
MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

October/November 2016

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

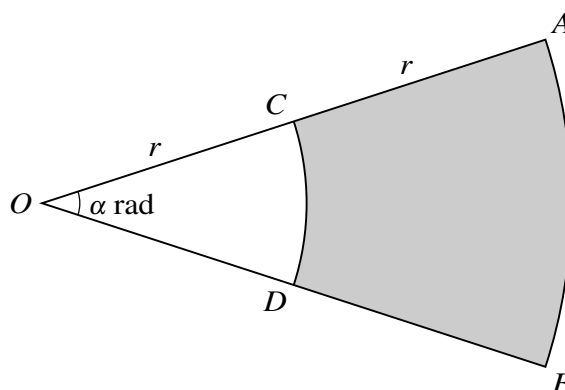
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

- 1 (i) Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]
- (ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

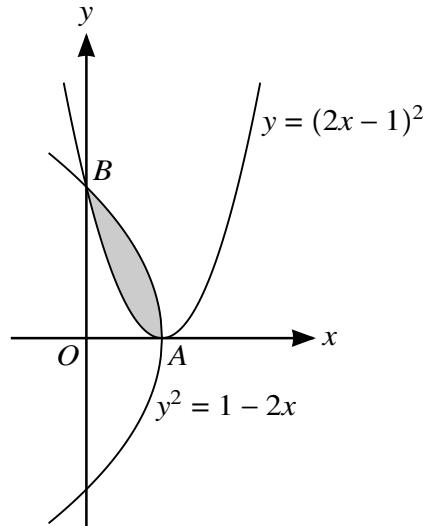
- 2 Find the term independent of x in the expansion of $\left(2x + \frac{1}{2x^3}\right)^8$. [4]

3



In the diagram OCA and ODB are radii of a circle with centre O and radius $2r$ cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

- (i) Find the value of α . [2]
- (ii) It is given that the area of the shaded region is 30 cm^2 . Find the value of r . [3]
- 4 C is the mid-point of the line joining $A(14, -7)$ to $B(-6, 3)$. The line through C perpendicular to AB crosses the y -axis at D .
- (i) Find the equation of the line CD , giving your answer in the form $y = mx + c$. [4]
- (ii) Find the distance AD . [2]
- 5 The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]
- 6 (i) Show that $\cos^4 x \equiv 1 - 2 \sin^2 x + \sin^4 x$. [1]
- (ii) Hence, or otherwise, solve the equation $8 \sin^4 x + \cos^4 x = 2 \cos^2 x$ for $0^\circ \leq x \leq 360^\circ$. [5]



The diagram shows parts of the curves $y = (2x - 1)^2$ and $y^2 = 1 - 2x$, intersecting at points A and B .

(i) State the coordinates of A . [1]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

8 The functions f and g are defined by

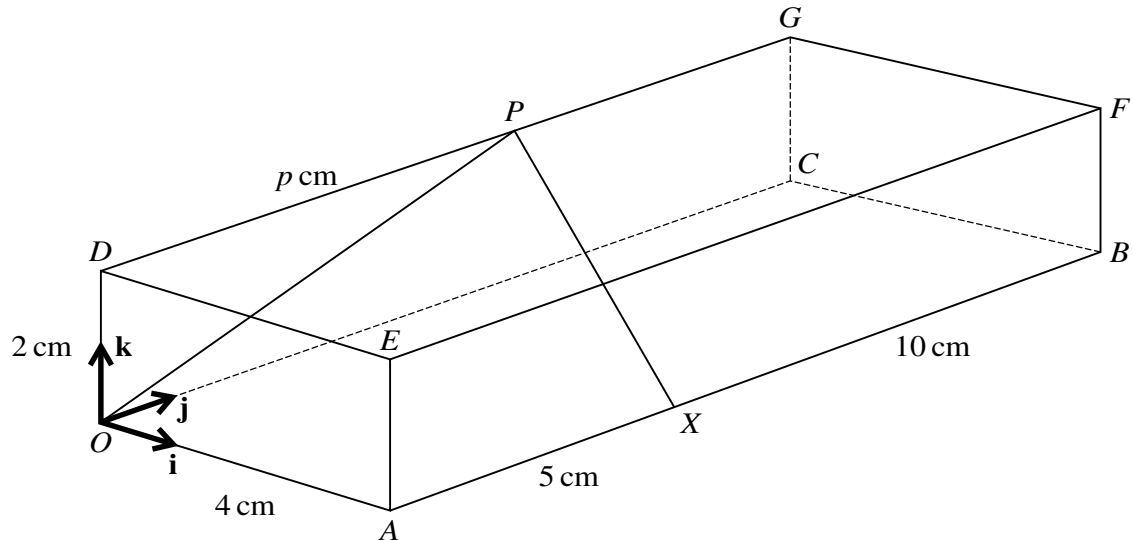
$$f(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

$$g(x) = \frac{4}{5x + 2} \quad \text{for } x \geq 0.$$

(i) Find and simplify an expression for $fg(x)$ and state the range of fg . [3]

