
MATHEMATICS

9709/11

Paper 1

October/November 2016

MARK SCHEME

Maximum Mark: 75

Published

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Page 2	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	11

Mark Scheme Notes

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	11

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AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\frac{1}{2}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	11

1	(i)	$(x+3)^2 - 7$	B1B1	[2]	For $a = 3$, $b = -7$
	(ii)	1, -7 seen $x > 1$, $x < -7$ oe	B1 B1	[2]	$x > 1$ or $x < -7$ Allow $x \leq -7, x \geq 1$ oe
2		$8C6(2x)^6 \left(\frac{1}{2x^3}\right)^2$ soi $28 \times 64 \times \frac{1}{4}$ oe (powers and factorials evaluated) 448	B1 B2,1,0 B1	[4]	May be seen within a number of terms May be seen within a number of terms Identified as answer
3	(i)	$2r\alpha + r\alpha + 2r = 4.4r$ $\alpha = 0.8$	M1 A1	[2]	At least 3 of the 4 terms required
	(ii)	$\frac{1}{2}(2r)^2 \cdot 0.8 - \frac{1}{2}(r^2) \cdot 0.8 = 30$ $(3/2)r^2 \times 0.8 = 30 \rightarrow r = 5$	M1A1 A1	[3]	Ft through on <i>their</i> α
4	(i)	$C = (4, -2)$ $m_{AB} = -1/2 \rightarrow m_{CD} = 2$ Equation of CD is $y + 2 = 2(x - 4)$ oe $y = 2x - 10$	B1 M1 M1 A1	[4]	Use of $m_1 m_2 = -1$ on their m_{AB} Use of <i>their</i> C and m_{CD} in a line equation
	(ii)	$AD^2 = (14 - 0)^2 + (-7 - (-10))^2$ $AD = 14.3$ or $\sqrt{205}$	M1 A1	[2]	Use <i>their</i> D in a correct method
5		$a(1+r) = 50$ or $\frac{a(1-r^2)}{1-r} = 50$ $ar(1+r) = 30$ or $\frac{a(1-r^3)}{1-r} = 30 + a$ Eliminating a or r $r = 3/5$ $a = 125/4$ oe $S = 625/8$ oe	B1 B1 M1 A1 A1 A1	[6]	Or otherwise attempt to solve for r Any correct method Ft through on <i>their</i> r and a ($-1 < r < 1$)

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6	(i)	$\cos^4 x = (1 - \sin^2 x)^2 = 1 - 2\sin^2 x + \sin^4 x$ AG	B1	[1]	Could be LHS to RHS or vice versa
	(ii)	$8\sin^4 x + 1 - 2\sin^2 x + \sin^4 x = 2(1 - \sin^2 x)$ $9\sin^4 x = 1$ $x = 35.3^\circ$ (or any correct solution) Any correct second solution from 144.7° , 215.3° , 324.7° The remaining 2 solutions	M1 A1 A1 A1✓ A1	[5]	Substitute for $\cos^4 x$ and $\cos^2 x$ or OR sub for $\sin^4 x \rightarrow 3\cos^2 x = 2$ $\rightarrow \cos x = (\pm)\sqrt{2/3}$ Allow the first 2 A1 marks for radians (0.616, 2.53, 3.76, 5.67)
7	(i)	$A = (1/2, 0)$	B1	[1]	Accept $x = 0$ at $y = 0$
	(ii)	$\int (1-2x)^{1/2} dx = \left[\frac{(1-2x)^{3/2}}{3/2} \right] [\div(-2)]$ $\int (2x-1)^2 dx = \left[\frac{(2x-1)^3}{3} \right] [\div 2]$ $[0 - (-1/3)] - [0 - (-1/6)]$ 1/6	B1B1 B1B1 M1 A1	[6]	May be seen in a single expression May use $\int_a^1 x dy$, may expand $(2x-1)^2$ Correct use of <i>their</i> limits
8	(i)	$fg(x) = 5x$ Range of fg is $y \geq 0$ oe	M1A1 B1	[3]	only Accept $y > 0$
	(ii)	$y = 4 / (5x + 2) \Rightarrow x = (4 - 2y) / 5y$ oe $g^{-1}(x) = (4 - 2x) / 5x$ oe 0, 2 with no incorrect inequality $0 < x \leq 2$ oe, c.a.o.	M1 A1 B1, B1 B1	[5]	Must be a function of x
9	(i)	$\mathbf{XP} = -4\mathbf{i} + (p-5)\mathbf{j} + 2\mathbf{k}$ $[-4\mathbf{i} + (p-5)\mathbf{j} + 2\mathbf{k}] \cdot (p\mathbf{j} + 2\mathbf{k}) = 0$ $p^2 - 5p + 4 = 0$ $p = 1$ or 4	B1 M1 A1 A1	[4]	Or $\mathbf{PX}$ Attempt scalar prod with $\mathbf{OP/PO}$ and set $= 0$ ($= 0$ could be implied)
	(ii)	$\mathbf{XP} = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \rightarrow \mathbf{XP} = \sqrt{16+16+4}$ Unit vector $= 1/6(-4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ oe	M1 A1	[2]	Expect 6
	(iii)	$\mathbf{AG} = -4\mathbf{i} + 15\mathbf{j} + 2\mathbf{k}$ $\mathbf{XQ} = \lambda \mathbf{AG}$ soi $\lambda = 2/3 \rightarrow \mathbf{XQ} = -\frac{8}{3}\mathbf{i} + 10\mathbf{j} + \frac{4}{3}\mathbf{k}$	B1 M1 A1	[3]	

