

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

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- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

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1	$(a+x)^5 = a^5 + {}^5C_1 a^4 x + {}^5C_2 a^3 x^2 + \dots$ soi $\left(-\frac{2}{a} \times (their\ 5a^4) + (their\ 10a^3)\right)(x^2)$ 0	M1 M1 A1 [3]	Ignore subsequent terms AG
2	$f(x) = x^3 - 7x + c$ $5 = 27 - 21 + c$ $c = -1 \rightarrow f(x) = x^3 - 7x - 1$	B1 M1 A1 [3]	Sub $x = 3, y = 5$. Dep. on c present
3	$4x^2 + x^2 = 1/2$ soi Solve as quadratic in x^2 $x^2 = 1/4$ $x = \pm 1/2$	B1 M1 A1 A1 [4]	E.g. $(4x^2 - 1)(2x^2 + 1)$ or $x^2 =$ formula Ignore other solution
4 (i)	$4 \cos^2 \theta + 15 \sin \theta = 0$ $4(1 - s^2) + 15s = 0 \rightarrow 4 \sin^2 \theta - 15 \sin \theta - 4 = 0$	M1 M1A1 [3]	Replace $\tan \theta$ by $\frac{\sin \theta}{\cos \theta}$ and multiply by $\sin \theta$ or equivalent Use $c^2 = 1 - s^2$ and rearrange to AG (www)
(ii)	$\sin \theta = -1/4$ $\theta = 194.5$ or 345.5	B1 B1B1 ✓ [3]	Ignore other solution Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)
5 (i)	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao $\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	B1B1 B1 [3]	
(ii)	$-\frac{8}{x^2} + 2 = 0 \rightarrow 2x^2 - 8 = 0$ $x = \pm 2$ $y = \pm 8$ $\frac{d^2y}{dx^2} > 0$ when $x = 2$ hence MINIMUM $\frac{d^2y}{dx^2} < 0$ when $x = -2$ hence MAXIMUM	M1 A1 A1 B1 ✓ B1 ✓ [5]	Set = 0 and rearrange to quadratic form If A0A0 scored, SCA1 for just (2, 8) { Ft for "correct" conclusion if $\frac{d^2y}{dx^2}$ incorrect or any valid method inc. a good sketch }

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6	(i)	$x^2 - x + 3 = 3x + a \rightarrow x^2 - 4x + (3 - a) = 0$	B1 [1]	AG
	(ii)	$5 + (3 - a) = 0 \rightarrow a = 8$ $x^2 - 4x - 5 = 0 \rightarrow x = 5$	B1 B1 [2]	Sub $x = -1$ into (i) OR B2 for $x = 5$ www
	(iii)	$16 - 4(3 - a) = 0$ (applying $b^2 - 4ac = 0$) $a = -1$ $(x - 2)^2 = 0 \rightarrow x = 2$ $y = 5$	M1 A1 A1 A1 [4]	OR $dy/dx = 2x - 1 \rightarrow 2x - 1 = 3$ $x = 2$ $y = 2^2 - 2 + 3 \rightarrow y = 5$ $5 = 6 + a \rightarrow a = -1$
7	(i)	$BC^2 = r^2 + r^2 = 2r^2 \rightarrow BC = r\sqrt{2}$	B1 [1]	AG
	(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ so	M1	Expect $\frac{1}{2}\pi r^2$
		Area $\triangle BCD = \frac{1}{2}(2r)r$	M1	Expect r^2 (could be embedded)
		Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$ oe	A1	
		Area semi-circle $CADE = \frac{1}{2}\pi r^2$	B1	
		Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$ or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$ $= r^2$	DM1 A1 [6]	Depends on the area $\triangle BCD$