(ii) Find an expression for $g^{-1}(x)$ and find the domain of g^{-1} . [5]

[Questions 9, 10 and 11 are printed on the next page.]



The diagram shows a cuboid $OABCDEFG$ with a horizontal base $OABC$ in which $OA = 4$ cm and $AB = 15$ cm. The height OD of the cuboid is 2 cm. The point X on AB is such that $AX = 5$ cm and the point P on DG is such that $DP = p$ cm, where p is a constant. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA , OC and OD respectively.

- (i) Find the possible values of p such that angle $OPX = 90^\circ$. [4]
 - (ii) For the case where $p = 9$, find the unit vector in the direction of $\overrightarrow{XP}$. [2]
 - (iii) A point Q lies on the face $CBFG$ and is such that XQ is parallel to AG . Find $\overrightarrow{XQ}$. [3]
- 10 A curve has equation $y = f(x)$ and it is given that $f'(x) = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$. The point A is the only point on the curve at which the gradient is -1 .
- (i) Find the x -coordinate of A . [3]
 - (ii) Given that the curve also passes through the point $(4, 10)$, find the y -coordinate of A , giving your answer as a fraction. [6]
- 11 The point $P(3, 5)$ lies on the curve $y = \frac{1}{x-1} - \frac{9}{x-5}$.
- (i) Find the x -coordinate of the point where the normal to the curve at P intersects the x -axis. [5]
 - (ii) Find the x -coordinate of each of the stationary points on the curve and determine the nature of each stationary point, justifying your answers. [6]

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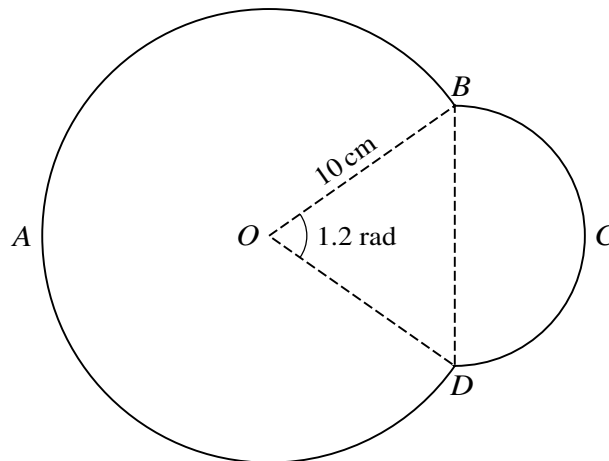
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- 1 A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{4x+1}}$. The point $(2, 5)$ lies on the curve. Find the equation of the curve. [4]
- 2 (i) Express the equation $\sin 2x + 3 \cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]
- (ii) Hence solve the equation for $-90^\circ \leq x \leq 90^\circ$. [3]
- 3 A curve has equation $y = 2x^2 - 6x + 5$.
- (i) Find the set of values of x for which $y > 13$. [3]
- (ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]
- 4 In the expansion of $(3 - 2x) \left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2 . [6]
- 5 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x - and y -axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b . [6]

6



The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

- (i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]
- (ii) Find the perimeter of the metal plate. [3]
- (iii) Find the area of the metal plate. [3]

7 The equation of a curve is $y = 2 + \frac{3}{2x-1}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x -coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y -coordinate as the point passes through P . [2]

8 (a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

9 Relative to an origin O , the position vectors of the points A , B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB . [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

10 A function f is defined by $f : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq \pi$.

(i) Find the range of f . [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Solve the equation $f(x) = 6$, giving answers in terms of π . [3]

The function g is defined by $g : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k , find an expression for $g^{-1}(x)$. [3]