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10 (i)	$3z - \frac{2}{z} = -1 \Rightarrow 3z^2 + z - 2 = 0$ $x^{1/2} \text{ (or } z) = 2/3 \text{ or } -1$ $x = 4/9 \text{ only}$	M1 A1 A1	[3]	Express as 3-term quad. Accept $x^{1/2}$ for z (OR $3x - 1 = -\sqrt{x}$, $9x^2 - 13x + 4 = 0$ M1, A1, A1 $x = 4/9$)
(ii)	$f(x) = \frac{3x^{3/2}}{3/2} - \frac{2x^{1/2}}{1/2} \quad (+c)$ Sub $x=4, y=10$ $10 = 16 - 8 + c \Rightarrow c = 2$ When $x = \frac{4}{9}$, $y = 2\left(\frac{4}{9}\right)^{3/2} - 4\left(\frac{4}{9}\right)^{1/2} + 2$ $-2/27$	B1B1 M1A1 M1 A1	[6]	c must be present Substituting x value from part (i)
11 (i)	$\frac{dy}{dx} = -(x-1)^{-2} + 9(x-5)^{-2}$ $m_{\text{tangent}} = -\frac{1}{4} + \frac{9}{4} = 2$ Equation of normal is $y - 5 = -\frac{1}{2}(x - 3)$ $x = 13$	M1A1 B1 M1 A1	[5]	May be seen in part (ii) Through (3, 5) and with $m = -1 / m_{\text{tangent}}$
(ii)	$(x-5)^2 = 9(x-1)^2$ $x-5 = (\pm)3(x-1) \text{ or } (8)(x^2 - x - 2) = 0$ $x = -1 \text{ or } 2$ $\frac{d^2y}{dx^2} = 2(x-1)^{-3} - 18(x-5)^{-3}$ When $x = -1$, $\frac{d^2y}{dx^2} = -\frac{1}{6} < 0$ MAX When $x = 2$, $\frac{d^2y}{dx^2} = \frac{8}{3} > 0$ MIN	B1 M1 A1 B1 B1 B1	[6]	Set $\frac{dy}{dx} = 0$ and simplify Simplify further and attempt solution If change of sign used, x values close to the roots must be used and all must be correct