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8	(i)	$x^2 - 4x = 12$ $x = -2$ or 6 $3^{\text{rd}} \text{ term} = (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	M1 A1 A1A1 [4]	$4x - x^2 = 12$ scores M1A0 SC1 for 16, 48 after $x = 2, -6$
	(ii)	$r^2 = \frac{x^2}{4x} \left(= \frac{x}{4} \right)$ soi $\frac{4x}{1 - \frac{x}{4}} = 8$ $x = \frac{4}{3}$ or $r = \frac{1}{3}$ $3^{\text{rd}} \text{ term} = \frac{16}{27}$ (or 0.593) ALT $\frac{4x}{1-r} = 8 \rightarrow r = 1 - \frac{1}{2}x$ or $\frac{4x}{1-r} = 8 \rightarrow x = 2(1-r)$ $x^2 = 4x \left(1 - \frac{1}{2}x \right)$ $r = \frac{2(1-r)}{4}$ $x = \frac{4}{3}$ $r = \frac{1}{3}$	M1 M1 A1 A1 [4] M1 M1 A1	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
9	(i)	$-(1)(x-3)^2 + 4$	B1B1B1 [3]	
	(ii)	Smallest (m) is 3	B1 [1]	Accept $m \geq 3$, $m = 3$. Not $x \geq 3$. Ft <i>their b</i>
	(iii)	$(x-3)^2 = 4 - y$ Correct order of operations $f^{-1}(x) = 3 + \sqrt{4-x}$ cao Domain is $x \leq 0$	M1 M1 A1 B1 [4]	Or x/y transposed. Ft <i>their a, b, c</i> Accept $y =$ if clear

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10	(i)	$PM = 2\mathbf{i} - 10\mathbf{k} + \frac{1}{2}(6\mathbf{j} + 8\mathbf{k})$ oe $PM = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ $\div \sqrt{4 + 9 + 36}$ Unit vector = $\frac{1}{7}(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$	M1 A1 M1 A1 [4]	Any valid method
	(ii)	$AT = 6\mathbf{j} + 8\mathbf{k}$, $PT = a\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ soi (or TA and TP) $(\cos ATP) = \frac{(6\mathbf{j} + 8\mathbf{k}) \cdot (a\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})}{\sqrt{36 + 64}\sqrt{a^2 + 36 + 4}}$ $= \frac{36 - 16}{\sqrt{36 + 64}\sqrt{a^2 + 36 + 4}}$ $\frac{20}{10\sqrt{a^2 + 40}}$ $\frac{2}{\sqrt{a^2 + 40}} = \frac{2}{7}$ oe and attempt to solve $a = 3$ ALT Alt (Cosine Rule) Vectors (AT, PT etc.) $\cos ATP = \frac{a^2 + 36 + 4 + 36 + 64 - (100 + a^2)}{2\sqrt{(a^2 + 40)}\sqrt{100}}$ then as above	B1 M1 A1 [5] B1 M1A1	Allow 1 vector reversed at this stage. (AM or MT could be used for AT) Ft from their AT and PT Withheld if only 1 vector reversed

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11	(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2} \right] \times [4]$ At $x=6$, $\frac{dy}{dx} = \frac{2}{5}$ Gradient of normal at $P = -\frac{1}{2}$ Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal, or $m_1 m_2 = -1$	B1B1 B1 B1 ^{1/2} B1 [5]	OR eqn of norm $y - 5 = \text{their } -\frac{5}{2}(x - 6)$ When $y = 0$, $x = 8$ hence result
	(ii)	Vol for curve $= (\pi) \int (1+4x)$ and attempt to integrate y^2 $= (\pi) [x + 2x^2] \text{ ignore '+ c'}$ $= (\pi) [6 + 72 - 0]$ $= 78(\pi)$ Vol for line $= \frac{1}{3} \times (\pi) \times 5^2 \times 2$ $= \frac{50}{3}(\pi)$ Total Vol $= 78\pi + 50\pi/3 = 94\frac{2}{3}\pi$ (or $284\pi/3$)	M1 A1 DM1 A1 M1 A1 A1 [7]	Apply limits $0 \rightarrow 6$ (allow reversed if corrected later) OR $(\pi) \left[\frac{\left(-\frac{5}{2}x + 20\right)^3}{3 \times -\frac{5}{2}} \right]_6^8$