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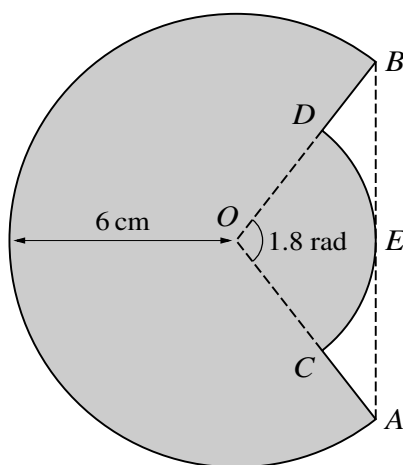
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- 1 Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]
- 2 The coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^5$ is 100. Find the value of the constant a . [4]
- 3 Showing all necessary working, solve the equation $6 \sin^2 x - 5 \cos^2 x = 2 \sin^2 x + \cos^2 x$ for $0^\circ \leq x \leq 360^\circ$. [4]
- 4 The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n . [4]

5



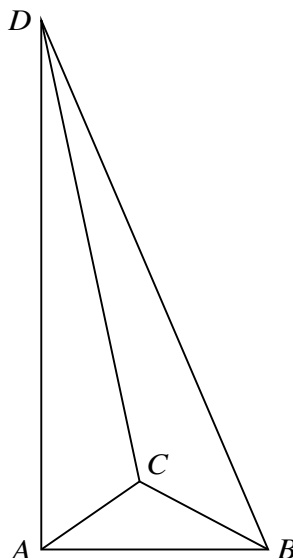
The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O . Angle $COD = 1.8$ radians.

- (i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]
- (ii) Find the area of the shaded region. [4]
- 6 Three points, A , B and C , are such that B is the mid-point of AC . The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

- (i) Find the coordinates of C in terms of m and n . [2]

The line $y = x + 1$ passes through C and is perpendicular to AB .

- (ii) Find the values of m and n . [5]



The diagram shows a triangular pyramid $ABCD$. It is given that

$$\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{AD} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(i) Verify, showing all necessary working, that each of the angles DAB , DAC and CAB is 90° . [3]

(ii) Find the exact value of the area of the triangle ABC , and hence find the exact value of the volume of the pyramid. [4]

[The volume V of a pyramid of base area A and vertical height h is given by $V = \frac{1}{3}Ah$.]

8 (i) Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a , b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $f(x) = x^2 + 1$ and $fg(x) = 4x^2 + 12x + 10$. Find $g(x)$. [1]

(iii) Find $(fg)^{-1}(x)$ and give the domain of $(fg)^{-1}$. [4]

9 (a) Two convergent geometric progressions, P and Q , have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos \theta$ and the second term is $\cos \theta + \sin^2 \theta$, where $0 \leq \theta \leq \pi$. The sum of the first 13 terms is 52. Find the possible values of θ . [5]

[Questions 10 and 11 are printed on the next page.]

- 10 A curve is such that $\frac{dy}{dx} = \frac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A(a^2, 3)$ lies on the curve. Find, in terms of a ,

(i) the equation of the tangent to the curve at A , simplifying your answer, [3]

(ii) the equation of the curve. [4]

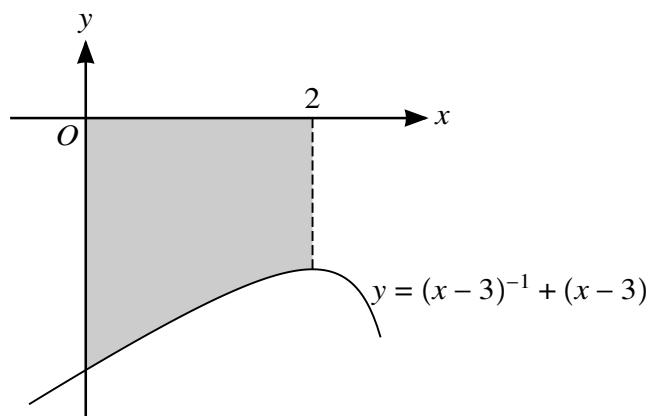
It is now given that $B(16, 8)$ also lies on the curve.

(iii) Find the value of a and, using this value, find the distance AB . [5]

- 11 A curve has equation $y = (kx - 3)^{-1} + (kx - 3)$, where k is a non-zero constant.

(i) Find the x -coordinates of the stationary points in terms of k , and determine the nature of each stationary point, justifying your answers. [7]

(ii)



The diagram shows part of the curve for the case when $k = 1$. Showing all necessary working, find the volume obtained when the region between the curve, the x -axis, the y -axis and the line $x = 2$, shown shaded in the diagram, is rotated through 360° about the x -axis. [5]

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