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1	$(y) = 8(4x+1)^{\frac{1}{2}} \div \frac{1}{2} \div 4 (+c)$ Uses $x = 2$ and $y = 5$ $c = -7$	B1 B1 M1 A1	[4]	Correct integrand (unsimplified) without $\div 4$. Ignore c . Substitution of correct values into an integrand to find c . $y = 4\sqrt{4x+1} - 7$
2 (i)	$2\sin 2x = 6\cos 2x$ $\tan 2x = k$ $\rightarrow \tan 2x = 3$ or $k = 3$	M1 A1	[2]	Expand and collect as far as $\tan 2x =$ a constant from $\sin \div \cos$ soi cwo
(ii)	$x = (\tan^{-1}(\text{their } k)) \div 2$ $(71.6^\circ \text{ or } -108.4^\circ) \div 2$ $x = 35.8^\circ, -54.2^\circ$ $x = 0.624^\circ, -0.946^\circ$ $x = 0.198\pi^\circ, -0.301\pi^\circ$	M1 A1 A1	[3]	Inverse then $\div 2$. soi. ∇ on 1st answer $\pm 90^\circ$ if in given range but no extra solutions in the given range. Both SR A1A0
3 (i)	$2x^2 - 6x + 5 > 13$ $2x^2 - 6x - 8 (> 0)$ $(x =) -1$ and 4 . $x > 4, x < -1$	M1 A1 A1	[3]	Sets to 0 + attempts to solve Both values required Allow all recognisable notation.
(ii)	$2x^2 - 6x + 5 = 2x + k$ $\rightarrow 2x^2 - 8x + 5 - k (= 0)$ Use of $b^2 - 4ac$ $\rightarrow -3$ OR $\frac{dy}{dx} = 4x - 6$ $4x - 6 = 2$ $x = 2$ $x = 2 \rightarrow y = 1$ Using <i>their</i> (2,1) in $y = 2x + k$ or $y = 2x^2 - 6x + 5$ $\rightarrow k = -3$	M1* DM1 A1 M1* DM1 A1	[3]	Equates and sets to 0. Use of discriminant Sets (their $\frac{dy}{dx}$) = 2 Uses <i>their</i> $x = 2$ and <i>their</i> $y = 1$

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4	Term in $x = \frac{nx}{2}$ $(3 - 2x)(1 + \frac{nx}{2} + \dots) \rightarrow 7 = \frac{3n}{2} - 2$ $\rightarrow n = 6$ Term in $x^2 = \frac{n(n-1)}{2} \left(\frac{x}{2}\right)^2$ Coefficient of $x^2 = \frac{3n(n-1)}{8} - \frac{2n}{2}$ $= \frac{21}{4}$	B1 M1 A1 B1 M1 A1	[6]	Could be implied by use of a numerical n. (Their 2 terms in x) = 7 May be implied by (their n) $\times$ (their $n-1$) $\div 8$. Considers 2 terms in x^2. aef
5	$A(a, 0)$ and $B(0, b)$ $a^2 + b^2 = 100$ M has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$ M lies on $2x + y = 10$ $\rightarrow a + \frac{b}{2} = 10$ Sub $\rightarrow a^2 + (20 - 2a)^2 = 100$ or $\left(10 - \frac{b}{2}\right)^2 + b^2 = 100$ $\rightarrow a = 6, b = 8.$	B1 M1* B1 $\frac{1}{2}$ M1* DM1 A1	[6]	soi Uses Pythagoras with their A & B. $\frac{1}{2}$ on their A and B. Subs into given line, using their M, to link a and b. Forms quadratic in a or in b. cao