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1	$f: x \mapsto 3x + 2, g: x \mapsto 4x - 12$ $f^{-1}(x) = \frac{x-2}{3}$ $gf(x) = 4(3x + 2) - 12$ Equate $\rightarrow x = \frac{2}{7}$	B1 B1 M1 A1 [4]	Equates, collects terms, +soln
2	$(x + 2k)^7$ Term in $x^5 = 21 \times 4k^2 = 84k^2$ Term in $x^4 = 35 \times 8k^3 = 280k^3$ Equate and solve $\rightarrow k = 0.3$ or $\frac{3}{10}$	B1 B1 M1 A1 [4]	Correct method to obtain k .
3 (i)	$\tan 60 = \frac{x}{h} \rightarrow x = h \tan 60$ $A = h \times x$ $V = 40\sqrt{(3h^2)}$	B1 M1 A1 [3]	Any correct unsimplified length Correct method for area ag
(ii)	$\frac{dV}{dh} = 80\sqrt{(3h)}$ If $h = 5, \frac{dh}{dt} = \frac{1}{2\sqrt{(3)}} \text{ or } 0.289$	B1 M1A1 [3]	B1 M1 (must be $\div$, not $\times$).
4 (i)	$\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \left(\frac{1}{s} - \frac{c}{s}\right)^2$ $\frac{(1-c)^2}{s^2} = \frac{(1-c)^2}{1-c^2}$ $= \frac{(1-c)(1-c)}{(1-c)(1+c)} \text{ or } \frac{(1-c)^2}{(1-c)(1+c)}$ $\equiv \frac{1-\cos x}{1+\cos x}$	M1 M1 A1 A1 [4]	Use of $\tan = \sin/\cos$ Use of $s^2 = 1 - c^2$ ag
(ii)	$\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ $\frac{1-\cos x}{1+\cos x} = \frac{2}{5} \rightarrow \cos x = \frac{3}{7}$ $\rightarrow x = 1.13 \text{ or } 5.16$	M1 A1 A1 [3]	Making $\cos x$ the subject $2\pi - 1^{\text{st}}$ answer.

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5	(i)	Length of $OB = \frac{6}{\cos 0.6} = 7.270$	M1 [1]	ag Any valid method
	(ii)	$AB = 6 \tan 0.6$ or 4.1 Arc length $= 7.27 \times (\frac{1}{2}\pi - 0.6) = (7.06)$ Perimeter $= 6 + 7.27 + 7.06 + 6 \tan 0.6 = 24.4$	B1 M1 A1 [3]	Sight of in (ii) Use of $s = r\theta$ with sector angle
	(iii)	Area of $AOB = \frac{1}{2} \times 6 \times 7.27 \times \sin 0.6$ Area of $OBC = \frac{1}{2} \times 7.27^2 \times (\frac{1}{2}\pi - 0.6)$ $\rightarrow$ area $= 12.31 + 25.65 = 38.0$	M1 M1 A1 [3]	Use of any correct area method Use of $\frac{1}{2}r^2\theta$.
6		$A(-3, 7), B(5, 1)$ and $C(-1, k)$		
	(i)	$AB = 10$ $6^2 + (k - 1)^2 = 10^2$ $k = -7$ and 9	B1 M1 A1 [3]	Use of Pythagoras
	(ii)	m of $AB = -\frac{3}{4}$ m perp $= \frac{4}{3}$ $M = (1, 4)$ Eqn $y - 4 = \frac{4}{3}(x - 1)$ Set y to 0, $\rightarrow x = -2$	B1 M1 B1 M1 A1 [5]	B1 M1 Use of $m_1 m_2 = -1$ Complete method leading to D .
7		$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}$		
	(i)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \overrightarrow{AC} = \begin{pmatrix} 3 \\ p - 2 \\ q + 3 \end{pmatrix} \overrightarrow{BC} = \begin{pmatrix} 1 \\ p - 5 \\ q + 2 \end{pmatrix}$ $\rightarrow p = 6\frac{1}{2}$ and $q = -1\frac{1}{2}$	B1B1 B1 B1 [4]	Any 2 of 3 relevant vectors
	(ii)	$6 + 3p - 6 + q + 3 = 0$ $\rightarrow q = -3p - 3$	M1 A1 [2]	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$
	(iii)	$AB^2 = 4 + 9 + 1$ $AC^2 = 9 + 1 + (q + 3)^2$ $\rightarrow (q + 3)^2 = 4$ $\rightarrow q = -1$ or -5	M1 A1 A1 [3]	For attempt at either

