

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

October/November 2014

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

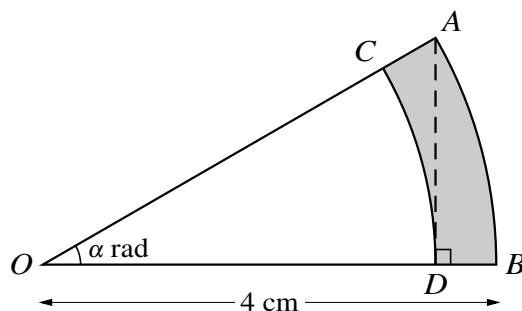
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.

- 1 In the expansion of $(2 + ax)^7$, the coefficient of x is equal to the coefficient of x^2 . Find the value of the non-zero constant a . [3]
- 2 Find the value of x satisfying the equation $\sin^{-1}(x - 1) = \tan^{-1}(3)$. [3]
- 3 Solve the equation $\frac{13 \sin^2 \theta}{2 + \cos \theta} + \cos \theta = 2$ for $0^\circ \leq \theta \leq 180^\circ$. [4]
- 4 The line $4x + ky = 20$ passes through the points $A(8, -4)$ and $B(b, 2b)$, where k and b are constants.
- (i) Find the values of k and b . [4]
- (ii) Find the coordinates of the mid-point of AB . [1]
- 5 Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points. [5]
- 6 Relative to an origin O , the position vector of A is $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and the position vector of B is $7\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.
- (i) Show that angle OAB is a right angle. [4]
- (ii) Find the area of triangle OAB . [3]
- 7 (i) A geometric progression has first term a ($a \neq 0$), common ratio r and sum to infinity S . A second geometric progression has first term a , common ratio $2r$ and sum to infinity $3S$. Find the value of r . [3]
- (ii) An arithmetic progression has first term 7. The n th term is 84 and the $(3n)$ th term is 245. Find the value of n . [4]



In the diagram, AB is an arc of a circle with centre O and radius 4 cm. Angle AOB is α radians. The point D on OB is such that AD is perpendicular to OB . The arc DC , with centre O , meets OA at C .

- (i) Find an expression in terms of α for the perimeter of the shaded region $ABDC$. [4]
- (ii) For the case where $\alpha = \frac{1}{6}\pi$, find the area of the shaded region $ABDC$, giving your answer in the form $k\pi$, where k is a constant to be determined. [4]
- 9 The function f is defined for $x > 0$ and is such that $f'(x) = 2x - \frac{2}{x^2}$. The curve $y = f(x)$ passes through the point $P(2, 6)$.
- (i) Find the equation of the normal to the curve at P . [3]
- (ii) Find the equation of the curve. [4]
- (iii) Find the x -coordinate of the stationary point and state with a reason whether this point is a maximum or a minimum. [4]
- 10 (i) Express $x^2 - 2x - 15$ in the form $(x + a)^2 + b$. [2]

The function f is defined for $p \leq x \leq q$, where p and q are positive constants, by

$$f : x \mapsto x^2 - 2x - 15.$$

The range of f is given by $c \leq f(x) \leq d$, where c and d are constants.

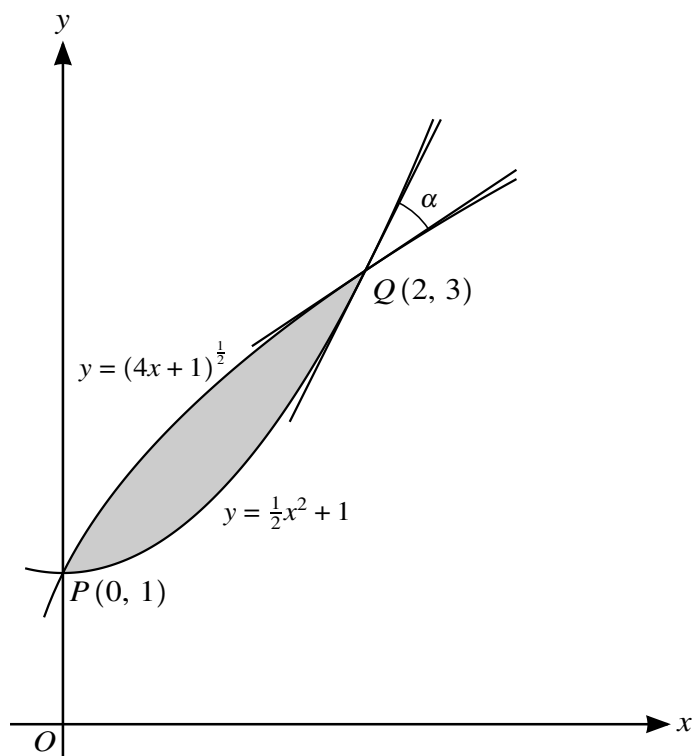
- (ii) State the smallest possible value of c . [1]

For the case where $c = 9$ and $d = 65$,

- (iii) find p and q , [4]
- (iv) find an expression for $f^{-1}(x)$. [3]

[Question 11 is printed on the next page.]