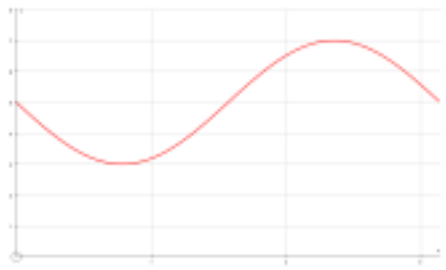
Page 6	Mark Scheme	Syllabus	Paper
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6	(i)	$\frac{r}{10} = \sin 0.6$ or $\frac{r}{10} = \cos 0.97$ or $BD = \sqrt{200 - 200 \cos 1.2} (= 11.3)$ $r = 10 \times 0.5646$, $r = 10 \times \sin 0.6$, $r = 10 \times \cos 0.971$ or $r = \frac{1}{2} BD$ $\rightarrow r = 5.646$	AG	M1 A1	[2]	Or other valid alternative.
	(ii)	Major arc = $10(\theta)$ (= 50.832) $\theta = 2\pi - 1.2$ (= 5.083) or $C = 2\pi \times 10$, Minor arc = 1.2×10 Semicircle = 5.646π (= 17.737) Major arc + semicircle = 68.6		M1 B1 A1	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Implied by 5.1
	(iii)	Area of major sector = $\frac{1}{2}10^2(\theta)$ (= 254.159) Area of triangle OBD = $\frac{1}{2}10^2 \sin 1.2$ (= 46.602) Area = semicircle + sector + triangle (= 50.1 + 254.2 + 46.6) = 351		M1 M1 A1	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Use of $\frac{1}{2}ab \sin C$ or other complete method
7	(i)	$\frac{dy}{dx} = \frac{-3}{(2x-1)^2} \times 2$		B1 B1	[2]	B1 for a single correct term (unsimplified) without $\times 2$.
	(ii)	e.g. Solve for $\frac{dy}{dx} = 0$ is impossible.		B1	[1]	Satisfactory explanation.
	(iii)	If $x = 2$, $\frac{dy}{dx} = \frac{-6}{9}$ and $y = 3$ Perpendicular has $m = \frac{9}{6}$ $\rightarrow y - 3 = \frac{3}{2}(x - 2)$ Shows when $x=0$ then $y=0$	AG	M1* M1* DM1 A1	[4]	Attempt at both needed. Use of $m_1 m_2 = -1$ numerically. Line equation using (2, their 3) and their m .
	(iv)	$\frac{dx}{dt} = -0.06$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -\frac{2}{3} \times -0.06 = 0.04$		M1 A1	[2]	

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8	(a) (i)	$200 + (15 - 1)(+/-5)$ $= 130$	M1 A1	[2]	Use of n th term with $a = 200$, $n = 14$ or 15 and $d = +/- 5$.
	(ii)	$\frac{n}{2}[400 + (n - 1)(+/-5)] = (3050)$ $\rightarrow 5n^2 - 405n + 6100 (= 0)$ $\rightarrow 20$	M1 A1 A1	[3]	Use of S_n $a=200$ and $d = +/- 5$.
	(b) (i)	$ar^2, ar^5 \rightarrow r = \frac{1}{2}$ $\frac{63}{2} = \frac{a(1 - \frac{1}{2}^6)}{\frac{1}{2}} \rightarrow a = 16$	M1 A1 M1 A1	[4]	Both terms correct. Use of $S_n = 31.5$ with a numeric r .
	(ii)	Sum to infinity $= \frac{16}{\frac{1}{2}} = 32$	B1	[1]	✓ for their a and r with $ r < 1$.
9	(i)	$-4 - 6 - 6 = -16$ $\sqrt{x_1^2 + y_1^2 + z_1^2}$ or $\sqrt{x_2^2 + y_2^2 + z_2^2}$ $3 \times 7 \times \cos \theta = -16$ $\rightarrow \theta = 139.6^\circ$ or 2.44° or 0.776π	M1 M1 M1 A1	[4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on their $\overrightarrow{OA}$ & $\overrightarrow{OB}$ Modulus once on either their $\overrightarrow{OA}$ or $\overrightarrow{OB}$ All linked using their $\overrightarrow{OA}$ & $\overrightarrow{OB}$
	(ii)	$\overrightarrow{AC} = c - a = \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix}$ Magnitude $= 10$ Scaling $\rightarrow \frac{15}{10} \times \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \\ 9 \end{pmatrix}$	B1 M1 A1	[3]	For $15 \times$ their unit vector.
	(iii)	$\begin{pmatrix} 2 + 2p \\ 6 - 2p \\ 5 - p \end{pmatrix}$ $\rightarrow -2(2 + 2p) + 3(6 - 2p) + 6(5 - p) = 0$ $\rightarrow p = 2\frac{3}{4}$	B1 M1 A1	[3]	Single vector solved by scalar product. Dot product of $(p \overrightarrow{OA} + \overrightarrow{OC})$ and $\overrightarrow{OB} = 0$.

