
MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2 (P2)

October/November 2015

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

- 1 Use logarithms to solve the equation

$$5^{x+3} = 7^{x-1},$$

giving the answer correct to 3 significant figures. [4]

- 2 A curve has equation

$$y = \frac{3x+1}{x-5}.$$

Find the coordinates of the points on the curve at which the gradient is -4 . [5]

- 3 (i) Express $8 \sin \theta + 15 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$8 \sin \theta + 15 \cos \theta = 6$$

for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 4 (i) By sketching a suitable pair of graphs, show that the equation

$$\ln x = 4 - \frac{1}{2}x$$

has exactly one real root, α . [2]

- (ii) Verify by calculation that $4.5 < \alpha < 5.0$. [2]

- (iii) Use the iterative formula $x_{n+1} = 8 - 2 \ln x_n$ to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 5 (a) Find $\int (\tan^2 x + \sin 2x) dx$. [3]

- (b) Find the exact value of $\int_0^1 3e^{1-2x} dx$. [4]

- 6 (i) Find the quotient and remainder when

$$x^4 + x^3 + 3x^2 + 12x + 6$$

is divided by $(x^2 - x + 4)$. [4]

- (ii) It is given that, when

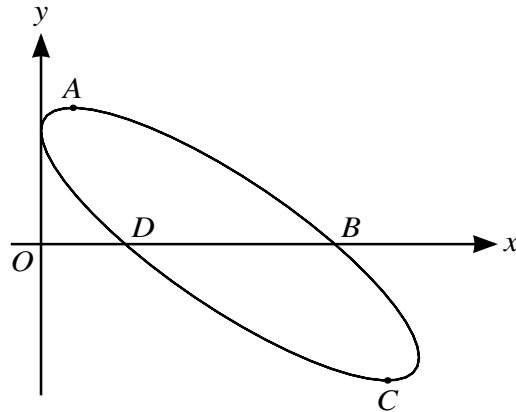
$$x^4 + x^3 + 3x^2 + px + q$$

is divided by $(x^2 - x + 4)$, the remainder is zero. Find the values of the constants p and q . [2]

- (iii) When p and q have these values, show that there is exactly one real value of x satisfying the equation

$$x^4 + x^3 + 3x^2 + px + q = 0$$

and state what that value is. [3]



The parametric equations of a curve are

$$x = 6 \sin^2 t, \quad y = 2 \sin 2t + 3 \cos 2t,$$

for $0 \leq t < \pi$. The curve crosses the x -axis at points B and D and the stationary points are A and C , as shown in the diagram.

(i) Show that $\frac{dy}{dx} = \frac{2}{3} \cot 2t - 1$. [5]

(ii) Find the values of t at A and C , giving each answer correct to 3 decimal places. [3]

(iii) Find the value of the gradient of the curve at B . [3]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2 (P2)

October/November 2015

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

- 1 (i) Solve the equation $|3x - 2| = 5$. [3]
- (ii) Hence, using logarithms, solve the equation $|3 \times 5^y - 2| = 5$, giving the answer correct to 3 significant figures. [2]

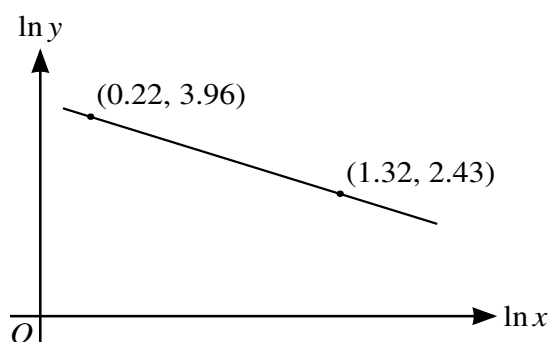
- 2 The sequence of values given by the iterative formula

$$x_{n+1} = 2 + \frac{4}{x_n^2 + 2x_n + 4},$$

with initial value $x_1 = 2$, converges to α .

- (i) Determine the value of α correct to 3 decimal places, giving the result of each iteration to 5 decimal places. [3]
- (ii) State an equation satisfied by α and hence find the exact value of α . [2]

3



The variables x and y satisfy the equation $y = Kx^m$, where K and m are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(0.22, 3.96)$ and $(1.32, 2.43)$, as shown in the diagram. Find the values of K and m correct to 2 significant figures. [6]

- 4 The polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + 11x^2 + ax + a,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $p(x)$.