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8	(i)	$f: x \rightarrow x^2 + ax + b,$ $x^2 + 6x - 8 = (x + 3)^2 - 17$ or $2x + 6 = 0 \rightarrow x = -3 \rightarrow y = -17$ $\rightarrow \text{Range } f(x) \geq -17$	B1 B1 B1 ✓ [3]	B1 for $(x + 3)^2$. B1 for -17 or B1 for $x = -3$, B1 $y = -17$ Following through visible method.
	(ii)	$(x - k)(x + 2k) = 0$ $\equiv x^2 + 5x + b = 0$ $\rightarrow k = 5$ $\rightarrow b = -2k^2 = -50$	M1 A1 A1 [3]	Realises the link between roots and the equation comparing coefficients of x
	(iii)	$(x + a)^2 + a(x + a) + b = a$ Uses $b^2 - 4ac \rightarrow 9a^2 - 4(2a^2 + b - a)$ $\rightarrow a^2 < 4(b - a)$	M1 DM1 A1 [3]	Replaces “ x ” by “ $x + a$ ” in 2 terms Any use of discriminant
9	(i)	$f''(x) = \frac{12}{x^3}$ $f'(x) = -\frac{6}{x^2} (+c)$ $= 0 \text{ when } x = 2 \rightarrow c = \frac{3}{2}$ $f(x) = \frac{6}{x} + \frac{3x}{2} (+A)$ $= 10 \text{ when } x = 2 \rightarrow A = 4$	B1 M1 A1 B1 ✓ B1 ✓ A1 [6]	Correct integration Uses $x = 2$, $f'(x) = 0$ For each integral
	(ii)	$-\frac{6}{x^2} + \frac{3}{2} = 0 \rightarrow x = \pm 2$ Other point is $(-2, -2)$	M1 A1 [2]	Sets their 2 term $f'(x)$ to 0.
	(iii)	At $x = 2$, $f''(x) = 1.5$ Min At $x = -2$, $f''(x) = -1.5$ Max	B1 B1 [2]	

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10 (i)	$y = \sqrt{9 - 2x^2} \quad P(2, 1)$ $\frac{dy}{dx} = \frac{1}{2\sqrt{9 - 2x^2}} \times -4x$ At P, $x = 2$, $m = -4$ Normal grad = $\frac{1}{4}$ Eqn AP $y - 1 = \frac{1}{4}(x - 2)$ $\rightarrow A(-2, 0)$ or $B(0, \frac{1}{2})$ Midpoint AP also $(0, \frac{1}{2})$	B1 B1 M1 M1 A1 A1 [6]	Without “$\times -4x$” Allow even if B0 above. For $m_1 m_2 = -1$ calculus needed Normal, not tangent Full justification.
(ii)	$\int x^2 dy = \int \left(\frac{9}{2} - \frac{y^2}{2} \right) dy$ $= \frac{9y}{2} - \frac{y^3}{6}$ Upper limit = 3 Uses limits 1 to 3 $\rightarrow$ volume = $4\frac{2}{3} \pi$	M1 A1 B1 DM1 A1 [5]	Attempt to integrate x^2 Correct integration Evaluates upper limit Uses both limits correctly