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10 (i)	$3 \leq f(x) \leq 7$	B1 B1	[2]	Identifying both 3 and 7 or correctly stating one inequality. Completely correct statement. NB $3 \leq x \leq 7$ scores B1B0
(ii)		B1* DB1	[2]	One complete oscillation of a sinusoidal curve between 0 and π . All correct, initially going downwards, all above $f(x)=0$
(iii)	$5 - 2\sin 2x = 6 \rightarrow \sin 2x = -\frac{1}{2}$ $\rightarrow 2x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}$ $\rightarrow x = \frac{7\pi}{12} \text{ or } \frac{11\pi}{12}$ $0.583\pi \text{ or } 0.917\pi$ $\frac{\pi + 0.524}{2} \text{ or } \frac{2\pi - 0.524}{2}$ $1.83^\circ \text{ or } 2.88^\circ$	M1 A1 A1✓	[3]	Make $\sin 2x$ the subject. ✓ for $\frac{3\pi}{2}$ – 1 st answer from $\sin 2x = -\frac{1}{2}$ only, if in given range SR A1A0 for both.
(iv)	$k = \frac{\pi}{4}$	B1	[1]	
(v)	$2\sin 2x = 5 - y \rightarrow \sin 2x = \frac{1}{2}(5 - y)$ $(g^{-1}(x)) = \frac{1}{2} \sin^{-1} \frac{(5 - x)}{2}$	M1 M1 A1	[3]	Makes $\pm \sin 2x$ the subject so i by final answer. Correct order of operations including correctly dealing with “–”. Must be a function of x

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	13

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\frac{1}{2}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	13

1	$kx^2 - 3x = x - k \Rightarrow kx^2 - 4x + k (= 0)$ $(-4)^2 - 4(k)(k)$ soi $k > 2$, $k < -2$ cao Allow $(2, \infty)$ etc. Allow $2 < k < -k$	M1 M1 A1	[3]	Eliminate y and rearrange into 3-term quad $b^2 - 4ac$.
2	$(+/-)20 \times 3^3(x^3)$, $10a^3(x^3)$ soi $-540 + 10a^3 = 100$ oe $a = 4$	B1B1 M1 A1	[4]	Each term can include x^3 Must have 3 terms and include a^3 and 100
3	$4\sin^2 x = 6\cos^2 x \Rightarrow \tan^2 x = \frac{6}{4}$ or $4\sin^2 x = 6(1 - \sin^2 x)$ $[\tan x = (\pm)1.225$ or $\sin x = (\pm)0.7746$ or $\cos x = (\pm)0.6325]$ $x = 50.8$ (Allow 0.886 (rad)) Another angle correct $x = 50.8^\circ, 129.2^\circ, 230.8^\circ, 309.2^\circ$ [0.886, 2.25/6, 4.03, 5.40 (rad)]	M1 A1 A1 A1	[4]	Or $4(1 - \cos^2 x) = 6\cos^2 x$ Or any other angle correct Ft from 1st angle (Allow radians) All 4 angles correct in degrees
4	$f'(x) = 3x^2 - 6x - 9$ soi Attempt to solve $f'(x) = 0$ or $f'(x) > 0$ or $f'(x) \geq 0$ soi $(3)(x-3)(x+1)$ or 3, -1 seen or 3 only seen Least possible value of n is 3. Accept $n = 3$. Accept $n \geq 3$	B1 M1 A1 A1	[4]	With or without equality/inequality signs Must be in terms of n
5 (i)	$\cos 0.9 = OE / 6$ or $= \sin\left(\frac{\pi}{2} - 0.9\right)$ oe $OE = 6 \cos 0.9 = 3.73$ oe AG	M1 A1	[2]	Other methods possible
(ii)	Use of $(2\pi - 1.8)$ or equivalent method Area of large sector $= \frac{1}{2} \times 6^2 \times (2\pi - 1.8)$ oe Area of small sector $\frac{1}{2} \times 3.73^2 \times 1.8$ Total area $= 80.7(0) + 12.5(2) = 93.2$	M1 M1 M1 A1	[4]	Expect 4.48 Or $\pi 6^2 - \frac{1}{2} 6^2 1.8$. Expect 80.70 Expect 12.52 Other methods possible
6 (i)	$\frac{2+x}{2} = n \Rightarrow x = 2n - 2$ $\frac{m+y}{2} = -6 \Rightarrow y = -12 - m$	B1 B1	[2]	No MR for $(\frac{1}{2}(2+n), \frac{1}{2}(m-6))$ Expect $(2n-2, -12-m)$