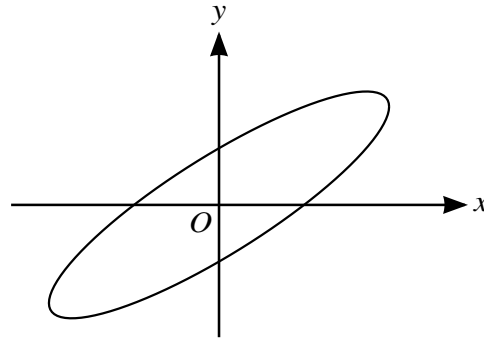
- (i) Use the factor theorem to show that $a = -4$. [2]
- (ii) When $a = -4$,
- (a) factorise $p(x)$ completely, [3]
- (b) solve the equation $6 \sec^3 \theta + 11 \sec^2 \theta + a \sec \theta + a = 0$ for $0^\circ \leq \theta \leq 180^\circ$. [2]

- 5 Find the x -coordinates of the stationary points of the following curves:

(i) $y = 4xe^{-3x}$; [3]

(ii) $y = \frac{4x^2}{x+1}$. [5]

6



The diagram shows the curve with parametric equations

$$x = 3 \cos t, \quad y = 2 \cos\left(t - \frac{1}{6}\pi\right),$$

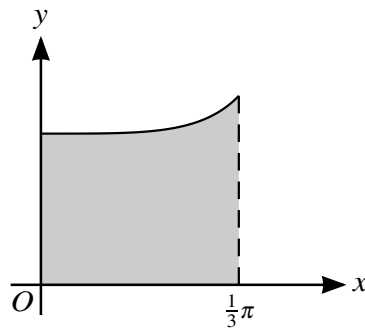
for $0 \leq t < 2\pi$.

(i) Show that $\frac{dy}{dx} = \frac{1}{3}(\sqrt{3} - \cot t)$. [5]

(ii) Find the equation of the tangent to the curve at the point where the curve crosses the positive y-axis. Give the answer in the form $y = mx + c$. [4]

7 (i) Show that the exact value of $\int_0^{\frac{1}{3}\pi} \left(\cos^2 x + \frac{1}{\cos^2 x} \right) dx$ is $\frac{1}{6}\pi + \frac{9}{8}\sqrt{3}$. [6]

(ii)



The diagram shows the curve $y = \cos x + \frac{1}{\cos x}$ for $0 \leq x \leq \frac{1}{3}\pi$. The shaded region is bounded by the curve and the lines $x = 0$, $x = \frac{1}{3}\pi$ and $y = 0$. Find the exact volume of the solid obtained when the shaded region is rotated completely about the x -axis. [4]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

MATHEMATICS

9709/23

Paper 2 Pure Mathematics 2 (P2)

October/November 2015

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **2** printed pages and **2** blank pages.

1 Find the exact value of $\int_{-1}^{35} \frac{3}{2x+5} dx$, giving the answer in the form $\ln k$. [5]

2 (i) Solve the equation $|2x+3| = |x+8|$. [3]

(ii) Hence, using logarithms, solve the equation $|2^{y+1} + 3| = |2^y + 8|$. Give the answer correct to 3 significant figures. [2]

3 The parametric equations of a curve are

$$x = (t+1)e^t, \quad y = 6(t+4)^{\frac{1}{2}}.$$

Find the equation of the tangent to the curve when $t = 0$, giving the answer in the form $ax + by + c = 0$ where a , b and c are integers. [6]

4 (i) Find the quotient when $3x^3 + 5x^2 - 2x - 1$ is divided by $(x-2)$, and show that the remainder is 39. [4]

(ii) Hence show that the equation $3x^3 + 5x^2 - 2x - 40 = 0$ has exactly one real root. [3]

5 It is given that $\int_0^a (3e^{3x} + 5e^x) dx = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{3} \ln(106 - 5e^a)$. [5]

(ii) Use an iterative formula based on the equation in part (i) to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

6 (i) Express $(\sqrt{5}) \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence

(a) solve the equation $(\sqrt{5}) \cos \theta - 2 \sin \theta = 0.9$ for $0^\circ < \theta < 360^\circ$, [4]

(b) state the greatest and least values of

$$10 + (\sqrt{5}) \cos \theta - 2 \sin \theta$$

as θ varies. [2]

7 The equation of a curve is $y = \frac{\sin 2x}{\cos x + 1}$.

(i) Show that $\frac{dy}{dx} = \frac{2(\cos^2 x + \cos x - 1)}{\cos x + 1}$. [7]

(ii) Find the x -coordinate of each stationary point of the curve in the interval $-\pi < x < \pi$. Give each answer correct to 3 significant figures. [3]

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.