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	$x^2 - 4x + c = 2x - 7 \rightarrow x^2 - 6x + c + 7 (= 0)$ $36 - 4(c + 7) < 0$ $c > 2$	M1 DM1 A1 [3]	All terms on one side Apply $b^2 - 4ac < 0$. Allow $\leq$.
2	$[7C2] \times \left[\left(\frac{x}{3} \right)^5 \right] \times \left[\left(\frac{9}{x^2} \right)^2 \right]$ soi $21 \times \frac{1}{3^5} (x^5) \times 81 \left(\frac{1}{x^4} \right)$ soi 7	B2,1,0 B1 B1 [4]	Seen Identified as required term Accept $7x$
3 (i)	$[3] [(x-1)^2] [-1]$	B1B1B1 [3]	
(ii)	$f'(x) = 3x^2 - 6x + 7$ $= 3(x-1)^2 + 4$ > 0 hence increasing	B1 B1 ✓ DB1 [3]	Ft <i>their (i)</i> + 5 Dep B1 ✓ unless other valid reason
4 (i)	Sector $OCD = \frac{1}{2}(2r)^2 \theta (= 2r^2 \theta)$ Sector(s) $OAB/OEF = (2) \frac{1}{2} r^2 (\pi - \theta)$ Total $= r^2 (\pi + \theta)$	B1 B1 B1 [3]	$2r^2 \theta$ seen somewhere Accept with/without factor (2) AG www
(ii)	Arc $CD = 2r\theta$ Arc(s) $AB/EF = (2)r(\pi - \theta)$ Straight edges $= 4r$ Total $2\pi r + 4r$ (which is independent of θ)	B1 B1 B1 B1 [4]	Accept with/without factor (2) Must be simplified

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6	(i) (a)	$1.92 + 1.84 + 1.76 + \dots$ oe $\frac{20}{2}[2 \times 1.92 + 19 \times (-0.08)]$ oe 23.2 cao	B1 M1 A1 [3]	OR $a=0.96$, $d=-.04$ & ans doubled/adjusted Corr formula used with corr d & <i>their</i> a, n $a = 1, n = 21 \rightarrow 12.6$ (25.2), $a = 0.96, n = 21 \rightarrow 11.76$ (23.52)
	(b)	$1.92 + 1.92(.96) + 1.92(.96)^2 + \dots$ $\frac{1.92(1 - .96^{20})}{1 - .96}$ 26.8 cao	B1 M1 A1 [3]	OR $a=.96$, $r=.96$ & ans /doubled/adjusted Corr formula used with $r=.96$ & <i>their</i> a, n $a = .96, n = 21 \rightarrow 13.82$ (27.63) $a = 1, n = 21 \rightarrow 14.39$ (28.78)
	(ii)	$\frac{1.92}{1 - .96} = 48$ or $\frac{0.96}{1 - 0.96} = 24$ & then Double AG	M1A1 [2]	$a = 1 \rightarrow 25$ (50) but must be doubled for M1 $1.92 \frac{(1 - 0.96^n)}{1 - 0.96} < 48 \rightarrow 0.96^n > 0$ (www) 'which is true' scores SCB1
7	(a)	$1 + 3\sin^2 \theta + 4\cos \theta = 0$ $1 + 3(1 - \cos^2 \theta) + 4\cos \theta + 0$ $3\cos^2 \theta - 4\cos \theta - 4 = 0$ AG $\cos \theta = -2/3$ $\theta = 131.8$ or 228.2	M1 M1 A1 B1 B1B1 ✓ [6]	Attempt to multiply by $\cos \theta$ Use $c^2 + s^2 = 1$ Ignore other solution Ft for $360 - 1^{\text{st}}$ soln. -1 extra solns in range Radians 2.30 & 3.98 scores SCB1
	(b)	$c = b/a$ cao $d = a - b$	B1 B1 [2]	Allow $D = (0, a - b)$
8	(i)	$3x + 1 \leq -1$ (Accept $3x + 1 = -1, 3a + 1 = -1$) $x \leq -2/3 \Rightarrow$ largest value of a is $-2/3$ (in terms of a)	M1 A1 [2]	Do not allow gf in (i) to score in (iii) Accept $a \leq -2/3$ and $a = -2/3$
	(ii)	$fg(x) = 3(-1 - x^2) + 1$ $fg(x) + 14 = 0 \Rightarrow 3x^2 = 12$ oe (2 terms) $x = -2$ only	B1 B1 B1 [3]	No marks in this part for gf used
	(iii)	$gf(x) = -1 - (3x + 1)^2$ oe $gf(x) \leq -50 \Rightarrow (3x + 1)^2 \geq 49$ (Allow $\leq$ or $=$ $3x + 1 \geq 7$ or $3x + 1 \leq -7$ (one sufficient) www $x \leq -8/3$ only www	B1 M1 A1 A1 [4]	No marks in this part for fg used OR attempt soln of $9x^2 + 6x - 48 + / \leq / \geq 0$ OR $x - 2 \geq$ or $3x + 8 \leq 0$ (one suffic)