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	13

	(ii)	Sub <i>their</i> x, y into $y = x + 1 \rightarrow -12 - m = 2n - 2 + 1$ $\frac{m+6}{2-n} = -1$ oe Not nested in an equation Eliminate a variable $m = -9, n = -1$	M1* B1 DM1 A1A1	[5]	Expect $m + 2n = -11$ Expect $m - n = -8$ Note: other methods possible
	7 (i)	AB.AC = $3 - 2 - 1 = 0$ hence perpendicular or 90° AB.AD = $3 + 4 - 7 = 0$ hence perpendicular or 90° AC.AD = $1 - 8 + 7 = 0$ hence perpendicular or 90° AG	B1 B1 B1	[3]	$3 - 2 - 1$ or sum of prods etc must be seen Or single statement: mutually perpendicular or 90° seen at least once .
	(ii)	Area $ABC = (\frac{1}{2})\sqrt{3^2 + 1^2 + 1^2} \times \sqrt{1^2 + (-2)^2 + (-1)^2}$ $= \frac{1}{2}\sqrt{11} \times \sqrt{6}$ Vol. = $\frac{1}{3} \times \text{their } \Delta ABC \times \sqrt{1^2 + 4^2 + (-7)^2}$ $= \frac{1}{6}\sqrt{66} \times \sqrt{66} = 11$	M1 A1 M1 A1	[4]	Expect $\frac{1}{2}\sqrt{66}$ Not 11.0
8	(i)	$(2x+3)^2 + 1$ Cannot score retrospectively in (iii)	B1B1B1	[3]	For $a = 2, b = 3, c = 1$
	(ii)	$g(x) = 2x + 3$ cao	B1	[1]	In (ii),(iii) Allow if from $4\left(x + \frac{3}{2}\right)^2 + 1$
	(iii)	$y = (2x+3)^2 + 1 \Rightarrow 2x+3 = (\pm)\sqrt{y-1}$ or ft from (i) $x = (\pm)\frac{1}{2}\sqrt{y-1} - \frac{3}{2}$ or ft from (i) $(fg)^{-1}(x) = \frac{1}{2}\sqrt{x-1} - \frac{3}{2}$ cao Note alt. method $g^{-1}f^{-1}$ Domain is $(x) > 10$ ALT. method for first 3 marks: Trying to obtain $g^{-1}[f^{-1}(x)]$ $g^{-1} = \frac{1}{2}(x-3), f^{-1} = \sqrt{x-1}$ A1 for $\frac{1}{2}\sqrt{x-1} - \frac{3}{2}$	M1 M1 A1 B1 *M1 DM1 A1	[4]	Or with x/y transposed. Or with x/y transposed Allow sign errors. Must be a function of x . Allow $y = \dots$ Allow $(10, \infty), 10 < x < \infty$ etc. but not with y or f or g involved. Not ≥ 10 Both required

