegible]



The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A(1, 0)$ and $B(5, 2)$ lying on the curve.

- (i) Find the equation of the line AB , giving your answer in the form $y = mx + c$. [2]

- (ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB . [5]

[illegible]

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- (iii) Find the perpendicular distance between the line AB and the tangent parallel to AB . Give your answer correct to 2 decimal places. [3]

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