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9	(i)	At $x = 4$, $\frac{dy}{dx} = 2$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 2 \times 3 = 6$	B1 M1A1 [3]	Use of Chain rule
	(ii)	$(y) = x + 4x^{\frac{1}{2}} (+c)$ Sub $x = 4$, $y = 6 \rightarrow 6 = 4 + (4 \times 4^{\frac{1}{2}}) + c$ $c = -6 \rightarrow (y = x + 4x^{\frac{1}{2}} - 6$	B1 M1 A1 [3]	Must include c
	(iii)	Eqn of tangent is $y - 6 = 2(x - 4)$ or $(6 - 0)/(4 - x) = 2$ $B = (1, 0)$ (Allow $x = 1$) Gradient of normal = $-1/2$ $C = (16, 0)$ (Allow $x = 16$) Area of triangle = $\frac{1}{2} \times 15 \times 6 = 45$	M1 A1 M1 A1 A1 [5]	Correct eqn thru (4, 6) & with $m =$ their 2 [Expect eqn of normal: $y = -\frac{1}{2}x + 8$] Or $AB = \sqrt{45}$, $AC = \sqrt{180} \rightarrow$ Area = 45.0
10	(i)	$f'(x) = 2 - 2(x + 1)^{-3}$ $f''(x) = 6(x + 1)^{-4}$ $f'0 = 0$ hence stationary at $x = 0$ $f''0 = 6 > 0$ hence minimum	B1 B1 B1 [4]	AG www. Dependent on correct $f''(x)$ except $-6(x + 1)^{-4} \rightarrow < 0$ MAX scores SC1
	(ii)	$AB^2 = (3/2)^2 + (3/4)^2$ $AB = 1.68$ or $\sqrt{45/4}$ oe	M1 A1 [2]	
	(iii)	Area under curve = $\int f(x) = x^2 - (x + 1)^{-1}$ $= \left(1 - \frac{1}{2}\right) - \left(\frac{1}{4} - 2\right) = 9/4$ (Apply limits $- \frac{1}{2} \rightarrow 1$) Area trap. = $\frac{1}{2}(3 + \frac{9}{4}) \times \frac{3}{2}$ $= 63/16$ or 3.94 Shaded area $63/16 - 9/4 + 27/16$ or 1.69 ALT eqn AB is $y = -\frac{1}{2}x + 11/4$ Area = $\int -\frac{1}{2}x + 11/4 - \int 2x + (x + 1)^{-2}$ $= \left[-\frac{1}{4}x^2 + \frac{11}{4}x\right] - \left[x^2 - (x + 1)^{-1}\right]$ Apply limits $- \frac{1}{2} \rightarrow 1$ to both integrals $27/16$ or 1.69	B1 M1A1 M1 A1 A1 [6] B1 M1 A1A1 M1 A1	Ignore $+c$ even if evaluated Do not penalise reversed limits Allow reversed subtn if final ans positive Attempt integration of at least one Ignore $+c$ even if evaluated Dep. on integration having taken place Allow reversed subtn if final ans positive