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	13

9	(a)	$\frac{6}{1-r} = \frac{12}{1+r}$ $r = \frac{1}{3}$ $S = 9$	M1 A1 A1	[3]	
	(b)	$\frac{13}{2}[2\cos\theta + 12\sin^2\theta] = 52$ $2\cos\theta + 12(1 - \cos^2\theta) = 8 \rightarrow 6\cos^2\theta - \cos\theta - 2 (= 0)$ $\cos\theta = 2/3 \text{ or } -1/2 \text{ so i}$ $\theta = 0.841, 2.09 \text{ Dep on previous A1}$	M1* DM1 A1 A1A1	[5]	Use of correct formula for sum of AP Use $s^2 = 1 - c^2$ & simplify to 3-term quad Accept $0.268\pi, 2\pi/3$. SRA1 for $48.2^\circ, 120^\circ$ Extra solutions in range -1
	10 (i)	at $x = a^2, \frac{dy}{dx} = \frac{2}{a^2} + \frac{1}{a^2}$ or $2a^{-2} + a^{-2} \left(= \frac{3}{a^2} \text{ or } 3a^{-2} \right)$ $y - 3 = \frac{3}{a^2}(x - a^2)$ or $y = \frac{3}{a^2}x + c \rightarrow 3 = \frac{3}{a^2}a^2 + c$ $y = \frac{3}{a^2}x$ or $3a^{-2}x$ cao	B1 M1 A1	[3]	$\frac{2}{a^2} + \frac{1}{a^2}$ or $2a^{-2} + a^{-2}$ seen anywhere in (i) Through $(a^2, 3)$ & with <i>their</i> grad as $f(a)$
	(ii)	$(y) = \frac{2}{a}x^{1/2} + \frac{ax^{-1/2}}{-1/2} (+c)$ sub $x = a^2, y = 3$ into $\int dy / dx$ $c = 1 \left(y = \frac{4x^{1/2}}{a} - 2ax^{-1/2} + 1 \right)$	B1B1 M1 A1	[4]	c must be present. Expect $3 = 4 - 2 + c$
	(iii)	sub $x = 16, y = 8 \rightarrow 8 = \frac{4}{a} \times 4 - 2a \times \frac{1}{4} + 1$ $a^2 + 14a - 32 (= 0)$ $a = 2$ $A = (4, 3), B = (16, 8) \quad AB^2 = 12^2 + 5^2 \rightarrow AB = 13$	*M1 A1 A1 DM1A1	[5]	Sub into <i>their</i> y Allow -16 in addition

Page 7	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – October/November 2016	9709	13

11	(i)	Attempt diffn. and equate to 0 $\frac{dy}{dx} = -k(kx-3)^{-2} + k = 0$ $(kx-3)^2 = 1$ or $k^3x^2 - 6k^2x + 8k (= 0)$ $x = \frac{2}{k}$ or $\frac{4}{k}$ $\frac{d^2y}{dx^2} = 2k^2(kx-3)^{-3}$ When $x = \frac{2}{k}$, $\frac{d^2y}{dx^2} = (-2k^2) < 0$ MAX All previous When $x = \frac{4}{k}$, $\frac{d^2y}{dx^2} = (2k^2) > 0$ MIN working correct	*M1 DM1 *A1*A1 B1 $\frac{1}{4}$ DB1 DB1	Must contain $(kx-3)^{-2}$ + other term(s) Simplify to a quadratic Legitimately obtained Ft must contain $Ak^2(kx-3)^{-3}$ where $A > 0$ Convincing alt. methods (values either side) must show which values used & cannot use $x = 3/k$	[7]
	(ii)	$V = (\pi) \int [(x-3)^{-1} + (x-3)]^2 dx$ $= (\pi) \int [(x-3)^{-2} + (x-3)^2 + 2] dx$ $= (\pi) \left[-(x-3)^{-1} + \frac{(x-3)^3}{3} (+2x) \right]$ Condone missing 2x $= (\pi) \left[1 - \frac{1}{3} + 4 - \left(\frac{1}{3} - 9 + 0 \right) \right]$ $= 40\pi / 3$ oe or 41.9	*M1 A1 A1 DM1 A1	Attempt to expand y^2 and then integrate Or $\left[-(x-3)^{-1} + \frac{x^3}{3} - 3x^2 + 9x + 2x \right]$ Apply limits $0 \rightarrow 2$ 2 missing $\rightarrow 28\pi / 3$ scores M1A0A1M1A0	[5]