11



The diagram shows parts of the curves $y = (4x + 1)^{\frac{1}{2}}$ and $y = \frac{1}{2}x^2 + 1$ intersecting at points $P(0, 1)$ and $Q(2, 3)$. The angle between the tangents to the two curves at Q is α .

- (i) Find α , giving your answer in degrees correct to 3 significant figures. [6]
- (ii) Find by integration the area of the shaded region. [6]

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MATHEMATICS

9709/12

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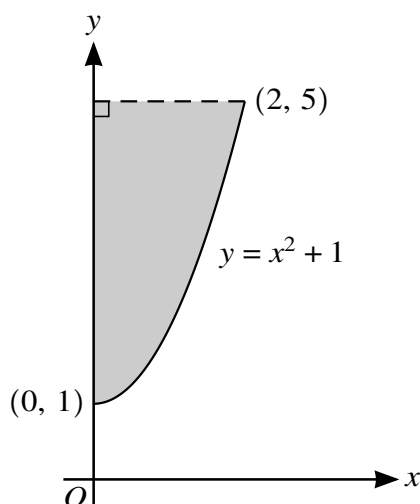
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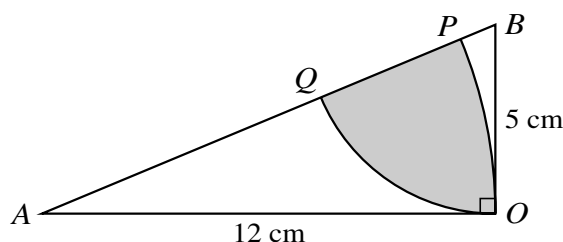
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1



The diagram shows part of the curve $y = x^2 + 1$. Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [4]

2



The diagram shows a triangle AOB in which OA is 12 cm, OB is 5 cm and angle AOB is a right angle. Point P lies on AB and OP is an arc of a circle with centre A . Point Q lies on AB and OQ is an arc of a circle with centre B .

(i) Show that angle BAO is 0.3948 radians, correct to 4 decimal places. [1]

(ii) Calculate the area of the shaded region. [5]

3 (i) Find the first 3 terms, in ascending powers of x , in the expansion of $(1 + x)^5$. [2]

The coefficient of x^2 in the expansion of $(1 + (px + x^2))^5$ is 95.

(ii) Use the answer to part (i) to find the value of the positive constant p . [3]

4 A curve has equation $y = \frac{12}{3 - 2x}$.

(i) Find $\frac{dy}{dx}$. [2]

A point moves along this curve. As the point passes through A , the x -coordinate is increasing at a rate of 0.15 units per second and the y -coordinate is increasing at a rate of 0.4 units per second.

(ii) Find the possible x -coordinates of A . [4]

- 5 (i) Show that the equation $1 + \sin x \tan x = 5 \cos x$ can be expressed as

$$6 \cos^2 x - \cos x - 1 = 0. \quad [3]$$

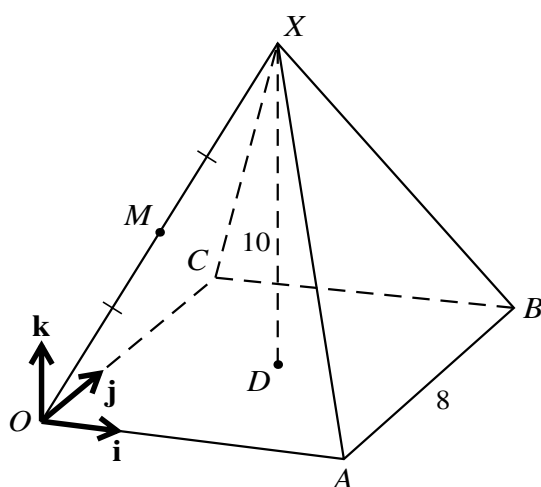
- (ii) Hence solve the equation $1 + \sin x \tan x = 5 \cos x$ for $0^\circ \leq x \leq 180^\circ$. [3]

- 6 The equation of a curve is $y = x^3 + ax^2 + bx$, where a and b are constants.

- (i) In the case where the curve has no stationary point, show that $a^2 < 3b$. [3]

- (ii) In the case where $a = -6$ and $b = 9$, find the set of values of x for which y is a decreasing function of x . [3]

7



The diagram shows a pyramid $OABCX$. The horizontal square base $OABC$ has side 8 units and the centre of the base is D . The top of the pyramid, X , is vertically above D and $XD = 10$ units. The mid-point of OX is M . The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector $\mathbf{k}$ is vertically upwards.

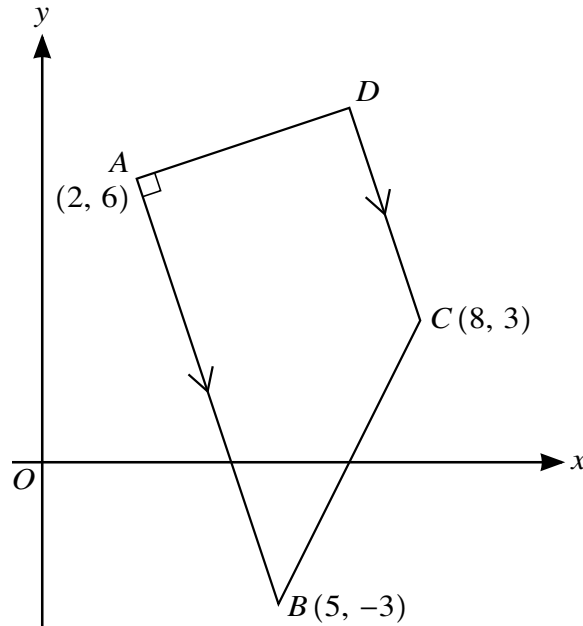
- (i) Express the vectors $\overrightarrow{AM}$ and $\overrightarrow{AC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

- (ii) Use a scalar product to find angle MAC . [4]

- 8 (a) The sum, S_n , of the first n terms of an arithmetic progression is given by $S_n = 32n - n^2$. Find the first term and the common difference. [3]

- (b) A geometric progression in which all the terms are positive has sum to infinity 20. The sum of the first two terms is 12.8. Find the first term of the progression. [5]

[Questions 9, 10 and 11 are printed on the next page.]



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC and angle BAD is 90° . The coordinates of A , B and C are $(2, 6)$, $(5, -3)$ and $(8, 3)$ respectively.

(i) Find the equation of AD . [3]

(ii) Find, by calculation, the coordinates of D . [3]

The point E is such that $ABCE$ is a parallelogram.

(iii) Find the length of BE . [2]

10 A curve is such that $\frac{d^2y}{dx^2} = \frac{24}{x^3} - 4$. The curve has a stationary point at P where $x = 2$.

(i) State, with a reason, the nature of this stationary point. [1]

(ii) Find an expression for $\frac{dy}{dx}$. [4]

(iii) Given that the curve passes through the point $(1, 13)$, find the coordinates of the stationary point P . [4]

11 The function $f : x \mapsto 6 - 4 \cos\left(\frac{1}{2}x\right)$ is defined for $0 \leq x \leq 2\pi$.

(i) Find the exact value of x for which $f(x) = 4$. [3]

(ii) State the range of f . [2]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Find an expression for $f^{-1}(x)$. [3]

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MATHEMATICS

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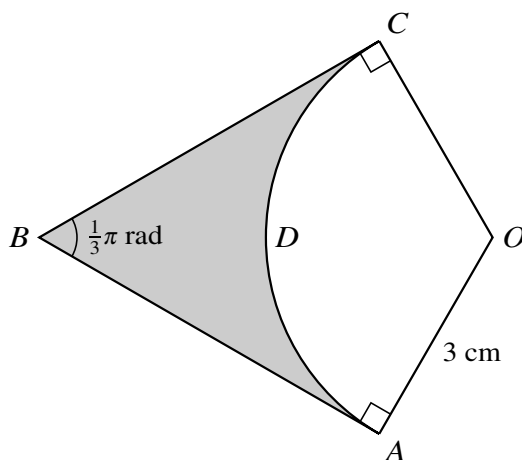
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- 1 In the expansion of $(2 + ax)^6$, the coefficient of x^2 is equal to the coefficient of x^3 . Find the value of the non-zero constant a . [4]

2



In the diagram, $OADC$ is a sector of a circle with centre O and radius 3 cm. AB and CB are tangents to the circle and angle $ABC = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and π ,

- (i) the perimeter of the shaded region, [3]
 (ii) the area of the shaded region. [3]
- 3 (i) Express $9x^2 - 12x + 5$ in the form $(ax + b)^2 + c$. [3]
 (ii) Determine whether $3x^3 - 6x^2 + 5x - 12$ is an increasing function, a decreasing function or neither. [3]

- 4 Three geometric progressions, P , Q and R , are such that their sums to infinity are the first three terms respectively of an arithmetic progression.

Progression P is $2, 1, \frac{1}{2}, \frac{1}{4}, \dots$

Progression Q is $3, 1, \frac{1}{3}, \frac{1}{9}, \dots$

- (i) Find the sum to infinity of progression R . [3]
 (ii) Given that the first term of R is 4, find the sum of the first three terms of R . [3]
- 5 (i) Show that $\sin^4 \theta - \cos^4 \theta \equiv 2 \sin^2 \theta - 1$. [3]
 (ii) Hence solve the equation $\sin^4 \theta - \cos^4 \theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 6 A is the point $(a, 2a - 1)$ and B is the point $(2a + 4, 3a + 9)$, where a is a constant.

- (i) Find, in terms of a , the gradient of a line perpendicular to AB . [3]
 (ii) Given that the distance AB is $\sqrt{260}$, find the possible values of a . [4]

- 7 Three points, O , A and B , are such that $\vec{OA} = \mathbf{i} + 3\mathbf{j} + p\mathbf{k}$ and $\vec{OB} = -7\mathbf{i} + (1-p)\mathbf{j} + p\mathbf{k}$, where p is a constant.

(i) Find the values of p for which $\vec{OA}$ is perpendicular to $\vec{OB}$. [3]

(ii) The magnitudes of $\vec{OA}$ and $\vec{OB}$ are a and b respectively. Find the value of p for which $b^2 = 2a^2$. [2]

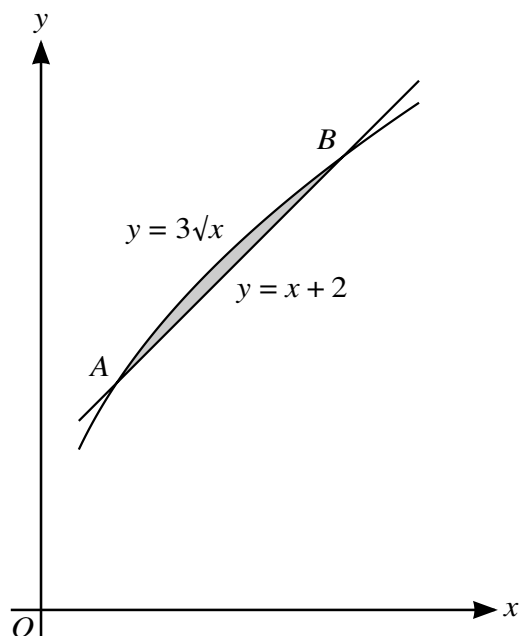
(iii) Find the unit vector in the direction of $\vec{AB}$ when $p = -8$. [3]

- 8 A curve $y = f(x)$ has a stationary point at $(3, 7)$ and is such that $f''(x) = 36x^{-3}$.

(i) State, with a reason, whether this stationary point is a maximum or a minimum. [1]

(ii) Find $f'(x)$ and $f(x)$. [7]

9



The diagram shows parts of the graphs of $y = x + 2$ and $y = 3\sqrt{x}$ intersecting at points A and B .

(i) Write down an equation satisfied by the x -coordinates of A and B . Solve this equation and hence find the coordinates of A and B . [4]

(ii) Find by integration the area of the shaded region. [6]

[Question 10 is printed on the next page.]

- 10 (a)** The functions f and g are defined for $x \geq 0$ by

$$f : x \mapsto (ax + b)^{\frac{1}{3}}, \text{ where } a \text{ and } b \text{ are positive constants,}$$

$$g : x \mapsto x^2.$$

Given that $fg(1) = 2$ and $gf(9) = 16$,

- (i) calculate the values of a and b , [4]
 - (ii) obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]
- (b)** A point P travels along the curve $y = (7x^2 + 1)^{\frac{1}{3}}$ in such a way that the x -coordinate of P at time t minutes is increasing at a constant rate of 8 units per minute. Find the rate of increase of the y -coordinate of P at the instant when P is at the point $(3, 4)$. [5]